MATHS4
Primary teachers

# Adding and subtracting

2026

# MATHS 4 PRIMARY TEACHERS

**Name:** _____

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This **Adding and Subtracting** module aligns to the following PrimTEd standards

| | | |
|---|---|---|
| **General pedagogical standards for mathematic teaching** | GPM1 | **Plan** effective learning experiences |
| | GPM 2 | Take **learner knowledge** into account |
| | GPM 3 | **Engage learners productively** in mathematics |
| | GPM 4 | Teach a **balanced** mathematics curriculum |
| **Mathematical acting and thinking** | MAT 1 | **Playful engagement** to search for and develop mathematical insight |
| | MAT 2 | **Represent and use** mathematics |
| | MAT 3 | **Reason** mathematically |
| | MAT 4 | **Reflect** for action |
| **Number and algebra** | NA 1 | Knowledge and use of **emergent number awareness** of learners |
| | NA 2.1 | Demonstrate a knowledge of key ways of representing or modelling numbers, and moves between these **representations** |
| | NA 2.2 | Show how **counting** is used as a fundamental mediating resource to relate operations on collections to the basic arithmetic operations. |
| | NA 3.1 | Demonstrate an understanding of **part-part-whole** relationships and the link between the operations of addition and subtraction. |
| | NA 3.2 | Describe and use addition and subtraction **problem types** and the range of **representations** and increasingly sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems. |
| | NA 3.2 | Explain and identify **sets**, set membership, set relations, set operations, and set partitions. |
| | NA 7 | **Early algebra** |
| | NA 8 | **Overarching mathematical ideas** for school arithmetic |

## ABOUT THE COVER

The cover refers to an English nursery rhyme called **Five Little Ducks**

"5 little ducks went swimming one day,
Over the hills and far away.
Mother duck said: quack quack quack
But only 4 little ducks came back.

4 little ducks went swimming
one day...

★ It is important that children learn to count backwards and forwards.

# INTRODUCTION TO ADDITION AND SUBTRACTION

This *Maths 4 Primary Teachers* course for Bachelor of Education students is undertaken as design-based research initiative.

2026 Fourth intervention cycle

Date of publication: January 2026



**Suggested citation:**
Roberts, N., Porteus, K., Morrison, S., Jooste, Z. & Westaway, L. (2025) *Adding and Subtracting: Maths 4 Primary teachers.* Kelello Consulting: Johannesburg

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# ACKNOWLEDGEMENTS

An initiative such as this takes a team effort.

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| Rhodes University | Lise Westaway<br>Bruce Brown | Samu Chikiwa<br>Tabisa Booi |
| The University of Johannesburg | Emmanuel Libusha | Jerry Maseko |
| The University of Fort Hare | Nondwe Ngiba<br>Bev Williams<br>Noludwe Bambiso | Karin Hackmack<br>Kim Porteus |
| Tshwane University of Technology | Jeanette Ramollo<br>Victoria Chilengi | Anil Kanjee |
| University of Kwazulu Natal | Zanele Ngcobo | Suryakumari Rajah |
| University of Witwatersrand | Samantha Morrison<br>Corin Mathews | Lynn Bowie<br>Hamsa Venkat |
| University of Zululand | Lynn Kok | Nozipho Luthuli<br>Duduzile Sibaya |
| Water Sisulu University | Faith Hlungulu<br>Nomaroma Kumanda | Nelisiwe Tengwane<br>Nozuko Nqabeni |
| University of Stellenbosch | Nicky Roberts | |
| University of Free State | Msebenzi Rabaza | Lerato Ramp hele |
| University of Limpopo | Manare Setati | |

Extensive use is made of the learner activity books designed by Dr Porteus and colleagues and used in the Magic Classroom Collective by the Nelson Mandela Institute. We are grateful to Maskew Miller Learning for sponsoring the printing, binding and distribution costs for this module.

**Adding and subtracting songs**

- 5 little ducks, is an English song with subtraction.
- What are other songs using counting?
- What are some songs in African languages which use addition and subtraction?

10 green bottles.

SCAN ME

| | Key ideas | Mental fluencies | Representations |
|---|---|---|---|
| **UNIT 1 INTRODUCTION** | | | |
| **1.1 How chidlren learn to add and subtract** | Classroom norms and mathematical powers<br>**Counting and measurement pathways** | Adding and subtracting 1s, 10s, 100s<br>Re-ordering for addition | Number pictures in 1s,10s and 100s<br>Number lines in 1s,10s,100s<br>Number sentences |
| **UNIT 2: CALCULATION STRATEGIES** | | | |
| **2.1 Early calculation strategies** | **Count all**<br>**Count on** | Small number song in 1s.<br>Start at 1 | Number pictures in 1s<br>Number lines in 1s |
| **2.2 Jump: a got-to strategy** | **Break down the second number** into 10s and 1s<br>Break down the second number into 100s, 10s and 1s | Small and medium number songs. Count forwards and backwards in 10s and 1s.<br>Start at any number. | Number pictures structured in 10s and 1s<br>Number lines structured in 10s and 1s<br>Number sentences |
| **2.3 Visit the tens** | **Break down the second number** to visit the 10s. This is also known as bridging through 10. | All the above and<br>Add to visit the next 10.<br><br>Subtract to visit the previous 10.<br>Bonds of 10 and 5<br>All bonds to 10 | Next and previous ten<br>Number pictures: Structured in 1s,10s and 100s<br>Number lines, structured in 100s, 10s and 1s<br>Number sentences. |
| **UNIT 3: BIG IDEAS FOR FLEXIBILITY** | | | |
| **3.1 Equals means "is the same as"** | Equals means 'is the same as', Missing number problems, Number talks | Missing number problems, functions | Equals as balance<br>Number sentences<br>Missing number problems |
| **3.2 Part-part-whole structure** | Family of 8 number sentences, link subtraction to addition, vary the position of the unknown | Re-ordering I | Whole-part-part diagram<br>Family of 8 number sentence |
| **3.3 Additive relations word problems** | 2 models: Static and dynamic. 4 problem types: change increase, change describes, collection and compare | Re-ordering II | Number pictures, number lines, number sentences |
| | | | |

| | Key ideas | Mental fluencies | Representations |
|---|---|---|---|
| **UNIT 4: CALCULATING FLEXIBLY** | | | |
| **4.1 Learning number facts and place value** | 100 addition facts for Foundation Phase<br>Break up using expanded notation<br>Break up flexibly | All mental fluencies | The addition table<br>Flard cards<br>Number pictures<br>Bundling sticks |
| **4.2 Near doubles, round and adjust** | Doubles and near doubles<br>Round off<br>Round and adjust for 9 and 99 | Doubles<br>Near doubles<br>9 as 1 less than 10<br>99 as 1 less than 100 | A t-table for doubling and halving<br><br>Number lines<br>Number sentences |
| **4.3 Break up both numbers (Split)** | Break up both numbers (split):<br>• Write all totals<br>• Condensed column method<br>• Shift down for subtraction | All mental fluencies | Working in columns |


The approaches in this module may be new to you. These approaches are needed to build number sense.

Flexible mental calculation is a 21st century skill.

We first want you to understand and feel confident doing the mathematics yourself.

You might not get a particular strategy at first. You just don't know it, yet.

We encourage you to try the new strategies, and make sense of the ways of working yourself.

Only when you are secure, will you feel able to teach children.

The strategies are what are used for flexible mental calculations, which are needed for high schools.

The ways of working are designed to help children learn algebra later.

This module deals with aspects such as place, value, addition, and subtraction, using approach that encourages the **development of number sense** rather than the mastery of discrete calculation skills. Calculators can add and subtract numbers accurately.

We want children to be able to **add and subtract, with understanding**. We want them to **reason mathematically, notice patterns and explain their thinking.** We want children to understand other people's ways of working. We want them to see the same calculation in different ways, and work **efficiently and flexibly.** These are the 21st Century skills which mathematics teaches.

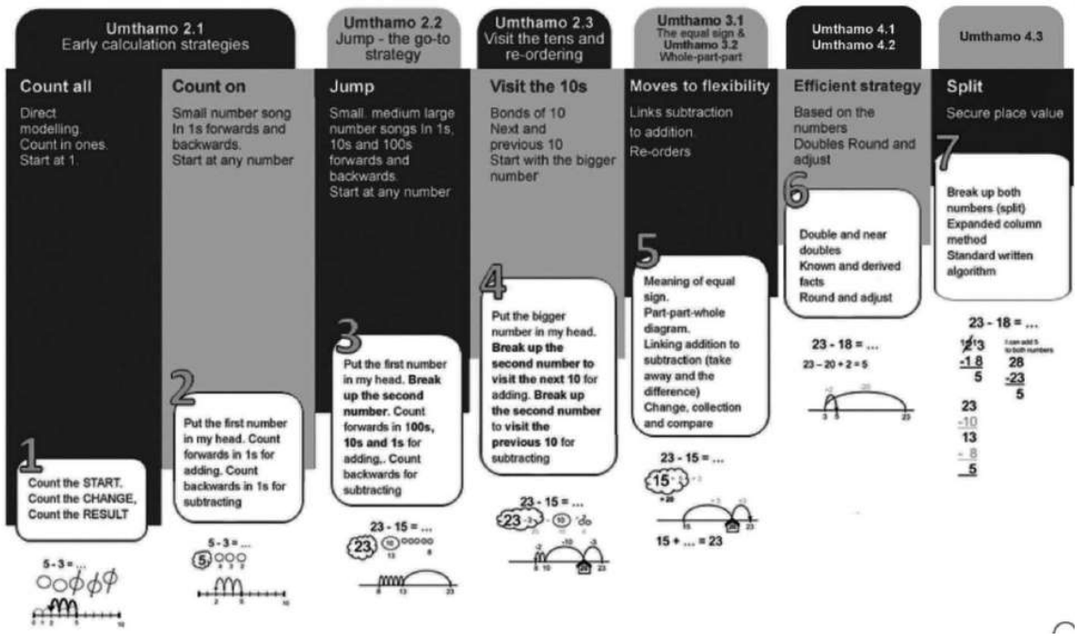
So, in our experience, it is better to help children develop their own reasoning, number sense and flexibility in how they calculate. We think it is more important for children to have a strong number sense, than to be able to follow one set of rules every time.


This Maths4Primary Teachers *Adding and Subtracting module* follows on from *Emergent Number Sense.*

If you did not complete Emergent Number Sense, then there is some revision of key concepts to help you.

# Progression for 7 levels of calculation strategies

**Why so many calculation strategies? What is the point?**




Some learning and teaching support materials think that children should move from level 2 straight to level 7. Teachers start preparing children to add and subtract in columns where they break up both numbers, from very early in Foundation Phase (in about Grade 2).

This can be difficult for children as for Level 7, they need to have a deep understanding of place value, and also know 100 addition facts, to do this well and with understanding. But in Foundation Phase, they are still learning about place value, and they don't yet know all the addition facts.

**We are not just teaching children to add and to subtract.** We are using the ideas of adding and subtracting to teach children to think mathematically. We want our children to have strong **number senses**, to be flexible and able to justify their reasoning. And so we need you – as a teacher – to be confident and secure adding and subtracting mentally and flexibly.

Remember that in primary school teachers are preparing children to succeed in mathematics (including algebra, graphs, functions, geometry etc) in high school. When the primary school children reach high school, they will not be asked to add and subtract large numbers. They will use a calculator or Excel for that. In high schools they will need to:

- Have excellent **number sense** (mental maths and flexibility).
- Understand **whole-part part relationships** and find **equivalent relationships** (families of number sentences).
- Understand **functions and inverse functions** (fizz pop and funky the function machine are important).
- Connect **subtraction to addition** (subtraction is either take away, or find the difference).
  - » If $a + b = c$, then $a - c = b$ and $a - c = b$
- Know that the **commutative property** holds for addition, but not for subtraction.
  - » $a + b = b + a$ but $a + b \neq b + a$
- Know that to add a big amount they can break it up, and then add each part.
  - » $a + (b + c + d) = a + b + c + d$
- Know that to subtract a big amount they can break it up, and then subtract each part.
  - » $a - (b + c + d) = a - b - c - d$.


mistakes in MATH are OK!
MATH helps me become a THINKER.
The PROCESS is more important than the ANSWER!
my MISTAKES can help my PEERS!
MATH can be FUN!
I have a POSITIVE math esteem!
I can be a RISK-TAKER and try new STRATEGIES in math!
I CAN DO MATH!
I CAN DO difficult things - even MATH!

# UNIT 1: LEARNING TO ADD AND SUBTRACT

Adding and Subtracting focuses on **how primary school children** (from Grade R to Grade 7 and beyond) **learn about addition and subtraction**. Children make meaning from the world around them. They come to school (even pre-school/creche and grade R) with many mathematical experiences from their life experience. Over time they are inducted – learning from adults and their peers – into a tradition of more formal mathematics.

Key Ideas

By the end of this unit you should be able to

1. Explain in broad terms how **children learn to add and subtract**
2. Know that for mathematics to make sense, we need **a clear mental map or picture** for numbers and relationships. There are **two main pathways for learning about numbers:**
    a. A **discrete pathway;** and
    b. A **continuous pathway.**
3. Know **how to succeed in this maths course**: what to do each week and how you are assessed.
4. Explain the concept **'additive relation'**.

There are **100 number facts** which children should come to know.
By Grade 2 adding single digits should be fluent. These are for rapid recall.
Complete the 100 addition facts in the ADDITION TABLE.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | | 5 | | 7 | | | 10 | 11 |
| 2 | 3 | 4 | | | | 8 | | | | 12 |
| 3 | 4 | 5 | 6 | 7 | | 9 | | | 12 | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |
| 10 | | | | | | | | | | |

# HOW CHILDREN LEARN TO ADD AND SUBTRACTION

Key Ideas

By the end of this unit you should be able to

1. Explain in broad terms how **children learn to add and subtract**
2. Know **how to succeed in this maths course**: what to do each week and how you are assessed.
3. Know that for mathematics to make sense, we need **a clear mental map or picture** for numbers and relationships. There are **two main pathways for learning about numbers:**
    a. A **discrete pathway;** and
    b. A **continuous pathway.**

## How to succeed in this course

### Weekly rhythm for learning

To be successful in this university course, you will need to work every week. We want you to pass and to pass well. We give you lots of opportunities to

- learn how to teach maths and
- improve your own maths.

If your own maths is shaky and if you hate maths you wont be able to teach it well. Now is your chance to work hard to develop a better relationship with maths, be better at doing maths (make should make sense); and learn to teach primary maths well.

Each week is an umthamo (bite sized chunk) of work. Each umthamo consists of lectures, tutorials, portfolio tasks which you and your peers mark, fluency practice and a quiz. If you work each week – in this rhythm for each week – you will succeed.

You will get feedback on your work through tutorials, marking and reflecting on tasks, and from your quizzes. Use this feedback to improve.

### Mistakes are good

In this course there are many opportunities to practice, check what you know, get feedback and improve.

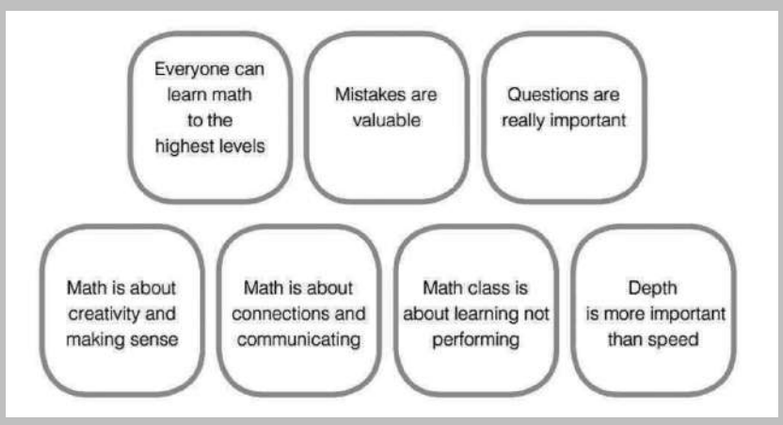

All mathematicians make mistakes. No one gets everything correct at first. Mathematicians learn from their mistakes. If you get everything correct you are not learning (you already knew the work).
To learn something new you must try, practice, get things wrong, and learn from your mistakes.

### Some classroom norms

These are some examples of classroom norms. Your class might agree to some of these, add some more. You may want to elect a class representative. Start a whatsapp group to help each other.

Diagram

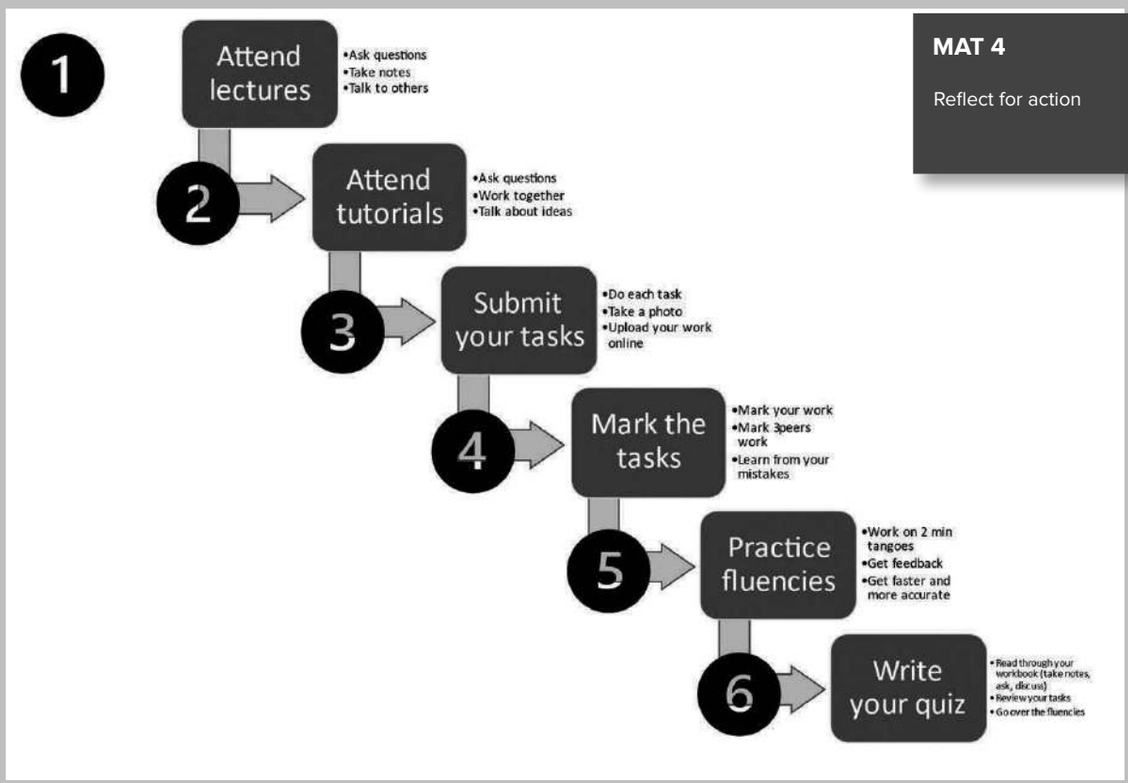
## Positive maths norms




You get feedback when you mark your own work, and when you mark other students work. You also get feedback when you practice 2 min tangoes online (the computer tells you if you are correct or incorrect). You can practice until you are getting most questions correct. You also get feedback when you write a weekly quiz. All of this helps you to learn and become more fluent with mental calculations.

Diagram

## Weekly 6 steps to success

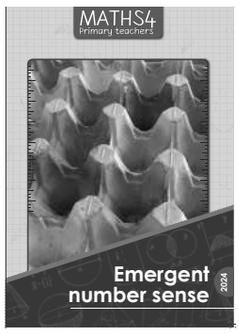



**MAT 4**

Reflect for action

If you follow these 6 steps, for every umthamo each week, you will succeed.

## Revisiting 'emergent number sense'




Some of you may have completed the *Emergent Number Sense* course which is also part of Maths4Primary teachers. That module focuses on how young children learn to mathematics. Here are the course highlights.

We know that many students taking this course have not liked mathematics at school. Many students find maths difficult and stressful. We believe everyone can do mathematics. Even if a person has been taught badly, and hated maths before, with effort and opportunity that person can do maths.

But, what is mathematics? Are we all talking about the same thing when we say "maths"? **Mathematics involves thinking about and working with numbers, shape and space.** Working with numbers is about **quantifying things:** comparing, ordering, sequencing "things". The "things" we quantify can either be:

- **Discrete objects** or **discrete events** that can be separated from each other which can be counted using a counting unit; or
- **Continuous quantities or extents**, that can be measured by focusing on a particular attribute and using standard or non-standard units of measurement.

A big idea in mathematics is finding and describing **patterns, structures or functions**.

The South African curriculum and assessment policy (CAPS) defines mathematics as follows:

> Mathematics is a language that makes use of symbols and notations for describing **numerical, geometric and graphical relationships.** It is a **human activity** that involves observing, representing and investigating **patterns and qualitative relationships** in physical and social phenomena and between mathematical objects themselves. It helps to develop **mental processes** that enhance logical and critical thinking, accuracy and **problem-solving** that will contribute to decision-making. (DBE, 2011).

We know that we can think about numbers as a number of things like apples or stars!

We can also think about numbers like walking along a path.

We can think about a number line.
0 is the starting point of the path. We have taken no steps.

We mark each equal step on the number line.

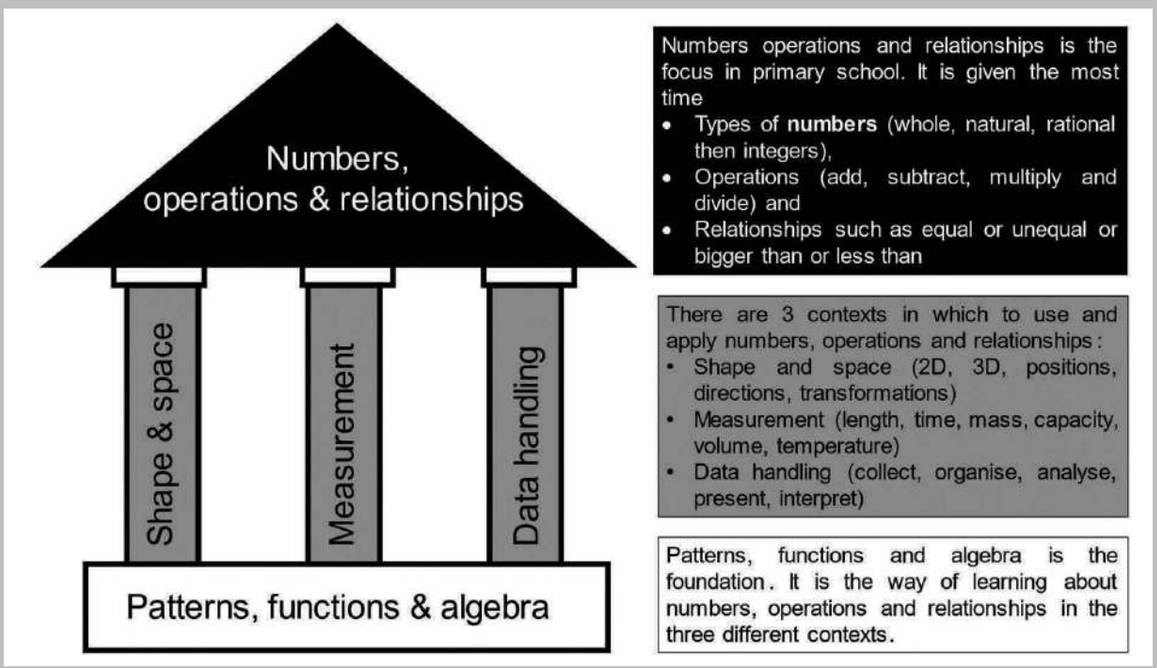
The South African Curriculum and Assessment Policy (CAPS) identifies five content areas for primary school mathematics.

**NA 1**

Knowledge and use of **emergent number** awareness of learners

Diagram

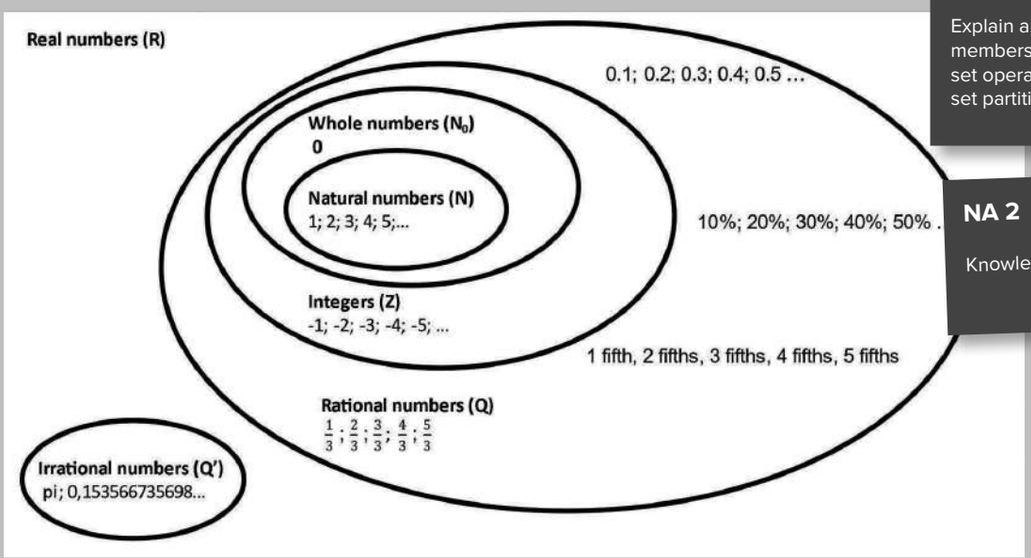
## South African curriculum for primary school




As teachers you need to know the **different types of numbers** which children encounter in primary and secondary school. This diagram shows how the different **number systems** fit together.

Diagram

## Number systems




**NA 3.2**

Explain and identify **sets**, set membership, set relations, set operations and set partitions

**NA 2**

Knowledge of **number system**

This course is designed to help you to learn the mathematics you need to teach primary school children, well.

Learning mathematics involves learning ways of thinking. Carpenter, Franke et al. (2003) write that learning mathematics

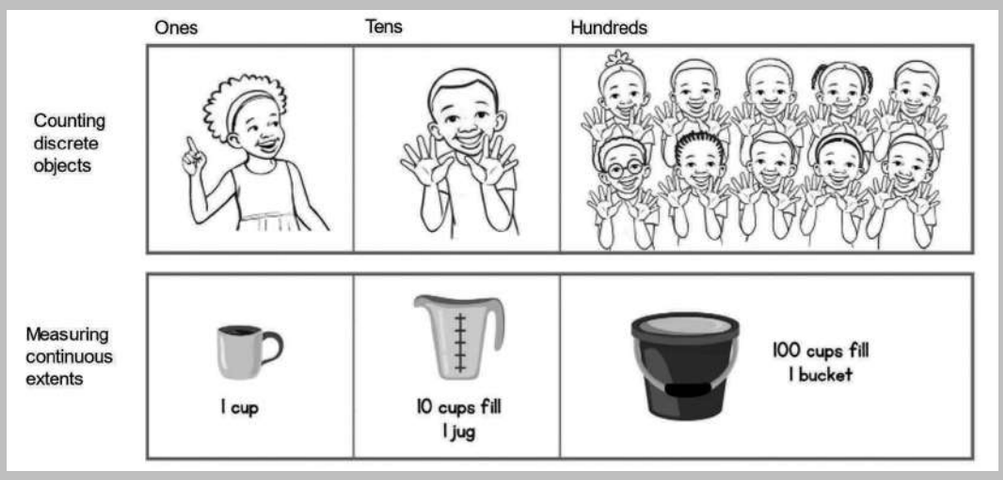
> involves learning **powerful mathematical ideas** rather than a collection of disconnected procedures for carrying out calculations. But it also entails **learning to generate those ideas, how to express them using words and symbols, and how to justify to oneself and others** that those ideas are true. (Carpenter, Franke et al. 2003, p. 1)

Learning mathematics is learning **ways of thinking** about **quantities (discrete objects/events** and **continuous quantities** which we can **measure).** Learning mathematics is **learning ways of thinking** about **shapes,** their arrangement, how they are positioned and can change (transform) in **space**.

Many of us have learnt maths by memorising. Many of us don't have a mental map for maths which goes beyond counting in ones. **Our first job in this course to see maths as meaningful.** Maths is not just a random collection of things to memorise. For maths to make sense we each need a coherent mental map or picture which builds systematically across concepts and over time. **Our second job in this course is to help children build meaningful mental maps for learning mathematics.**

Diagram

### Discrete objects and continuous extents




There are two main routes to developing children's number sense:

- **A counting pathway** where the starting point is discrete quantities (discrete objects or discrete events); and
- **A measurement pathway** where the starting point is continuous quantities that can be measured using a particular attribute such as length or volume.

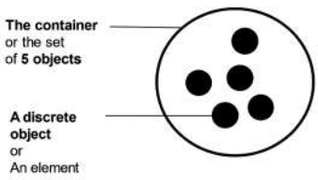
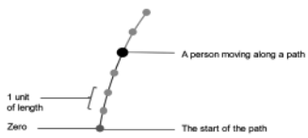
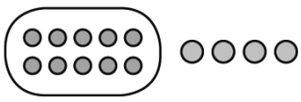
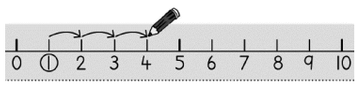
There are 8 mathematical powers which children use:

- imagine and express;
- organise and classify;
- conjecture and convince; and
- specialise and generalise.

**NA 8**

**Overarching ideas** for school arithmetic

In Maths4Primary teachers we consider both pathways, and an underlying approach of structuring numbers to identify, describe and generalise the patterns of 2s, 5s and 10s.

| **Quantities** | **Discrete quantities that can be counted** | **Continuous quantities that can be measured** |
|---|---|---|
| **Pathway into number sense** | Counting pathway | Measurement pathway |
| **Unit** | Counting unit (often in 1s, 2s, 5s or 10s) | Unit of measurement (standard or non-standard) |
| **Metaphor** | A container   | Movement along a path   |
| **Representations** | Number pictures  | Number lines   |
| **Number meaning** | **Cardinality** Number is the size of a set/collection of discrete quantities. | **Ordinality** Numbers are in a sequence arranged from smallest (first) to biggest (last) |

The two pathways are related. Children need to learn both meanings of the number: **cardinal** (size of a set) and **ordinal** (the sequence of numbers). They need to use and move between different **representations** of number.

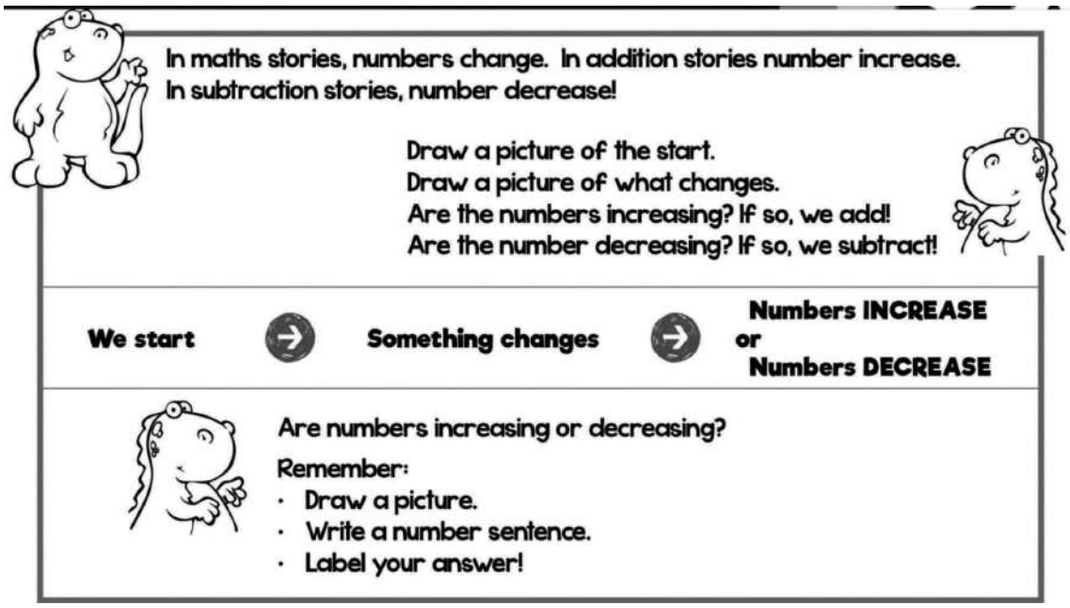
In both pathways children quantify. In both pathways children first choose a unit and then repeat that unit. Underlying both pathways – the counting pathway and the measurement pathway - is structured approach to **patterns, functions and algebra.**

## What is an additive relation?

**Additive relations (AR)** refers situations and calculations involving adding or subtracting.
In many additive relations situations there is a main story about change:

- There is a **start**.
- Something happens (there is a **change**).
- Then there is a **result.**

A story about change is not the only situation where we need to add or subtract.
But 'change' as a main story for addition and subtraction, is a good place to start.
The other situations for addition or subtraction are explained in UNIT 3.




## Addition

There are different situations where we need adding.

- There is a **start**.
- Something happens (there is a **change**).
- Then there is a **result.**

Two groups of objects join together.
Two parts are combined to make a new whole.

Activity for learners


Source: DBE grade 1.

The child may not yet know that the symbol + means add, or join or combine.
They may not yet know how to write the number symbols.
But they can solve the problem.

**NA 3.2**

Describe and use **addition and subtraction problem types**

Examples

### An additive relations word problem, involving addition

This is an example of a Grade 1 additive relation problem, involving addition.

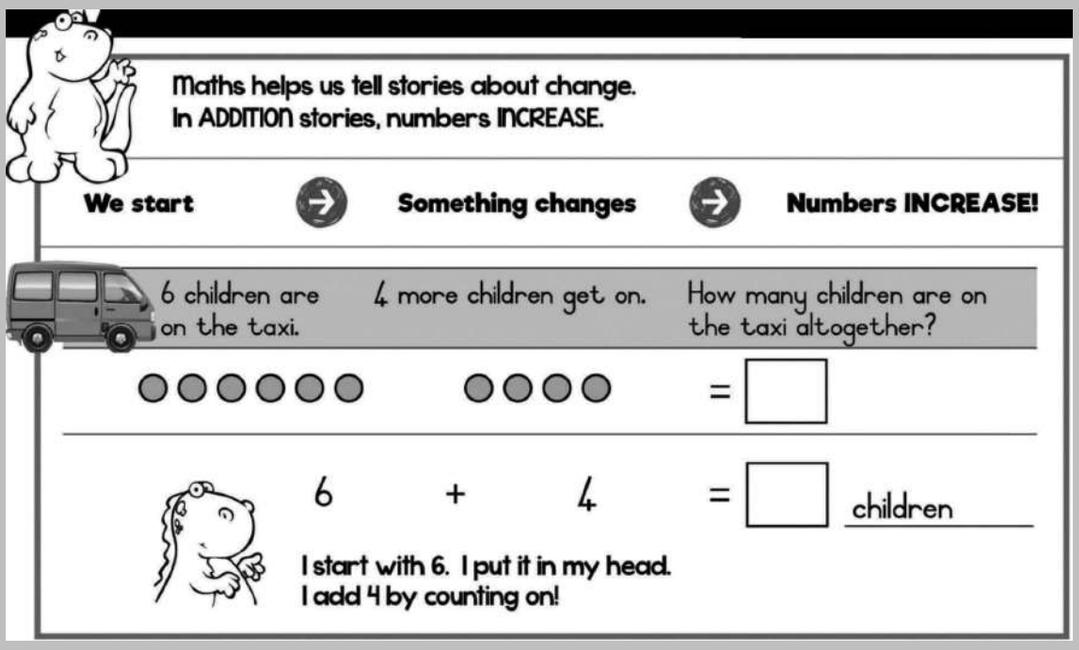
→ **Situation**
There are 8 passengers in a taxi. Three people get on.
There are now 11 passengers in the taxi.

→ **Calculation**
8 + 3 = 11

The units are **passengers**.
The three numbers 11 and 8 and 3 are an **additive relation**.

| 11 | |
|---|---|
| 8 | 3 |

This word problem has a **start–change–result** story.
As a teacher, you need to be able to create word problems.
You should be able to explain the situation, do the calculation, and identify the units.



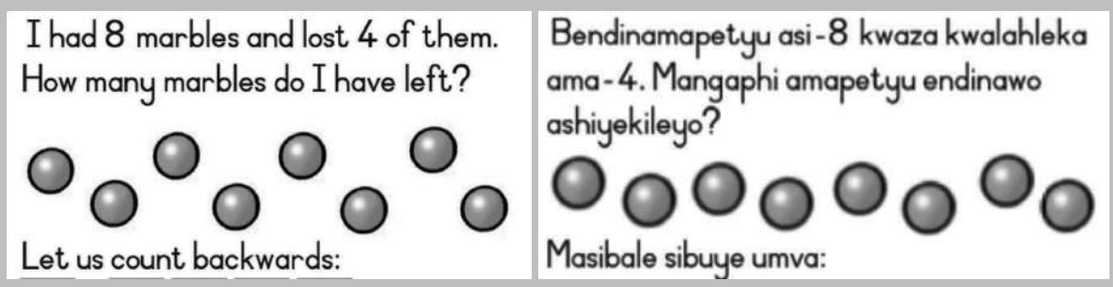

## Subtraction

There are different situations where we need subtracting.

- There is a **start**.
- Something happens (there is a **change**).
- Then there is a **result.**

There is a whole or one group.
One group is split into two parts.
One of the parts is removed, or taken away.

Activity for learners




Source: DBE grade 1.

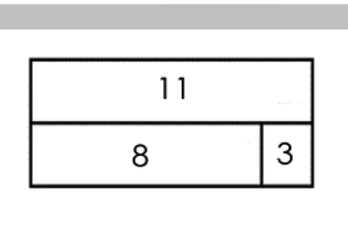
A child can solve this problem by arranging counters, or drawing and by talking.
They don't need to read or write words, numbers, or number symbols.

Examples

**An additive relations word problem, involving subtraction**

**Situation**
There are 11 passengers in a taxi. Three people get off.
There are 8 passengers left in the taxi.

**Calculation**
11 – 3 = 8

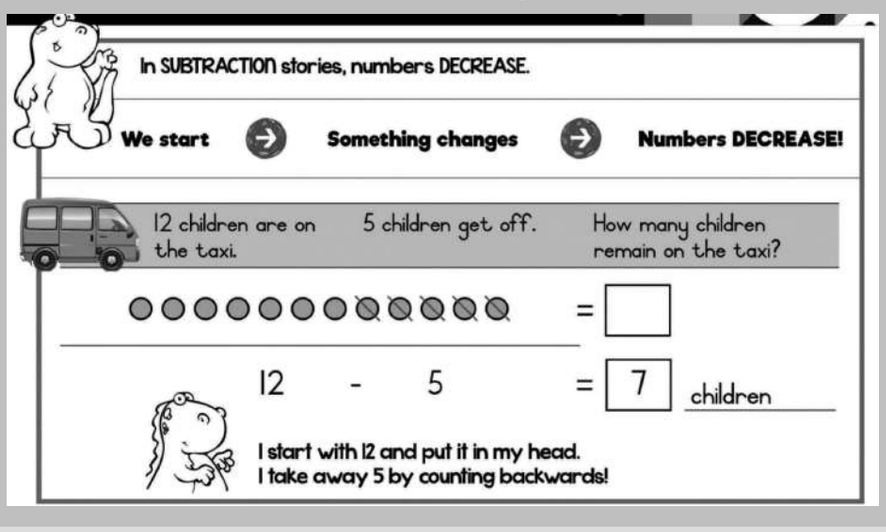



The units are **passengers**

There is a **start:** 11 passengers.
There is a **change:** 3 passengers get off.
There is a **result:** 8 passengers are left.

The three numbers are numbers are 11 and 3 and 8.
They have an additive relation structure: 11 – 3 = 8

This word problem also has a **start – change – result** structure as the main story.



**Additive relations** (AR) refers to situations and calculations involving **adding and subtracting**. Drawing a part-part-whole diagram helps to show the additive relations structure of the problem.

Part of learning to be a teacher involves knowing **what to teach**, and **when to teach it**. You also need to know **how to make a problem easier** – and **how to make it harder.** You will always have learners at different levels in your class.

**Additive relations problems** become more difficult to solve when you:

- Increase the size of the numbers;
- Change the type of numbers (for example from whole numbers to decimals/fractions/ percentages).
- Change where 'the missing number', or 'the box' is in the problem.
- Change the units from discrete objects to measurements (length, time, mass, capacity, volume, temperature)
- Change the main story for the word problem.

Examples

This is an example of a Grade 6 additive relations word problem.

**Grade 6 additive relation problem**

**→ Situation:**

When a baby elephant was born it weighs 118.9 kg.

After two years, his weight increased to 348.5 kg.
How much weight did he gain in the 2 years?

**→ Calculation:**

118.9 + ☐ = 348.5

348.5 – 118.9 = ☐

The units are **kilograms.**

| 348.5 kg | |
|---|---|
| 118.9 kg | |

Additive relations (AR) refers situations and calculations involving adding or subtracting. Adding and subtracting fit together in a part-part-whole structure.
Subtraction is the inverse of addition. Addition is the inverse of subtraction.

In the grade 6 example, the word problem also has a **start–change–result** story.
**Start:** When a baby elephant was born it weighs 118.99 kg.
**Change:** After two years, his weight increased
**Result:** to 348.5 kg.
We are looking for the change: How much weight did he gain in the 2 years?

Drawing a part-part-whole diagram helps to show the additive relations structure of the problem.

## How do children learn to add and subtract?

To add and subtract learners **count** and **structure** things (objects), and then numbers (mental objects). Over time they learn number facts. When they can automatically recall number facts (like 2 + 3 = 5) they start to use this known-fact as an object in their calculation

To help learners have a secure network of adding and subtracting facts there are some patterns or underlying **structures** that they should notice and which teachers should keep revisiting:

1. There are **two pathways** to learn about numbers, operations and relationships:
   a. **A counting pathway (with discrete objects) and**
   b. **A measurement pathway (for continuous extents)**

2. Children can **subitise or quickly see how many there** are for (up to 5 or 6 objects) **without counting.** They know how many there are just by looking (this is called subitising). They use their fingers to **structure numbers** by breaking up bigger numbers into smaller parts.

3. **Adding and subtracting are related.** For each addition fact, there are 7 other related facts. A part-part-whole diagram and families of 8 number sentences can help learners with this idea.

4. There are **100 addition facts which can be built up slowly.** Learners start to know the following as "known facts":
   a. 1 more than and 1 less than any number
   b. 2 more than and 2 less than any number
   c. Bonds of 5
   d. Bonds of 10
   e. Adding 10 (and later adding 100)
   f. Doubles

   They use these known facts, to work out:
   a. Adding 9 (and later adding 99) by compensating,
   b. Near doubles

5. As the numbers get bigger learners build on their **counting, structuring** and **known facts**, to become fluent with **calculation strategies. Different levels of calculating strategies** are used to help learners keep track of their thinking:
   Level 1: **Count all**
   Level 2: **Count on**
   Level 3: Count to calculate (**Break down the second number in 10s and 1s /Jump**)
   Level 4: Count to calculate (**Break down the second number to visit the ten**)
   Level 5: **Moves to flexibility** (Meaning of equal sign, part-part-whole, Re-ordering, linking addition to subtraction, take away and the difference)
   Level 6: **Calculate flexibly (**Doubles, near doubles, known and derived facts, round and adjust)
   Level 7: **Formal written methods**

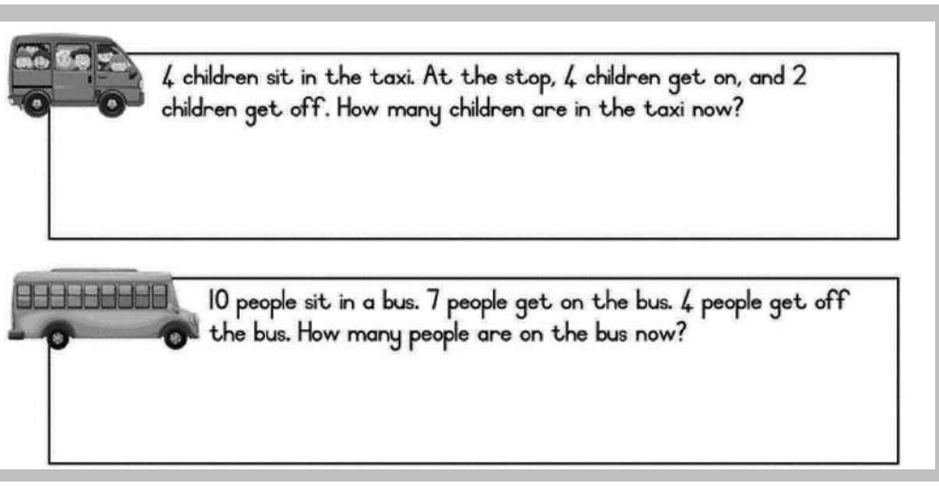
In this module we will focus on how learners learn to add and subtract and the first five levels of calculation strategies. You will learn to use and recognise each of these calculation strategies. You will also learn about and use the underlying fluencies and representations for these calculation strategies.

## SOME IDEAS FOR YOUR CLASSROOM

Children need lots of experience thinking about situations (involving discrete and continuous contexts) which demonstrate adding or subtracting. **Word problems are an important introduction to the ideas of adding** (as joining or combining) **and subtracting** (as separating or taking away).

Children have experiences. They know about people getting on and off taxis. This context has **discrete objects. It is in the counting pathway.** Before children can read, you can tell them a story and they act out the situation. They talk to solve the problem.

Activity for learners



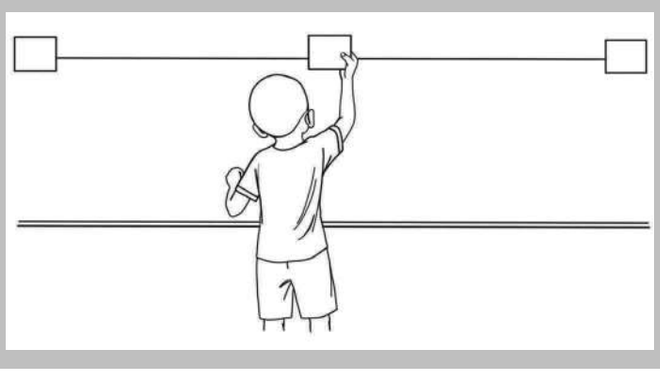

You also need to be supporting children to get a strong mental image for the measurement pathway. Child have experience of the world. They know about walking and taking steps, and comparing quantities. This context has **continuous objects. It is in the measurement pathway.** Children need to learn that 0 goes on the left and bigger numbers go to the right. They must be able to name, write and order the whole numbers from smallest (on the left) to biggest (on the right)

Activity for learners

Place number card 0 on the left, Number card 10 on the right.
Which number card goes in the middle?
Place number card 10 on the left, number card 20 on the right.
Which number card goes in the middle?

**NA 8**

**Overarching ideas** for school arithmetic

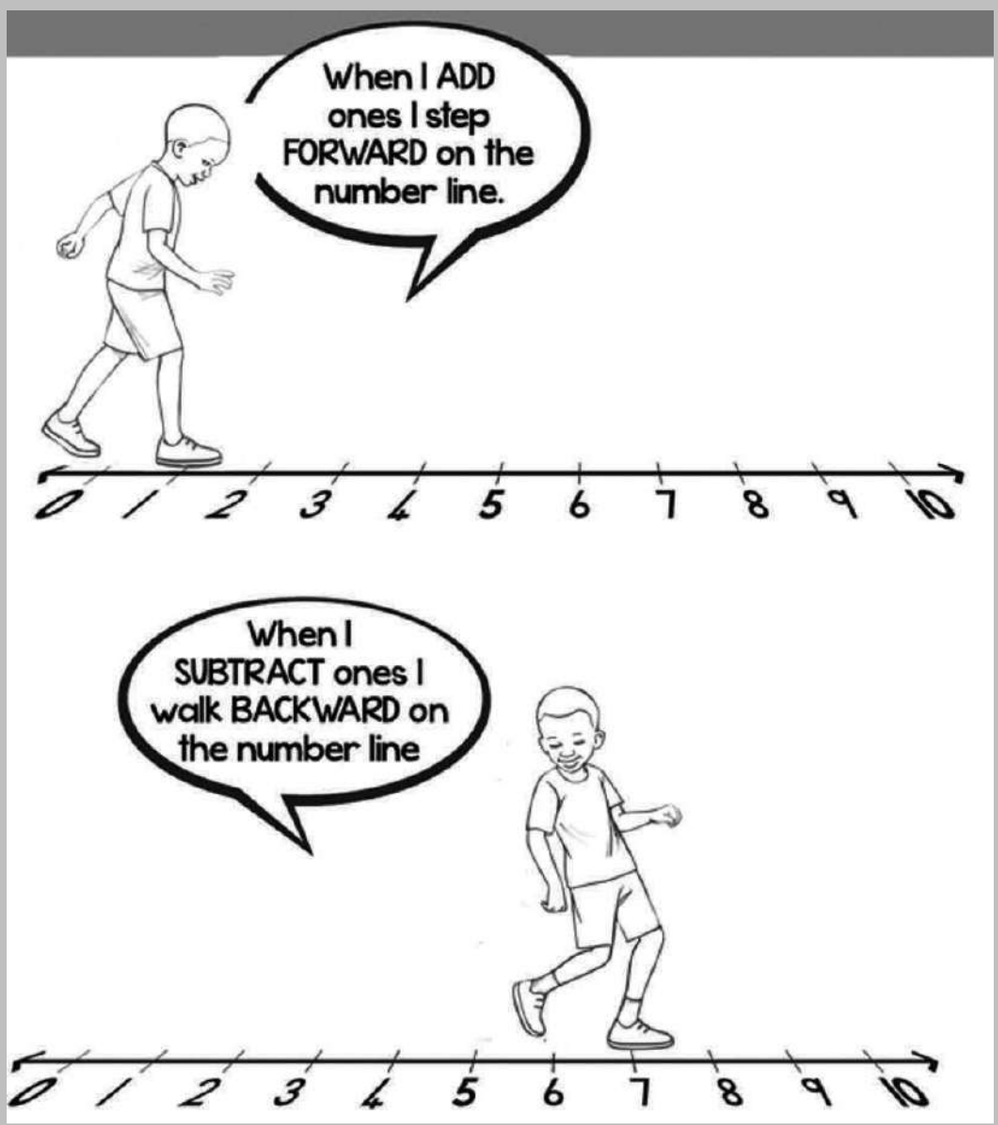


In the measurement pathway the numbers are a movement. You start at 0. And count forwards each step away from 0.

Young children are still learning about left and right. It is helpful to put 0 on the left of the board, and 10 on the right. We always start on the left (when reading or when counting forwards). Display a number line in your classroom. Put it on the front wall.

Activity for learners

Children should practice walking forwards (from left to right at the front of the classroom), while they count forwards in ones.



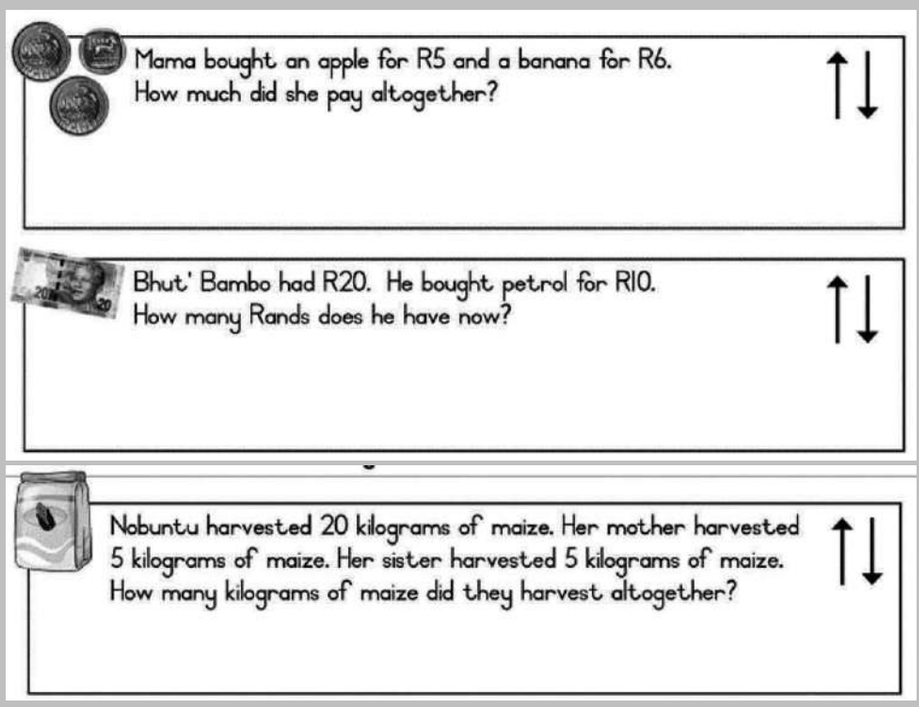

Children should practice walking backwards (from right to left in the front of the class) and counting backwards from 10.

Make sure that the adding and subtracting word problems you choose include **discrete objects** (like taxis or apples, or fingers) and ALSO include **money and measurement contexts.**

Before children can read, you can tell them a story and they act out the situation.
They talk to solve the problem. Is this problem about increase or decrease?
There is a start, then a change, then a result.

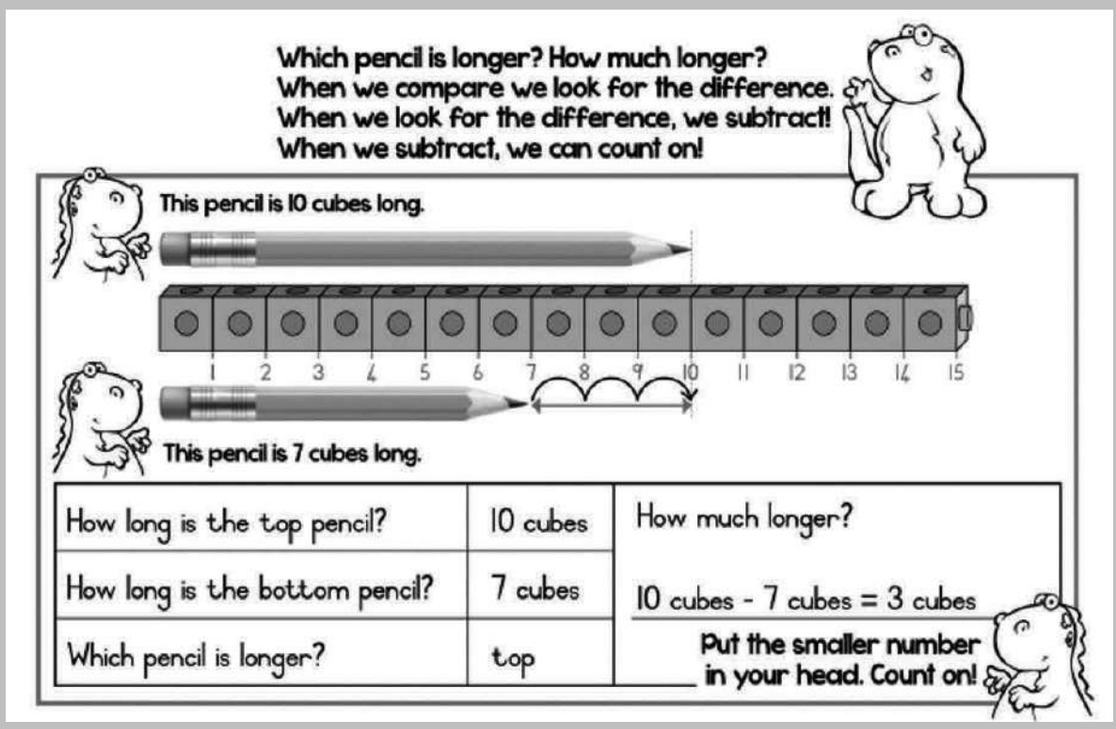
Activity for learners

These word problems have a **start-change-result** story.
Is the story about an increase or a decrease?




Activity for learners

This story is set in a measurement context. You compare lengths.
This is a compare (what is the difference?) story.



## Task: 1.1.1

**Show and explain a calculation**

Without using a calculator, show 2 DIFFERENT WAYS or STRATEGIES of how you could calculate: 3000 – 198 = ...

**FIRST STRATEGY**

3000 – 198 = ...

| BLACKBOARD (What do you write/draw?) |
| --- |
| TEACHER TALK (What do you say?) |

**SECOND STRATEGY**

3000 – 198 = ...

| BLACKBOARD (What do you write/draw?) |
| --- |
| TEACHER TALK (What do you say?) |

Reading

## Task: 1.1.2

**Reading - Porteus & Mostert (2022)**

When you study to be a teacher, you also learn to be a reader and a researcher.
Good teachers read. They take and use ideas from others. They share their ideas with their colleagues. They reflect on their teaching. They think deeply about what works and what does not work in their classroom. So, this course includes some readings. Read each article carefully. Read it more than once. Notice the authors and the date. Make notes. Highlight key ideas. Look up words that you don't understand. Discuss it with friends.

The literature review from Porteus & Mostert (2022) provides an overview of **what we know about how children learn additive relations**, and describes the **South African context of number work**. The theoretical framework explains how the idea of **mathematical representations** is defined and used.

## Literature Review

### How Do Children Learn Additive Relations?

Askew and Brown (2003) suggest that there is wide agreement in the literature that children learn additive relations between 0 and 30 through progressing through a relatively stable sequence. Children start by counting all, proceed to counting on from the first number, and (in the case of addition) further proceed to counting on from the biggest number. Children consolidate knowledge of additive relations between 0 and 30 using known facts to derive new facts (Thompson, 1999, 2000; Grey, 1991).

There is evidence that children can be taught through this sequence with directed instruction; there is less understanding of what and how long it takes to support children to build efficiency in this range in different instructional contexts. While the trajectory for learning additive relations of whole numbers from 0 to 30 is widely agreed, there is less agreement about how children best progress to learning additive relations from 30 to 100 (Askew & Brown, 2003: 8.)

There are two conditions recognised as conducive for the development of meaningful two-digit additive relations. The first condition is the "efficiency" of low number additive work, combining mental strategies with a child's speed of retrieval (He, 2014: 194). At the lower end of efficiency, children solve additive relations by counting (forwards or backwards) in ones. At a higher end of efficiency, children decompose, combine and structure number to enable faster retrieval. The second condition facilitating learning additive relations beyond 30 is a strong mental map for place value (Ladel & Kortenkamp, 2014). An understanding of the 10'ness of number is required to leverage unit counting to higher number ranges (if 3 + 4 = 7, then 53 + 4 = 57), to add and subtract 10s across 100 (58 – 30), and to establish a meaningful strategy to either add
or subtract two (or more) two-digit numbers.

Without the internalisation of an abstract 10, number symbols themselves metaphorically remain a long row of ones.

### South Africa Schooling and Counting in Ones

A binding constraint of mathematical performance in the early grades in South Africa is an over-reliance on counting in ones (Venkat, 2013; Ensor, Hoadley, Jacklin, Kuhn, Schnitte, Lombard & van

Heuvel-Panhuizen, 2009) often persevering into the intermediate phase and beyond (Schollar, 2008). As described by Venkat (2013, p. 31), notions of number remain temporal and literal rather than generalisable and abstractable. Five apples remain "1, 2, 3, 4, 5 apples" rather than 5 apples.

An overreliance on counting in ones, without successfully developing the necessary layers of abstraction that enable meaningful mathematics is not a South African phenomenon per se. Hoadley (2016: 6-7) highlights two meta studies describing early grade maths instruction in low- and middle-income countries. Two findings stand out. First, as emphasised by Atweh, Bose, Graven, Subramanian, and Venkat (2014), most classrooms in low- and middle-income countries have few instructional resources or tools upon which to leverage mathematical abstraction. Secondly, as emphasised by Nag, Chiat, Torgerson, and Snowling (2014), teaching in low-income nations overwhelmingly focuses on number facts, through recitation and rote learning, with little elaboration on the conceptual basis of early numeracy

## Early Number Structure to Place Value

The representations presented in this paper scaffold a learning trajectory from subitising and early structuring of number through to place value and two-digit additive relations.

- Subitising and Early Structuring of Number: We focus on maximising learner's ability to subitise, and establish an early mental map of number beyond counting in ones. Clements and Sarama (2009) define the ability to subitise as the direct perceptual apprehension and identification of the numerosity of a small group of items. It is the fast and highly accurate 182 way of quantifying collections up to six items, without counting. It is a key skill of seeing four fingers as a "4", rather than as 1-2-3-4. As numbers deeply embedded in our bodies, we aim to build learners' relationship with numbers (0 to 20) beyond counting in ones, focusing on a mental map of numbers organised by 2s and 5s.
- Place Value: Ladel and Kortenkamp (2014, p. 323-330) establish five number properties linked to place value: the positional property; the base-ten property; multiplicative property; the additive property, and the principle of bundling and unbundling.
- Two Digit Additive Relations: The progression to two-digit additive relations combines an understanding of place value, with an understanding of additive relations, whereby we add BOTH the tens and units in addition and subtract both the tens and units in subtraction.

## Theoretical Framing: Representations to Leverage Meaning Making In Early Grade Mathematics

Sociocultural theory, with roots in the early work of Vygotsky (Daniels, 2001) focuses attention on learning as mediating activity at the interface of a subject (e.g. the child), artefact (e.g. an instructional tool), and an object (e.g. the concept / what we hope a child will learn). In sociocultural learning theory, higher mental functions begin externally. The theory places emphasis on artefacts (tools and signs) as the basis to mediate transitions from the external to the internal. An artefact externalises the object through a suitable representation or visualisation. Children experience the object (in this case place value) by manipulating (acting to transform) the artefact (Ladel and Kortenkamp, 2014, p. 324.) By continuing to use the representational means, the child externalises her thinking, making her mental processes transparent to herself and others (and especially her teacher.) Whereas research previously placed an emphasis on "moving from the concrete to the abstract," research currently places emphasis on an iterative engagement between internal mental imagery (thinking) and external representations.

Sfard (2008) discusses the process of abstraction as mental "compressions." Through iterative engagements with meaningful tools, number concepts or processes are compressed into consolidated conceptual or number "objects," such as a reified 10.

Acting on these new mental objects allows for the development of (or compressions into) more complex mathematical objects (Sfard, 2008).

The use of mathematical representations as instructional tools has long been advocated to support early grade conceptual development (Roberts, 2014). Mathematical representations are visible or material objects that encode, stand for, or embody mathematical ideas or relationships. The notion of "representation" signifies a reference to meaning or conceptual significance. Representations are externalised tools, available for observation, discussion and manipulation (Goldin, 2014).

In their summary of research exploring classroom representations and children's mental imagery, Askew and Brown (2003 12-13) suggest that children's mental imagery of mathematics strongly reflects the mathematical representations used by their teachers (whether objects, verbal written, or pictoral.) They further suggest that learners who do well in mathematics, build an early mental imagery beyond counting in ones. Sociocultural theorists emphasise educational tools (such as representations) as "cultural;" tools may resonate in one schooling / social context, and not in another. If a representation is to be generative in a classroom, it must have a handle or grip aligned to the sociocultural context of teachers.

**Porteus and Mostert (2022) discuss two examples of common errors made when subtracting 2 digit numbers. See page 186 of their paper.**

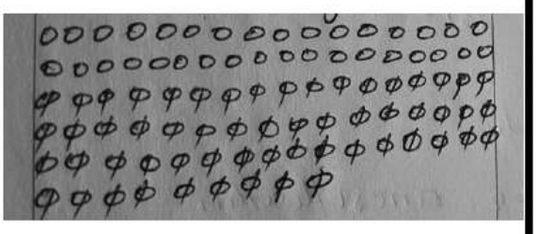
## Two Common Teacher Errors

Grade 2 and Grade 3 teachers, participating in a research design collective in the rural Eastern Cape were asked to imagine it is the final term of Grade 2. We presented the subtraction problem: 92 - 58. Teachers were asked to share what they would write on the board to support learners to approach this problem. Two of the errors that emerged are presented as Exemplar A and Exemplar B below.

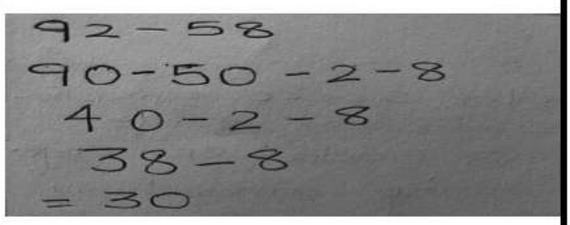
**Figure 1**

*Exemplars of Teacher Errors: Additive Relations*

Example 1: Counting in 1s



Ex. 2: "Breaking Down Both Numbers"




In Example A, we see a teacher use a small circle to represent a unit. She draws out 92 ones, and then crosses out (representing taking away) 58 of them...

In Example B, a teacher attempts to subtract using a method advocated in CAPS, namely to *break down both numbers written as a horizontal algorithm.*

**NA 3.2**

Describe the range of **representations** and increasingly sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## Task: 1.1.2

1. Who wrote this article?

2. What is the generally agreed sequence of how young children learn to add and subtract in the 0 to 30 number range for whole numbers

3. What is needed to develop meaningful calculations involving addition or subtraction?

4. What do Porteus and Mostert (2022) identify as a “binding constraint” in South African mathematics?

5. Explain what is meant by a “reified ten” (Sfard, 2008)

6. Show how to solve 92 - 38 = ..., using the breaking down both numbers method.

## Task: 1.1.3

### An learning trajectory to add and subtract with number pictures

SCAN ME

From page 187 onwards, Porteus and Mostert (2022) outline a learning trajectory for addition and subtraction using number pictures for South Africa.

There are 3 steps in the learning trajectory. For each step explain: (a) in which grade the step should be in focus, (b) what the step entails, and (c) give examples of games and activities for children.

Step 1 is **Subisiting and Early Number Structure.**

Step 2 is **Towards a Symbolic 10.**

Step 3 is **Adding and subtracting 2-digit numbers.**

## Tasks 1.1.4

### Investigate a "Two minute tango'

In this investigation, you explore and look for patterns in a **2 minute tango.**
This first time round, you won't be timed. In future, the idea is to practice working fluently with numbers. Over time you will get faster and more accurate. After this investigation you will only have 2 minutes to complete a **2 minute tango.**

- **Choose a partner. Both work on the first column of the 2 minute tango. Only do calculations Q1 to Q10 on the left hand column.**
- **Now compare answers. Talk about what you each did.**

| | | | |
|---|---|---|---|
| **1** | 2 + ____ = 5 | **11** | 81 + ____ = 90 |
| **2** | ____ + 5 = 5 | **12** | Double 68 |
| **3** | 9 + ____ = 10 | **13** | 392 + 19 + 8 = ____ |
| **4** | ____ + 1 = 10 | **14** | 100 = 37 + ____ |
| **5** | 33 + ____ = 40 | **15** | ____+ 10 = 53 |
| **6** | ____ + 39 = 40 | **16** | 644+393 ____ 646+391 Put <; > or = in place of ____ |
| **7** | 70 + 4 = ____ | **17** | One less than 300 is ____ |
| **8** | 80 + ____ = 87 | **18** | ____ = 47 + 80 |
| **9** | 80 + 7 = ____ | **19** | 7 + 7 = ____× 7 |
| **10** | 80 +____ = 85 | **20** | The number half-way between 900 and 1000 is ____ |

Did you notice that many of the calculations in the 2 minute tango fit together?

For example:
**Look at Q1 and Q2.** They both total to 5. You need to know bonds of 5. We use one hand to structure in 5s. Here are all the bonds of 5:

| | | |
|---|---|---|
| 0 + 5 = 5 | Don't forget that you | 5 = 0 + 5 |
| 1 + 4 = 5 | can also write the | 5 = 1 + 4 |
| 2 + 3 = 5 | bonds of 5 the other | 5 = 2 + 3 |
| 3 + 2 = 5 | way around too: | 5 = 3 + 2 |
| 4 + 1 = 5 | | 5 = 4 + 1 |
| 5 + 0 = 5 | | 5 = 5 + 0 |

**Look at Q3 and Q4.** They both total to 10. You need to know bonds of 10. We use two hands to structure in 10s.

1. **Write down all the bonds of 10.**

**Now look at Q5 and Q6.** They both total 40.
Forty is one of the tens in the medium number sequence. It is a multiple of 10.
You need to know that 3 + 7 = 10, to answer: 5. 33 + ☐ = 40

2. **Which bond of 10 do you need to know to answer : □ + 39 = 40**

**Now look at Q7.** 70 + 4 = ☐

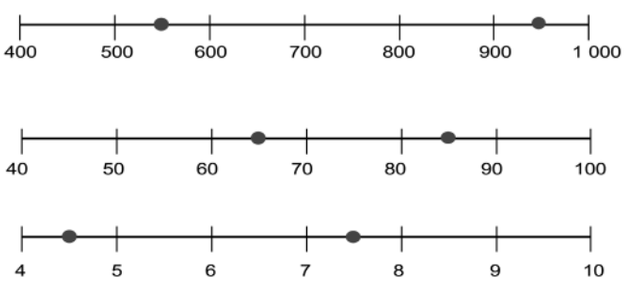
**Lets talk maths**

In English, seventy + four = 'seventy four'.
In isiXhosa it is almost the same:
amashumi asixhenxe + zine = amashumi asixhenxe anesine
*[tens, of which there are 7] + 4 = [tens, of which there are 7 and a 4]*
Seventy is a friendly multiple of ten. It is in the medium number sequence.
Learners need to know that when you add a single digit to a friendly ten (in the medium number sequence), you just put the two number names together.

3. **Now look at Q8, Q9 and Q10. What do they have in common? What can you do for those calculations?**

4. **Explain a quick way, using bonds of 10, to solve Q13, 392 + 19 + 8 = ...**

5. **Complete this drawing, and label all the red dots, which are exactly half way between the two numbers.**




6. **Explain how you calculated Q20:**
The number half-way between 900 and 1000 is ☐?

# SUMMARY

## Summary 1.1

- What are the **key ideas** from Emergent Number Sense, and how very young children learn about number?
- What are you hoping you learn from this course on addition and subtraction?
- What do we mean by "**additive relations**"?

# UNIT 2: CALCULATION STRATEGIES

In this unit you will be introduced to the idea of **progression in calculating strategies**. In the same way that babies first learn how to crawl, then walk and then run, learners first learn to calculate using simple strategies and gradually progress to using more advanced strategies. The purpose of this unit is to give you an overview of various additive strategies, and to place them in an hierarchical order, from the simplest to the most advanced.

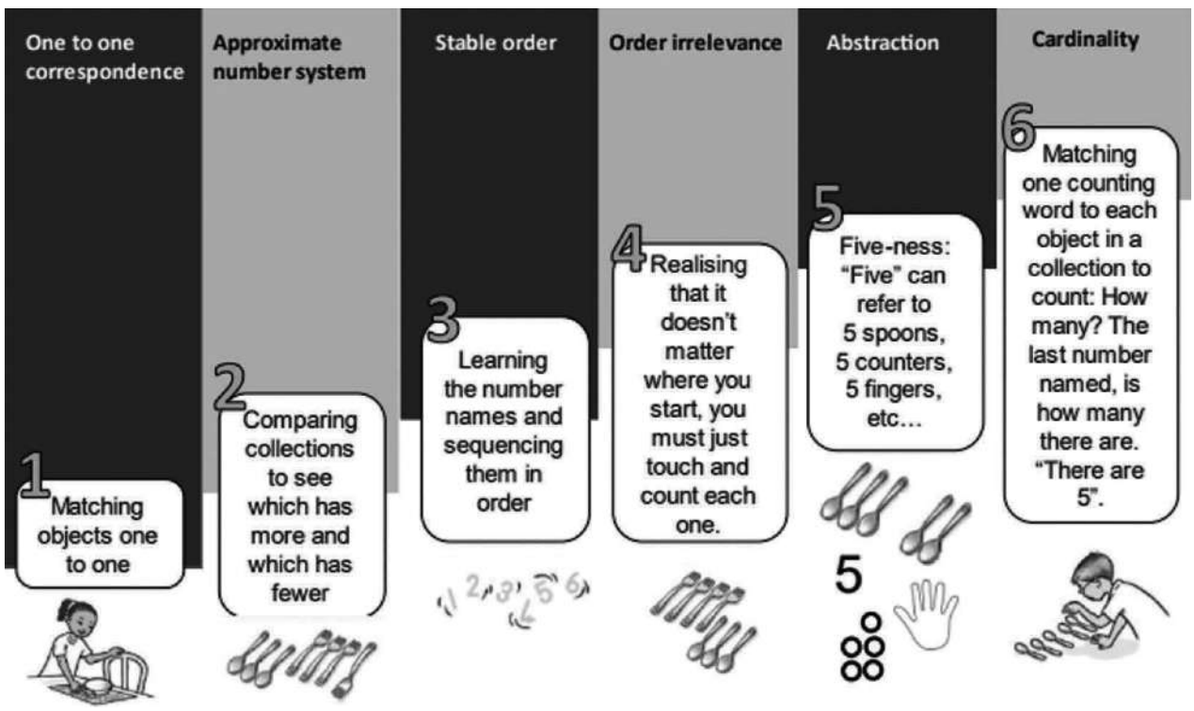
**Key Ideas**

At the end of this unit, you should:

1. Know that each calculation strategy requires particular **fluencies** (known facts or prior knowledge) to be done efficiently;
2. Explain and use the **count all** strategy for adding and subtracting;
3. Explain and use the **count on** strategy for adding and subtracting;
4. Explain and use the **'go-to' jump strategy** where you break down the second number using expanded notation)
5. Explain and use the '**visit the ten'** or **'bridge through ten'** strategy for adding and subtracting.

## Overview of calculation strategies

Before children learn to calculate, they learn to count, moving through different stages of the **counting trajectory.**



Once they have reached Stage 6 **cardinality** they start to use the **counting pathway for calculating**. They also start to use the m**easurement pathway** to order numbers and reason about numbers relative to each other (6 comes after 5 but before 7).

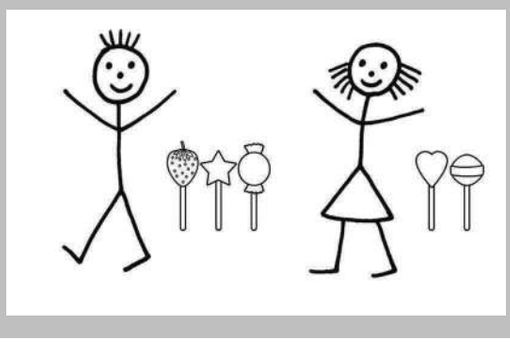
Learners develop **different calculation strategies** to solve addition and subtraction problems over time. To use each calculation strategy, learners need to be fluent with particular skills (such as counting forward and backward) and representations (number pictures, number lines, number sentences). When solving a problem, young learners intuitively try to model the situation (story) described or represented in the problem.

Examples

**Direct modelling for the lollipop problem**

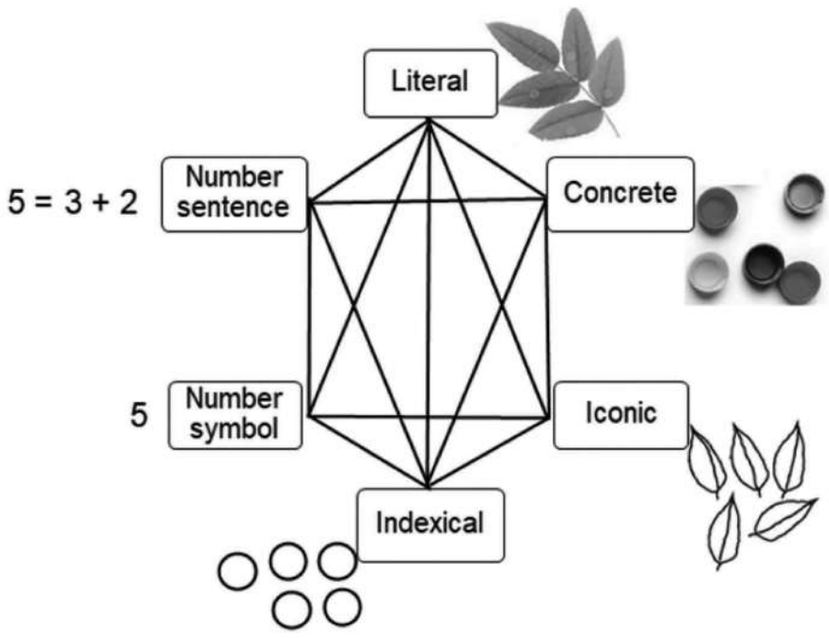
"Khanyi has 3 lollipops and Francina has 2 lollipops. How many lollipops do they have altogether?"

A young learner could draw a 'direct model' of the problem. The learner may draw Khanyi and Francina with 3 and 2 lollipops (respectively) and count all the lollipops to calculate the answer. The learners use discrete objects to directly model the situation.



They could use the actual objects (literal lollipops), concrete objects (like blocks or manipulatives), they could draw iconic pictures, or indexical pictures. They slowly learn to write a number sentence.

**Representations of discrete objects**




**NA 2.1**

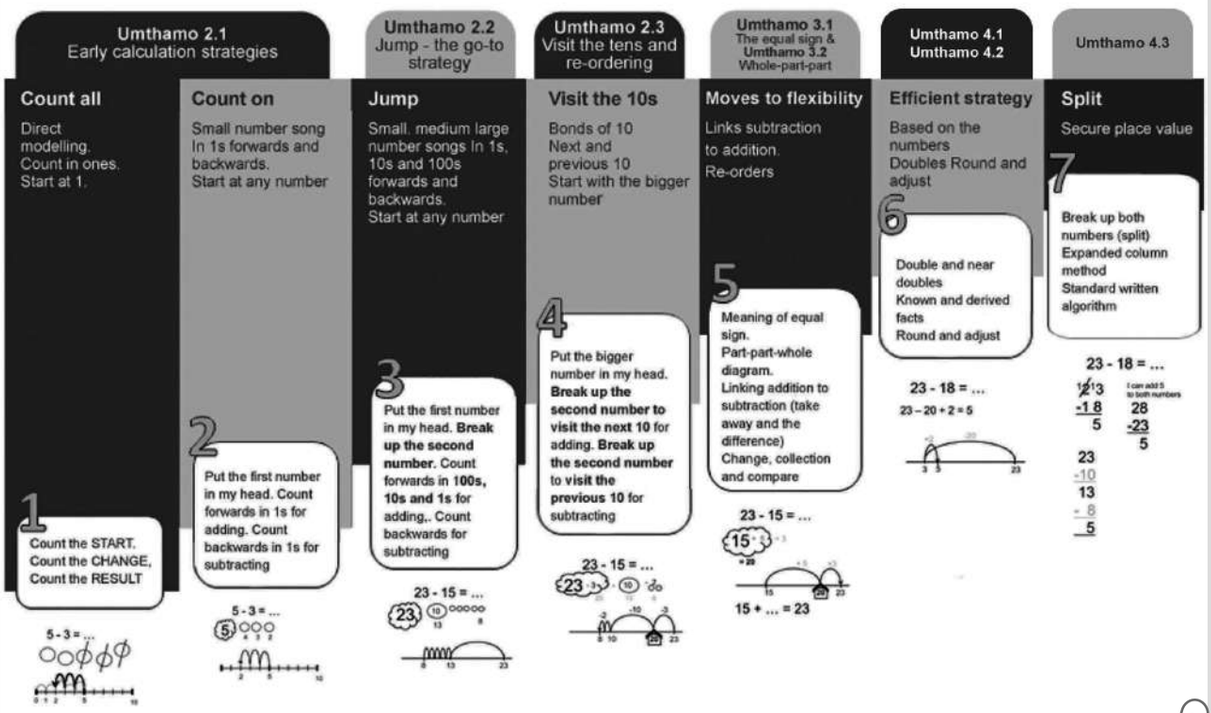
Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

When a learner calculates by depicting the literal elements in a problem (like boy, girl, lollipops) or uses counters (or other concrete items) to act out a problem, we call this **direct modelling.**

The table below provides an overview of **additive calculation strategies** and places them in hierarchical order, from level 1 (simplest) to level 7 (most advanced).

| Level | Strategy | Fluencies |
|---|---|---|
| **Level 1** | **Count all** is the simplest strategy, used by learners who are only able to count in ones using manipulatives or fingers | Count forwards in 1s. Start at 1. |
| **Level 2** | **Count on** with the small number song.<br>is used by learners who see numbers as objects, not only as part of an ordered list. Learners usually 'put the first number in their head' and then use the small number sequence to count on the second number. | Count forwards and backwards in 1s. Start from any any number |
| **Level 3** | **Count to calculate (Break down the second number)** is used with the small, medium and large counting songs. Learners can do this mentally, or write down their strategy using number pictures and number lines.<br><br>Learners break down 2-digit numbers into tens and ones to calculate efficiently using the medium and small number sequences. **This is a structured 'go-to' jump strategy.** | Count forwards and backwards in 1s,10s and 100s . Start from any number.<br><br>Small, medium, large number songs |
| **Level 4** | **Reorder.** Start with the bigger number (for example 3 + 11 = 11 + 3) and find the ten (for example 8 + 5 + 2 = 8 + 2 + 5)<br><br>**Visit the tens.** This is another **break down the second number strategy**. Rather than just counting in ones, tens and hundreds learners also start to break down the second number to visit the next ten (for adding) and visit the previous 10 (for subtracting). | Start with the bigger number<br>Bonds of 10<br>Visit the next 10<br>How far to the next 10?<br>Visit the previous 10. How far to the previous 10? |
| **Level 5** | **Moves to flexibility** is used when learners can work with number structure like 5-wise or 10-wise.<br>Rather than just counting in ones, tens and hundreds and visiting the tens, learners also start to use other forms of structuring numbers like<br>• **Meaning of equal sign;** families of number sentences, and part-part-whole<br>• **Link addition and subtraction:** take away and the difference | All of the above and<br>Part-part-whole<br>Meaning of equal sign |

| Level | Strategy | Fluencies |
|---|---|---|
| **Level 6** | **Calculate flexibly** is used by learners who can choose an efficient strategy based on the numbers involved. They might use all the above and:<br>• **Doubles and halves**,<br>• **Known or derived facts** (like 1 more/less than, 2 more/less than and near doubles),<br>• **Round and adjust,** round and estimate, | All of the above, and:<br>Doubles and halves<br>Bonds of 10<br>Known facts<br>Rounding off. |
| **Level 7** | Use all of the above and **formal written methods** with an efficient procedure for large numbers. A deep conceptual understanding of place value will allow learners to fluently **split** both numbers:<br>• solve multi-digit problems working with whole numbers through the expanded column method or the whole number-based algorithm;<br>• use a condensed digit-wise algorithm involving re-grouping | All of the above and conceptual place value is completely secure |




In this module we focus on Levels 1 to 5. In *Addition and Subtraction II*, we focus on levels 6 and 7.

**NA 3.2**

Describe the range of increasingly sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

# UMTHAMO 2.1

## EARLY CALCULATION STRATEGIES – LEVELS 1 & 2

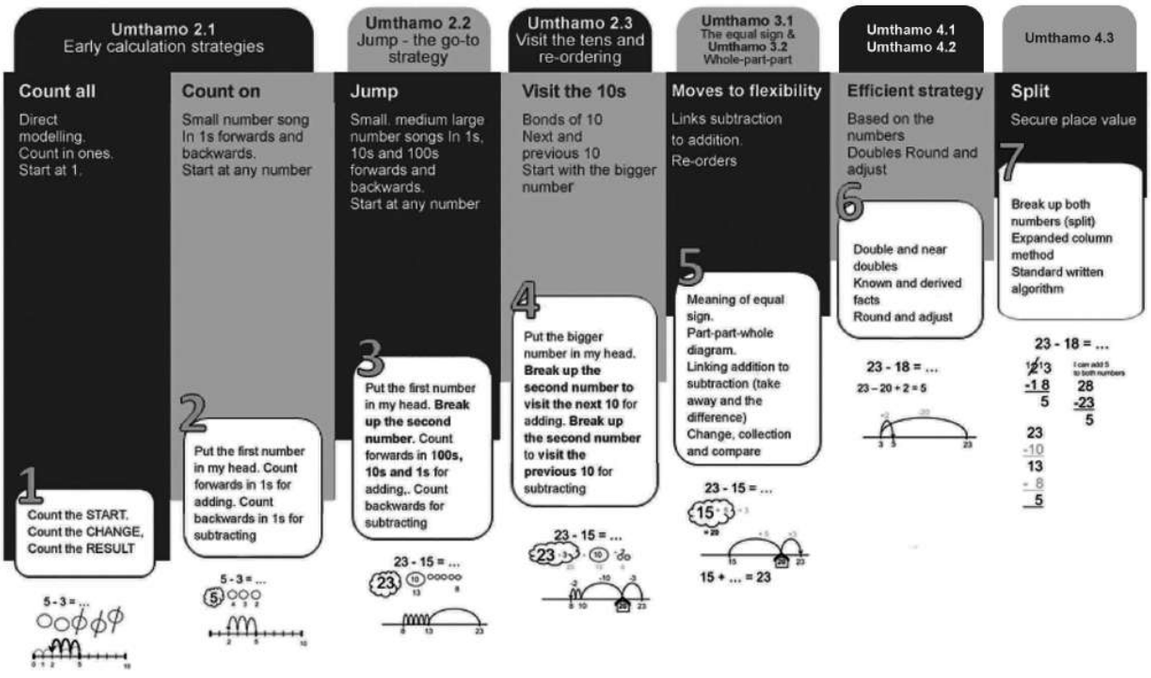
Key Ideas

At the end of this unit, you should:

1. Know that **each calculation strategy requires particular fluencies** (known facts or prior knowledge) to be done efficiently;
2. Explain and use the **count all** strategy for adding and subtracting;
3. Explain and use the **count on** strategy for adding and subtracting;

## LEVEL 1: COUNT ALL

In this umthamo we show how children use counting, for addition and subtraction.
The focus on **Level 1: Count all**.




To **count all,** learners need the following fluencies:

To count forwards using the small number sequence from one.

At this level children using count all with different sub levels of sophistication:

1. They **count all** for adding (usually with small numbers, less than 10)
2. They **count all or count out** of subtracting (usually with small numbers, less than 10)

### Count all for adding

Young learners use **count all** when adding.
This is called a **triple count** because the learner counts from one 3 times. This strategy is usually enacted with a concrete representation or a drawing.

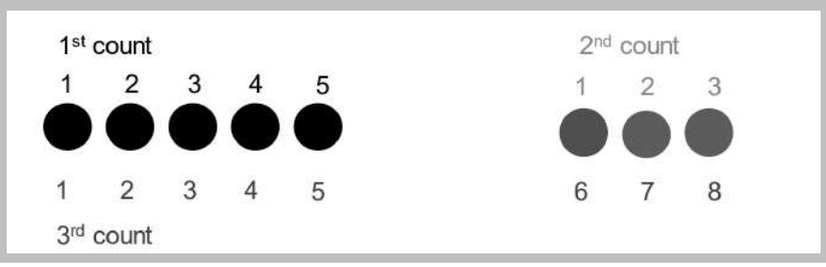
**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

Examples

### Count all for adding (Number picture)

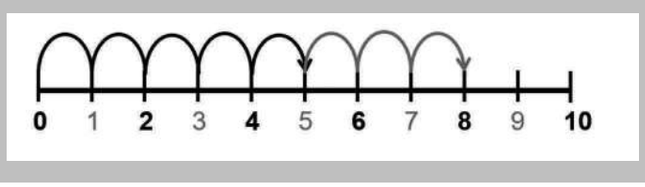
To solve 5 + 3 = ☐
1st count: The learner counts 5 blocks '1, 2, ... 5'
2nd count: She then counts another 3 blocks '1, 2, 3' and
3rd count: Finally she **counts all** the blocks saying '1, 2, ... 8, the answer is eight'.




Examples

### Count all for adding (Number line)

To solve 5 + 3 = ☐
1st count: The learner starts at 0 on the number line. She hops on 5, saying '1, 2, ... 5'
2nd count: She then hops on another 3 blocks '1, 2, 3' and
3rd count: Finally she goes back to 0 and **counts all** the hops saying '1, 2, ... 8, the answer is eight'.




**Count all** is a suitable strategy for young learners – in Grade R and Grade 1 and for small numbers (less than 10). The children need to know that

- adding refers to joining **number pictures**; or
- adding refers to hopping forwards on the **number line**.

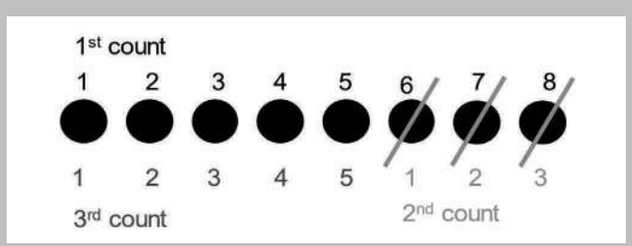
### Count all for subtracting

Young learners use **count all (or count out)** when subtracting. This also uses three counts.

Examples

### Count all for subtracting (Number picture)

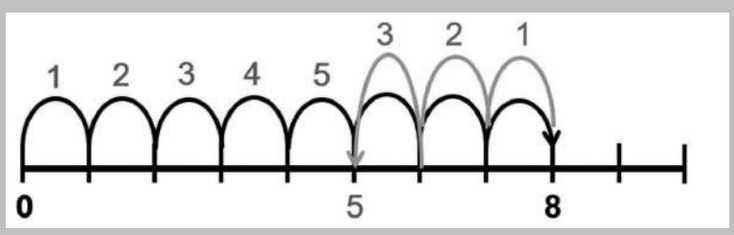
To solve 8 – 3 = ☐
1st count: The learner draws 8 circles '1, 2, 3, ... 8', and
2nd count: He crosses out 3 circles '1, 2, 3'.
3rd count: Then he **counts out** the remaining circles saying '1, 2, 3, 4, 5' the answer is five'.



Examples

**Count all for subtracting (Number lines)**

Count all is seldom used for subtraction on number lines. If it is used there are three counts.
1st count: The child starts at 0, counts on 8 hops,
2nd count: The child counts backwards 3 hops,
3rd count: They counts forwards from 0 to find their landing place.

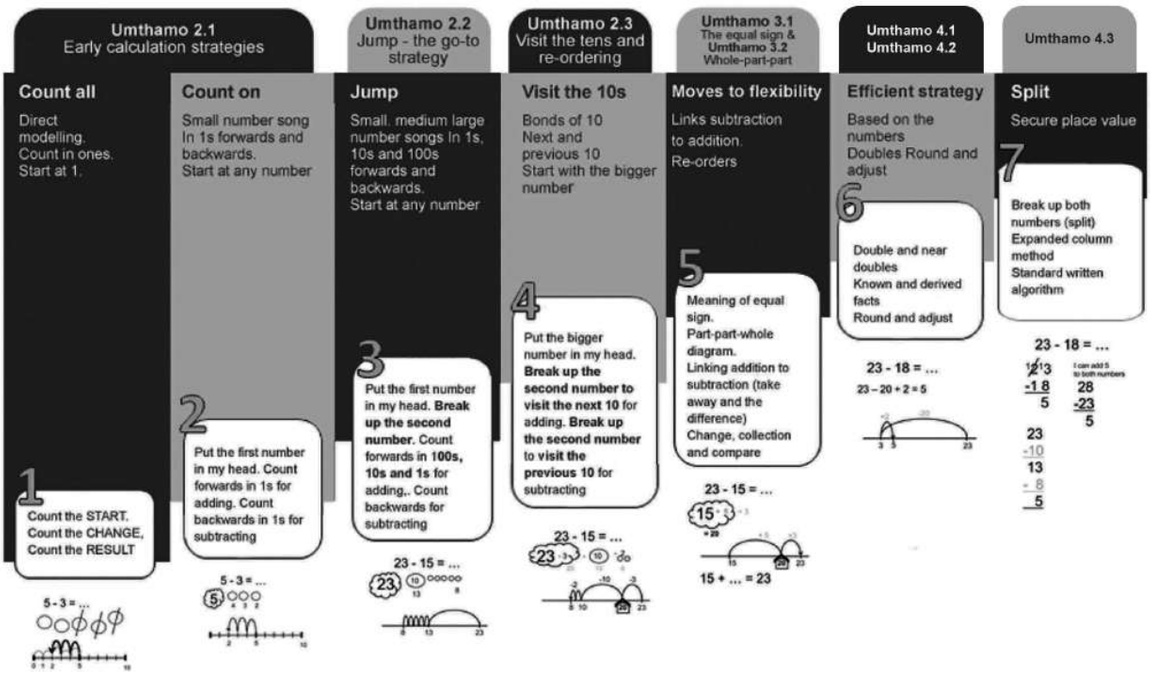



**Count all** is a suitable strategy for young learners – in Grade R and Grade 1 and for small numbers (less than 10). **The children need to know that:**

- **subtracting refers to** taking away or removing **number pictures.**
- **subtracting refers to** hopping backwards on the **number line.**

## LEVEL 2: COUNT ON

When children reach level 2, they also make use of level 1.




**Count on** is used for addition and subtraction.
To **count on,** learners need the following fluencies:

- To **count forwards using the small number sequence from one** (count all).
- To **count forwards and backwards using the small number sequence, from any number.**

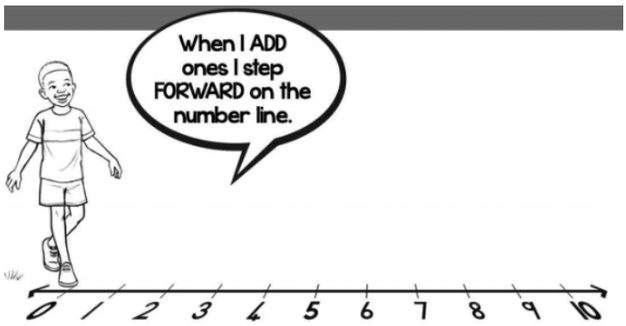
To be specific with our language we refer to **count forwards** for addition, and **count backwards** for subtraction.

Now, we briefly describe what is meant by **counting on** which includes:

- **counting forwards for addition**, and
- **counting backwards for subtraction**.

### Count forwards for adding

This strategy is referred to as **count forwards in ones** in CAPS. Learners put the first number in their head and then count on with the small number sequence.

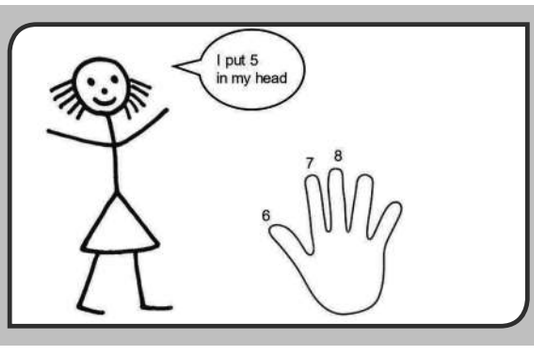



Examples

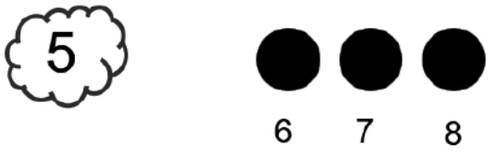
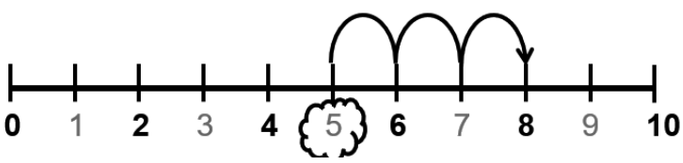
### Count forwards - addition

To solve 5 + 3 = ☐

The learner puts 5 'in her head' and counts forwards 3 saying '6, 7, 8'. They keep track of the 3 counts on their fingers. The answer is 8'.




What does **count forwards** look like on a number line or using number pictures?

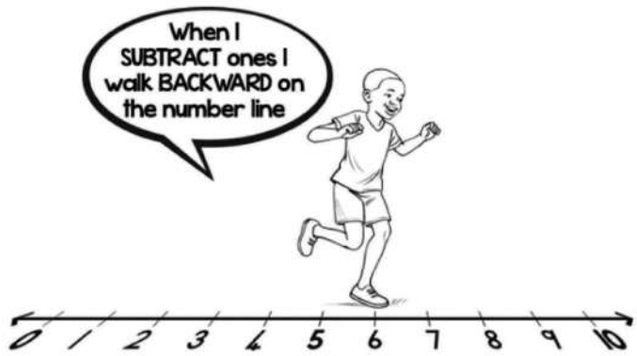
| | |
|---|---|
| **Number picture** | Put 5 in my head. Count forward 3.<br>  <br><br>5 + 3 = 8 |
| **Number line** | Start at 5. Hop forward 3.<br>  <br><br>5 + 3 = 8 |

**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

## Count backwards for subtracting

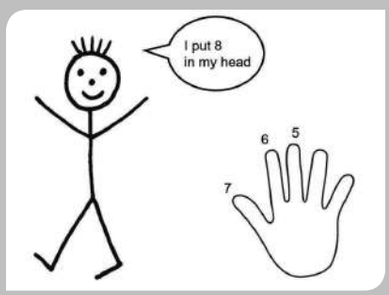
Count backwards is used for subtraction. Learners put the first number in their head, and then count back with the small number sequence.



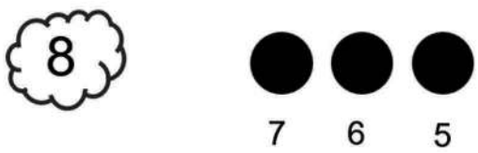
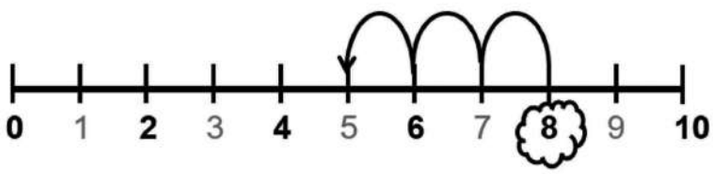

Examples

## Count backwards - subtraction

To solve 8 – 3 = ☐
The learner puts 8 in his head
And counts backwards 3, saying '7, 6, 5'.
They keep track of the 3 counts on their fingers
The answer is 5.




How do we show **count backwards** using a number picture or a number line?

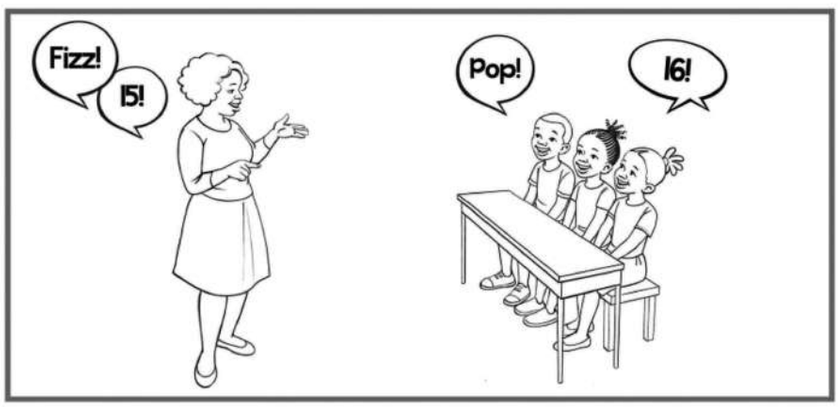
| | |
|---|---|
| **Number picture** |  <br>8 – 3 = 5<br><br>Put 8 in my head.<br>Count backwards 3. |
| **Number line** |  <br>8 – 3 = 5<br>Start at 8.<br>Hop backwards 3. |

## SOME IDEAS FOR YOUR CLASSROOM

Each maths lesson starts with ten minutes of **fast paced mental maths.** This time helps to build mental fluencies. There are three useful steps that you can to follow:
**Step 1:** Fizz pop;
**Step 2:** Strategic counting; and
**Step 3:** A fluency game for the week.

### Lesson starter Step 1: Fizz Pop

Activity for learners




**STEP 1: Fizz Pop**

**Play Fizz Pop: 1 MORE and 1 LESS**
Start with numbers from 0 to 20, go to 100.
Can you go higher?

You can also play fizz pop with: 2 more 2 less; 10 more 10 less, double, half.
It is a great way to get the class settled. You just start with Fizz, and they reply Pop.

### Lesson starter Step 2: Strategic counting

The next step is strategic counting. Counting with learners is strategic when it:

- Makes children think about number structure and relationships;
- Does not take too much time; and
- Keeps all children engaged.

The **small** counting song is in 1s.
The **medium** counting song is in 10s.
The **large** counting song is in 100s.

### The top three maths rules for strategic counting are:

1. Give a number to **begin** and **end.**
2. Give the **unit** (or group) that you are counting in (For example count in 1s, 2s, 5s, or 10s)
3. Give the **direction** for counting (count forwards or count backwards.

**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

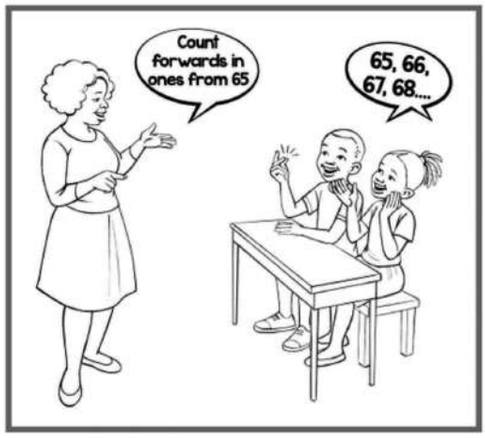
Activity for learners

## STEP 2: Strategic Counting

Count forwards in ones, starting and ending with any number between 0 and 100.

**Example:** Count forwards in ones from 65 to 78.

- When we count forward in ones, we are adding ones.
- We call 1 the little counting number.



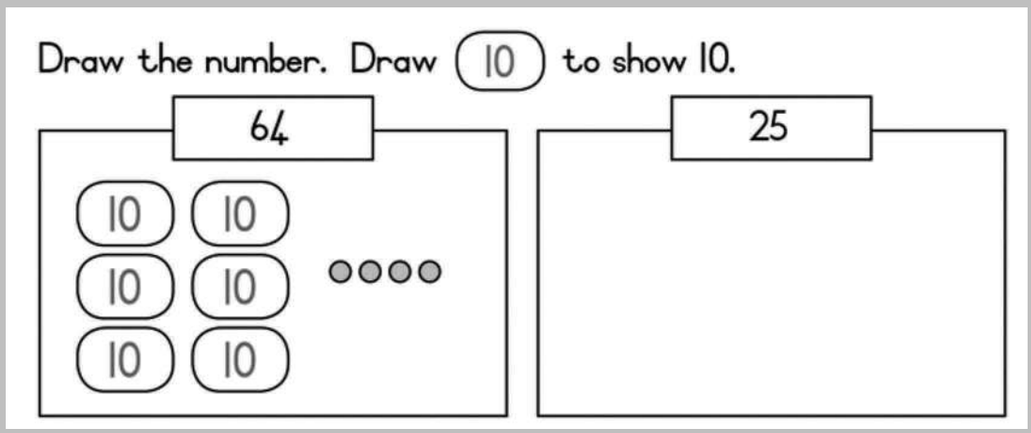

| Good Example | Not So Good Example |
|---|---|
| Count backwards in 1s from 135 to 123<br><br>Count backwards in 10s from 163 to 43 | Count from 150 to 100. |

### Lesson starter Step 3: Game of the week

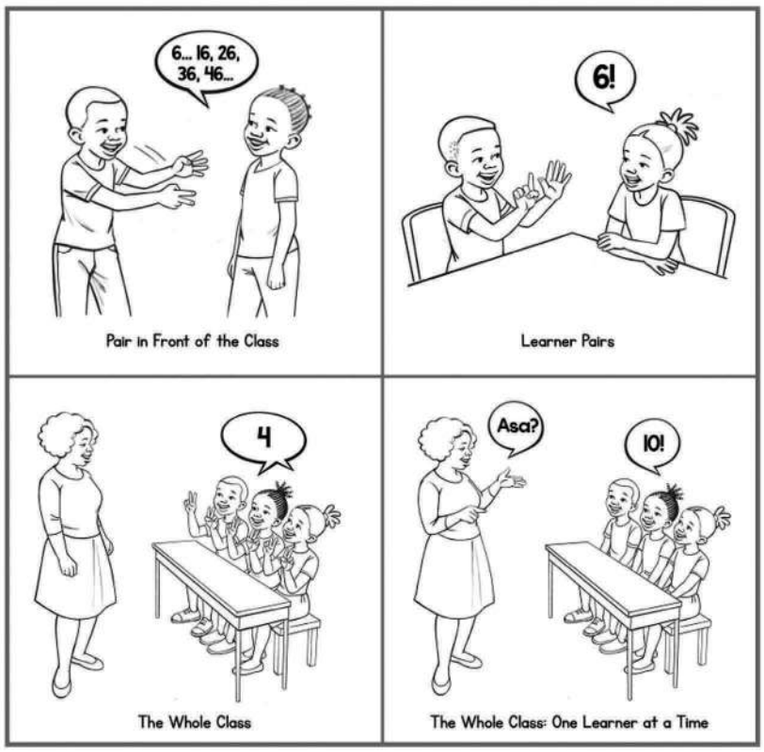
The third step each day is to play the **maths game of the week**. EACH week you can play a new maths game. You play the same game on Monday, Tuesday, Wednesday, Thursday and Friday. At the start of the week the game will be slow and hard. As you practice the learners become better and faster!

Game

### GAME OF THE WEEK




There are many different games, and you can also make up your own. You can play the games in different ways.




**GPM 3**

**Engage learners productively** in mathematics

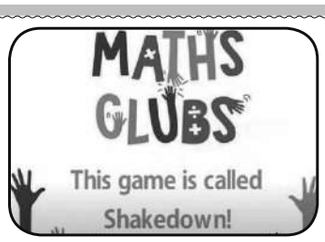
Activity for learners

**GAME OF THE WEEK: Count forward from 10 in 10's**

**Samp:** Roll the dice
**Bean:** Count on in tens from the dice value. Stop at the number closest to 100.

**Example**
Samp: Rolls a 6 on the dice
Bean: Says 6, 16, 26, 36, 46, 56, 66, 76, 86, 96.

Game




SCAN ME

**Shake down!**

Learners practice counting backwards in 1s from any number.

Game


SCAN ME

**Spill the Beans**

Learners practice finding the missing number.

Game

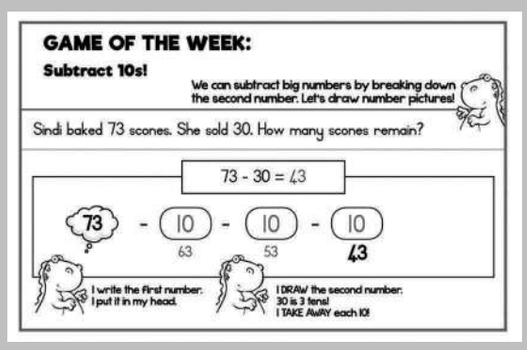
## Task: 2.1.1

### Planning number picture games

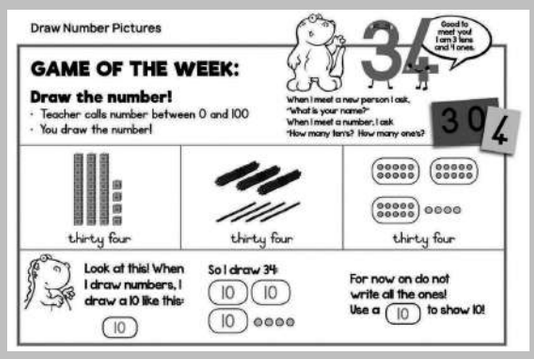
It takes time for children to become secure with drawing number pictures. Your head of department asks you to plan a sequence of games. Each game is played for the whole week.

Here are 6 suggested games to help children learn to draw number pictures and use them for adding and subtracting.

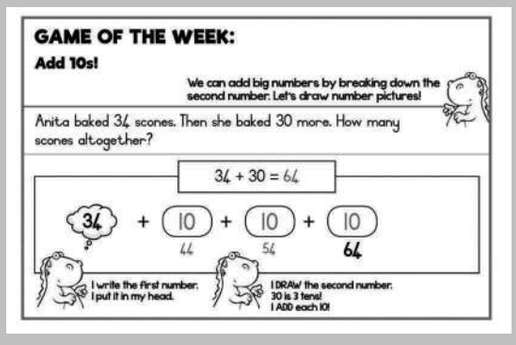
A. Subtract 10s



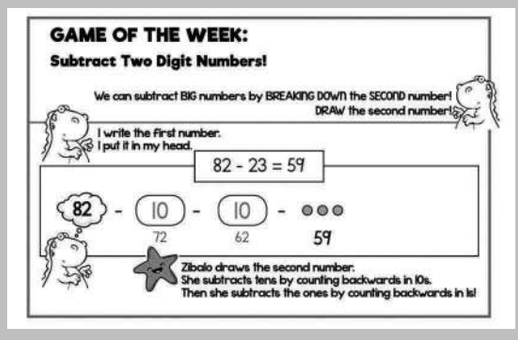

B. Draw the number



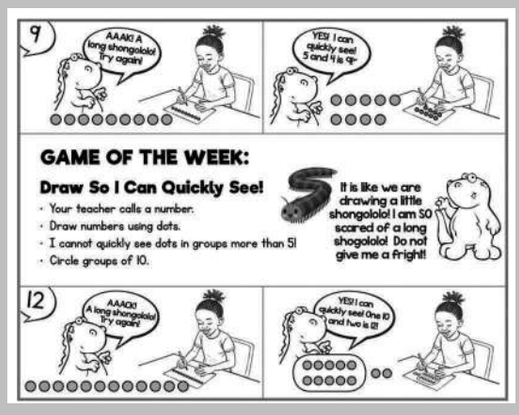

C. Add 10s



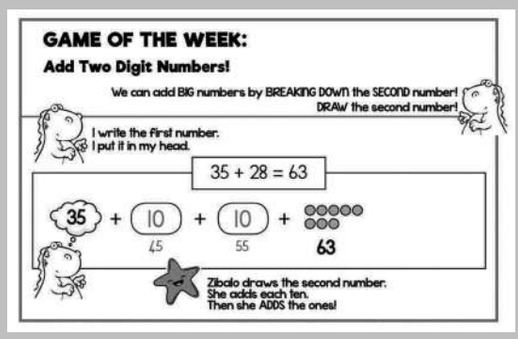

D. Subtract 2 digit numbers




E. Draw so I can quickly see




F. Add two digit numbers




You need to order them from easiest to hardest.
The easiest game is played in week 1.
The hardest game is played in week 6.

**GPM 1**

**Plan** effective learning experiences

1. Complete the table to plan for 6 weeks

| Week | Game of the week | Reason |
|---|---|---|
| 1. | | |
| 2. | | |
| 3. | | |
| 4. | | |
| 5. | | |
| 6. | | |

2. Solve by drawing a picture. **Bandile drove 34 kms to shop and then 25 kms to visit his mother. How many kms did he drive altogether?**

34 + 25 =

34 + 10 + 10 + ●●●●●

44 54 59

43 + 26 =

68 + 24 =

3. 68 - 32 =

68 -

61 - 23 =

4. Solve by breaking down the second number.

78 - 45 = 78 - 10 - 10 - 10 - 10 - 5 = _____

57 - 24 = 57 - _____ = _____

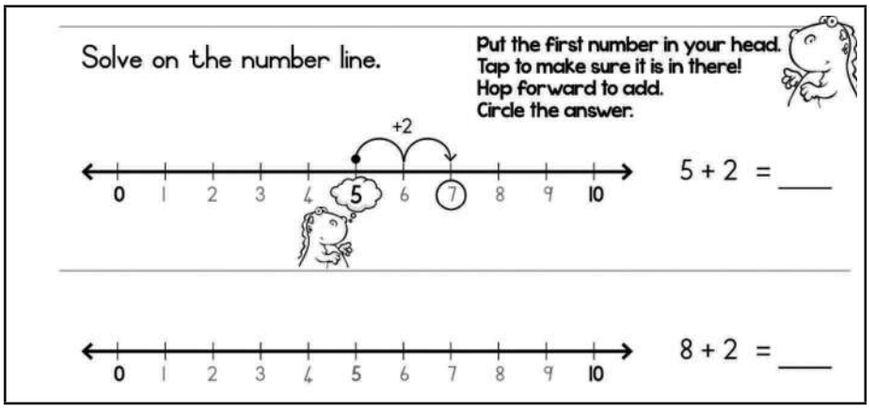
**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

## Task: 2.1.2

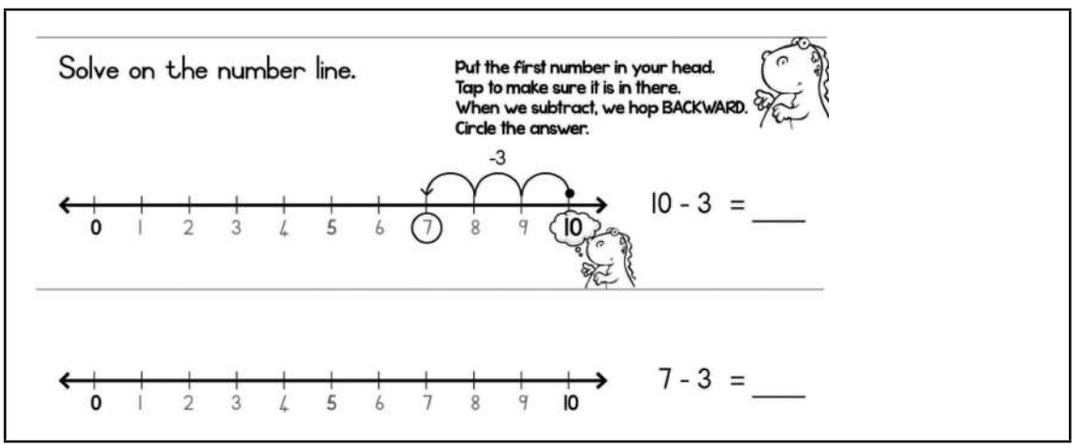
### Hopping forwards and backwards on a number line

1. Zibalo explains how to add by **counting forwards on a number line** (She generalises). You must specialise. Complete the number line for 8 + 2.

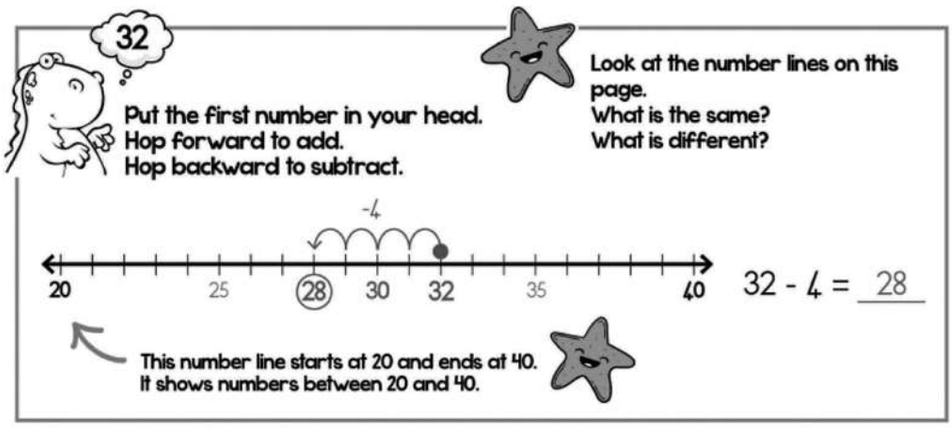



2. Write what you would say to explain how to solve 8 + 2 using a number line. Remember we use hops for ones, and jumps for bigger numbers.

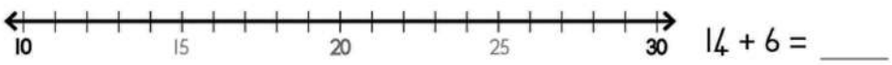
3. Zibalo explains how to subtract by **counting backwards on a number line** (She generalises). You must specialise. Complete the number line for 7 - 3

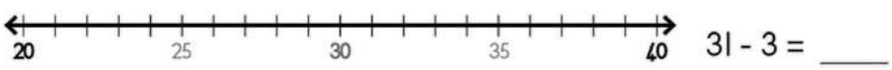


4. Number lines don't always start at 0.
   Complete the number line calculations using a **counting on (in 1s) strategy.**

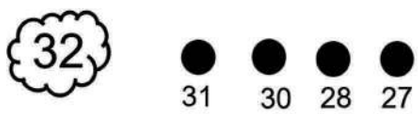



Draw big dot at the first number. Hop forward to add.
Hop backward to subtract.




5. Now answer the same calculations as in question 3, but use a **number picture** to show the **counting on (in 1s) strategy.**

| | |
|---|---|
|   | 32 – 4 = |
| | 14 + 6 = |
| | 31 – 3 = |

6. **Investigate: Magic Squares**
In a magic square the numbers in each row, column and diagonal add up to the same number.

Complete these magic squares.

The magic number is **27.**

| | | |
|---|---|---|
| | 5 | 12 |
| | 9 | |
| | | 8 |

The magic number is **30.**

| | | |
|---|---|---|
| | | |
| | 10 | |
| 4 | 18 | |

7. Complete the maths talk in an African language.

**Let's Talk Math in English!**

| NgesiXhosa sithi: | In English we say: |
|---|---|
| Sinokucinga ngamanani njengezinto, njengama-apile, abantu okanye iinkwenkwezi. | We can think about numbers as things like apples, people and stars. |
| | We can think about numbers as collections of things. Like apples in a bag. Like people in a taxi. |
| | We can add. We put the biggest number in our head. We count forward to add ones. |
| | We can subtract. We put the first number in our head. We count backwards to subtract ones. |

**MAT 1**

**Playful engagement** to search for and develop mathematical insight

**MAT 2**

**Reason mathematically**

## Task: 2.1.3

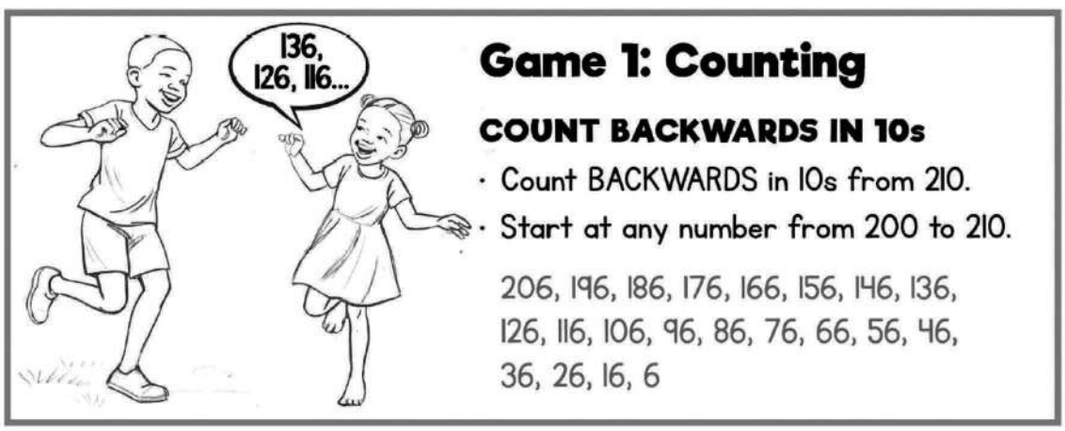
### Two minute tango – the small, medium and large number songs

In this *Addition and Subtraction I* module we expect you to **develop your own fluencies** so that you can rapidly recall known facts. As adults you can build your fluencies quickly. Children slowly develop their fluencies as they move from one grade to the next. **Each week you will practice two-minute tangos.** Each week the quiz will include timed mental fluency sections.

| | | | |
|---|---|---|---|
| **1** | 53 + 1 = _____ | **11** | _____ = 37 + 10 |
| **2** | 1 more than 47 is _____ | **12** | 100 more than 47 is _____ |
| **3** | 1 less than 47 is _____ | **13** | 100 less than 347 is _____ |
| **4** | 53 subtract 1 = _____ | **14** | One less than 800 is _____ |
| **5** | If I increase 69 by 1, I get _____ | **15** | 672; 772, 872, _____ , |
| **6** | 53 + 10 = _____ | **16** | 74; 64; _____ ; 44; 34 |
| **7** | 10 more than 47 is _____ | **17** | _____ + 10 = 33 |
| **8** | 10 less than 47 is _____ | **18** | 84 – _____ = 74 |
| **9** | Subtract 10 from 53 _____ | **19** | If I increase 69 by 100, I get _____ |
| **10** | If I increase 69 by 10, I get _____ | **20** | 53 + 100 = _____ |

For counting learners need to be able to use the:

- **Small number song** to count on and back in ones: 1, 2, 3, 4 ...
- **Medium number song** to count on an back in tens: 10, 20, 30, 40,...
- **Large number song** to count on and back in hundreds: 100, 200, 300, ...



## SUMMARY 2.1

- Explain how young children learn to add and subtract
- List the 5 levels of the calculation strategies.
- Explain the difference between **count on** and **count all**.
  Use an example of 17 – 4 = …You should use number pictures or number lines, and explain in words.

**MAT 4**

**Reflect** for action

# A 'GO-TO' STRATEGY FOR ADDING AND SUBTRACTING

Key Ideas

By the end of this umthamo you should be able to:

1. Describe a good go-to strategy for adding and subtracting
2. Explain and use the **jump strategy – where you break down the second number using expanded notation** - for adding and subtracting, using:
    a. Number pictures;
    b. Number lines; and
    c. Number sentences.

## What is a 'go-to' strategy?

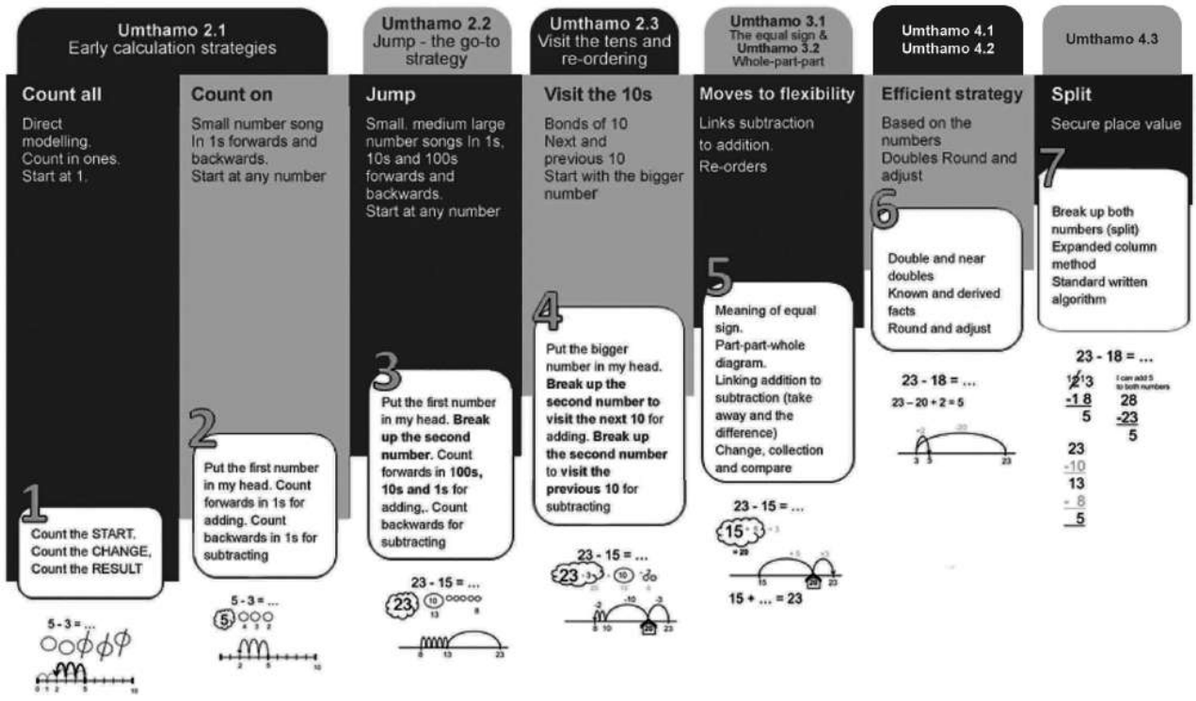
A 'go-to' is something that can be "relied on and regularly turned to for expert knowledge, skill, or performance". **A good go-to strategy:**

1. Can be used by the whole class (all learners);
2. Is easy to explain;
3. Works every time;
4. Works in a similar way for adding and for subtracting;
5. Builds a deep understanding of place-value;
6. Does not need special equipment or special tricks/skills.

**NA 3.2**

Describe the range of increasing sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

As a teacher you encourage children to work efficiently and to use different strategies. But it is important that there is **at least one calculation strategy that every learner in your class can use and can explain how to use**. The go-to strategy should be the strategy that they trust.



The **break down the second number strategy** (also known as the **jump strategy**) is an excellent go-to strategy for adding and subtracting. It builds on the **count on** strategy.

To use the **jump strategy** and **break down the second number,** learners need the following fluencies:

- To count forwards using the **small number song** from one (count all).
- To count forwards and backwards using the **small number song, from any number** (count on in 1s).
- To count forwards and backwards using the **medium and large number songs, from any number** (count on in 1s, 10s and 100s).
- To **break down a number** into 100s, 10s and 1s.

These are not difficult skills – and all children can learn them. This makes the **break down the second number (or jump)** strategy and good go-to strategy.


**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

Children should be able to start at any number and count on using:

- The **small number song** in 1s: 12, 13, 14, ....
- The **medium number song** in 10s: 12, 22, 23, ...
- The **large number song** in 100s: 12, 112; 212; 313.

Now they can use the same **count on** calculation strategy with big numbers. They put the first number in their heads, break down the second number to count on in 100s, 10s and 1s. To use the **break down a second number or jump strategy,** children need to be able to break down the second number. they need to know that 34 is 30 and 4.

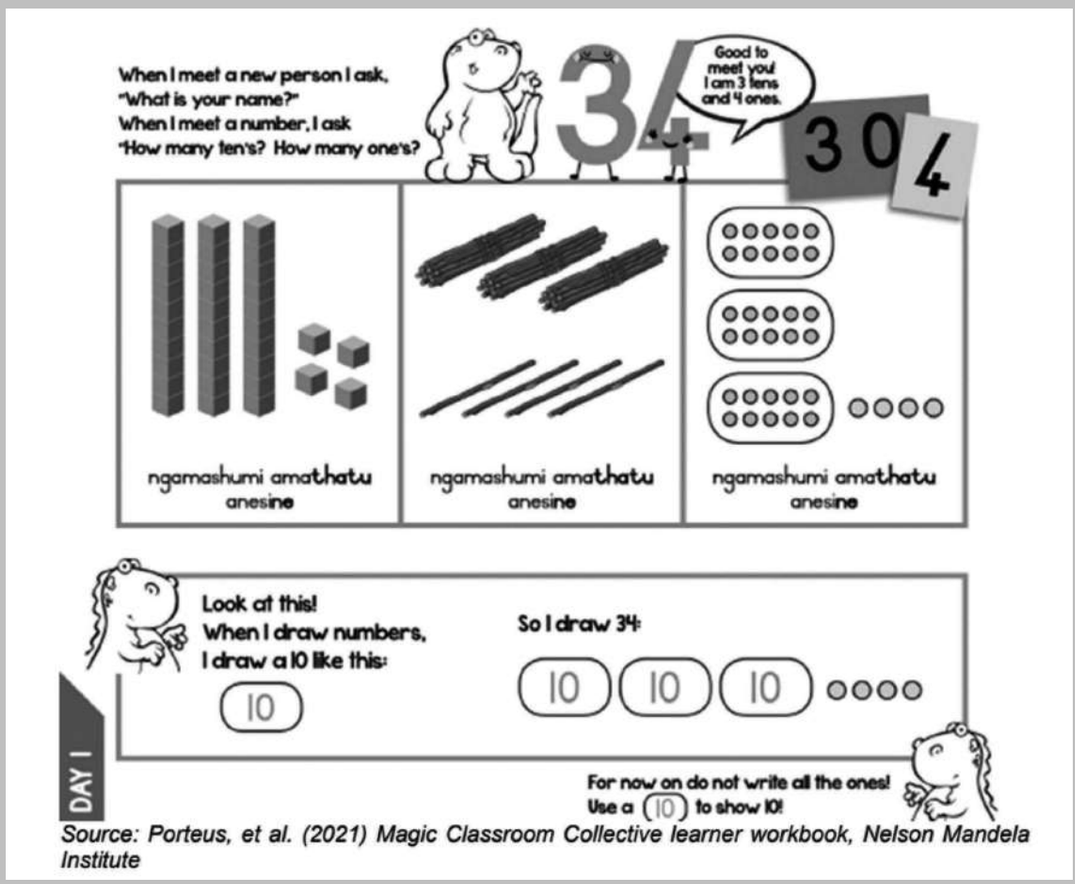
Game


SCAN ME

**Smart counting**

Learn more about smart of strategic counting. This video is in isiXhosa but has some English explanations too.

Activity for learners




*Source: Porteus, et al. (2021) Magic Classroom Collective learner workbook, Nelson Mandela Institute*

The **jump strategy** where you **break down the second number** works in the same way for subtracting and for adding. The only difference is that

- for **adding the counting is forwards**, or jumping is forwards,
- but for **subtracting the counting is backwards** or jumping is backwards.

To use the jump strategy, children need to be able to break down 2 digit numbers into 10s and 1s.

Break down into 10s and 1s.

| | |
|---|---|
| 34 = 10 + 10 + 10 + 4 | 26 = _____ |
| 34 = 30 + 4 | 26 = _____ |

**NA 2.1**

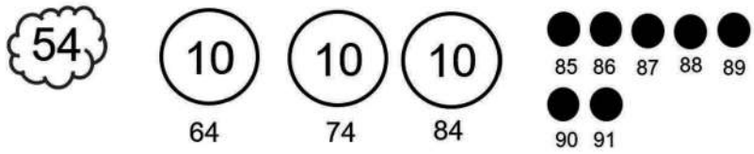
Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

## COUNT FORWARDS FOR ADDING

What does the **jump strategy** - where you **break down the second number -** look like in a number picture or a number line?

**Number picture**

54 + 37 = ...

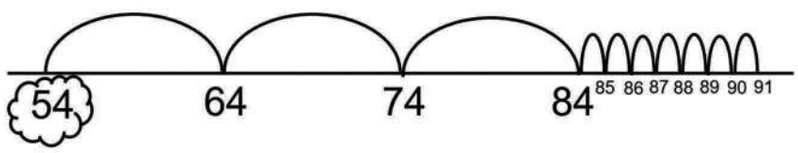



54 + 37 = 91

Put 54 in my head
Break down the second number into 30 and 7
Count forwards in 10s.
Count forwards in 1s

**Number line**

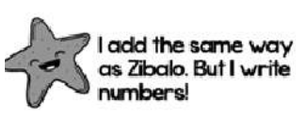
54 + 37 = ...




54 + 37 = 91

Start at 54. Leave space on the right.
Break down 37 into 30 and 7.
Jump forwards in 10s
Jump forwards in 1s

**Number sentences**




54 + 37 = 91

54 + 10 + 10 + 10 + 7
64 74 84 **91**

Write down 54.
Break down the 37 in 3 tens and 7 ones.
Count on in 10s.
Count on in 1s.

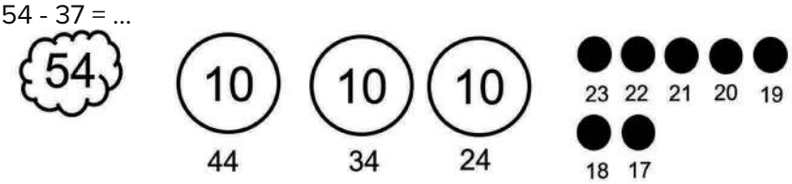
The **break down the second number strategy** is also called **the jump strategy** (as you hop in ones, and jump in 10s and leap in 100s).

## COUNT BACKWARDS FOR SUBTRACTING

What does the **jump strategy** where you **break down the second number** look like in a number picture or a number line?

**Number picture**

54 - 37 = ...

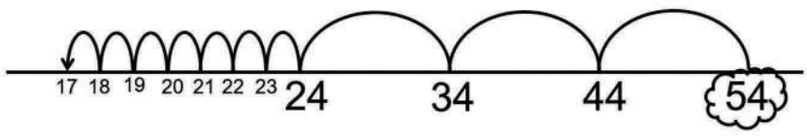



54 - 37 = 17

Put 54 in my head
Break down the second number into 30 and 7
Count backwards in 10s.
Count backwards in 1s

**Number line**

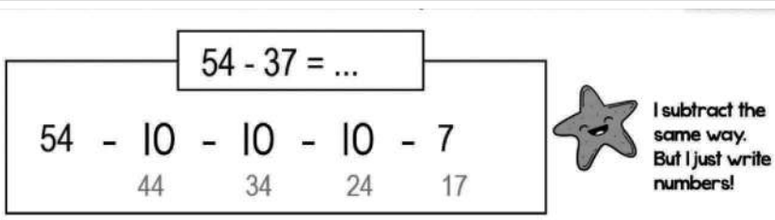
54 - 37 = ...




54 - 37 = 17

Start at 54 (on the right). Leave space on the left.
Break down the second number into 30 and 7.
Jump backwards in 10s.
Jump backwards in 1s.

**Number sentence**







It takes time for children to start to draw confidently and use number pictures and number lines to show their working.

Game

**1,2,3 Show!**

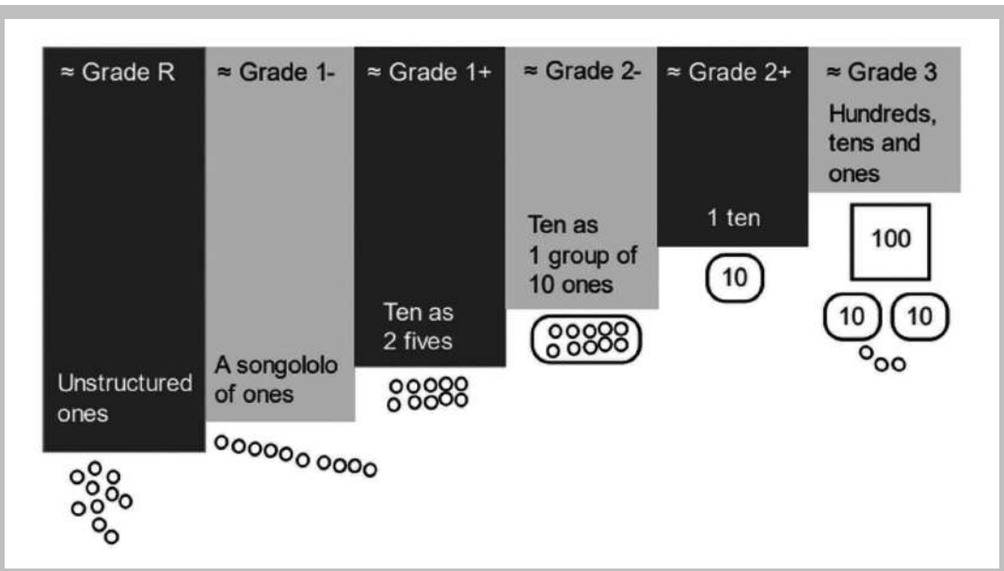
SCAN ME

Children practice quick addition with their fingers. You can also do subtraction.

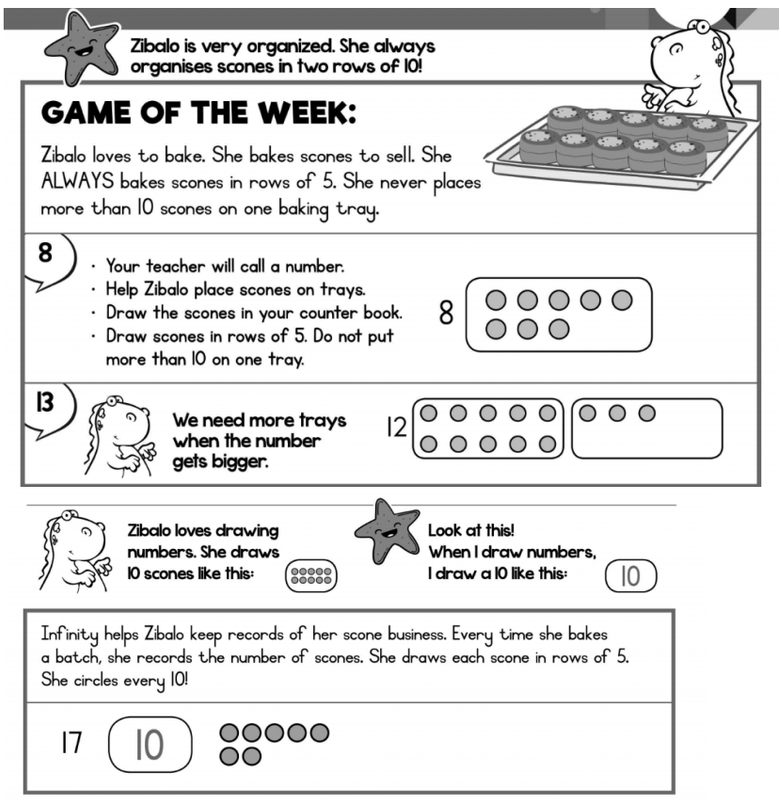
## Number pictures

It takes time for children to learn to use number pictures. Children slowly develop their **number pictures** to be more structured.

Diagram




At first children have to be coached to draw 1s using a ten frame. They can structure the numbers into 2s or 5s. They circle each group of ten 1s.



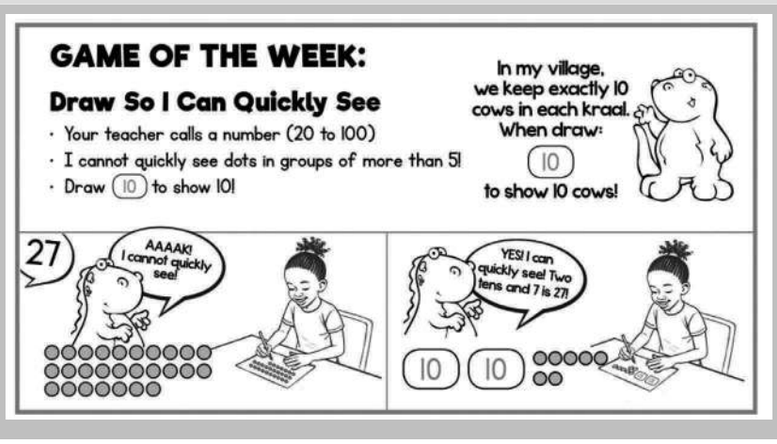

**MAT 2**

**Reason mathematically**

**NA 2.1**

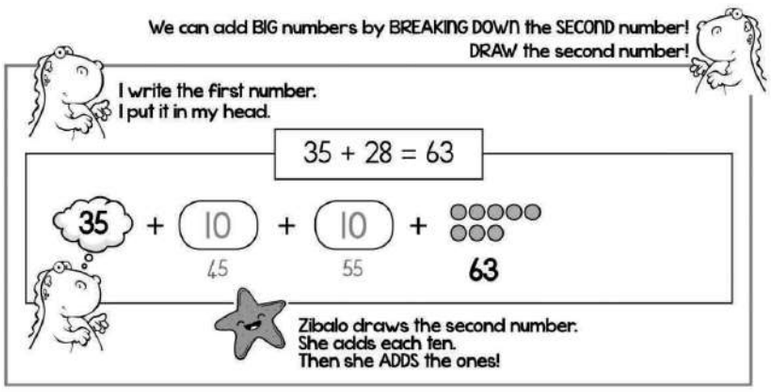
Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

Activity for learners

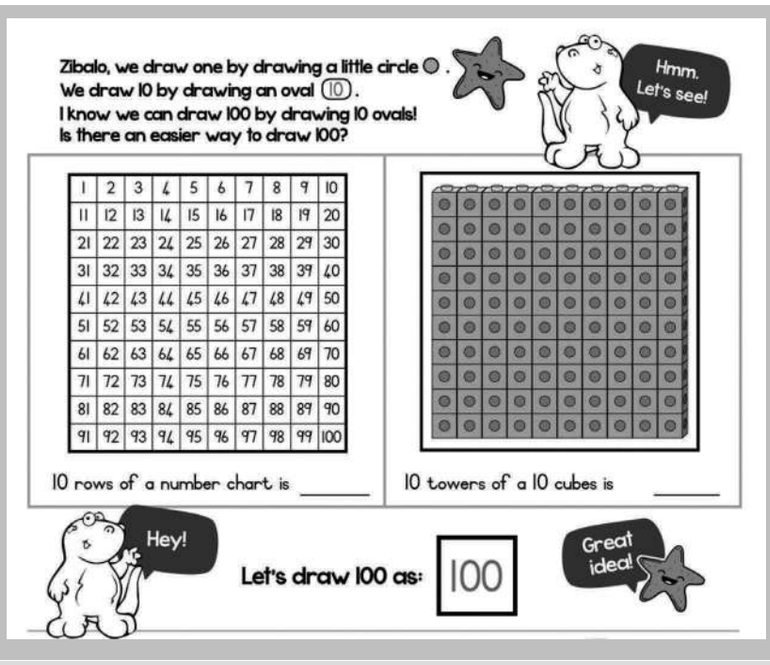



When they are used to this way of quickly showing a number, instead of drawing 10 ones, they draw 1 ten (and circle it). Later (Grade 3) they start using a square to show 100.

To use the **break down the second number** strategy with **number pictures**, children write the first number, then break down and draw the second number.



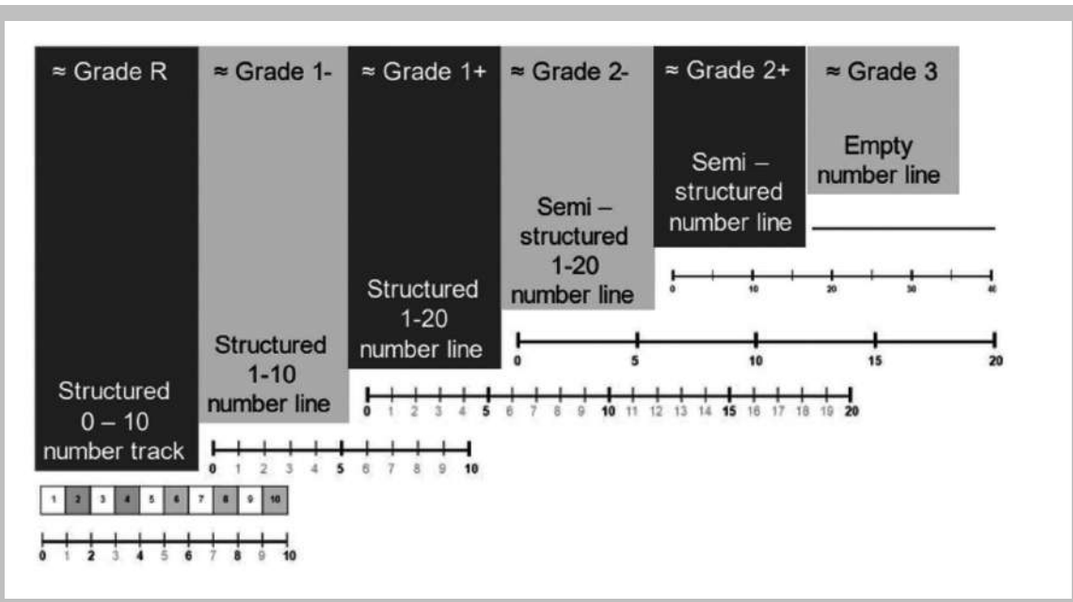

Activity for learners



## Number lines

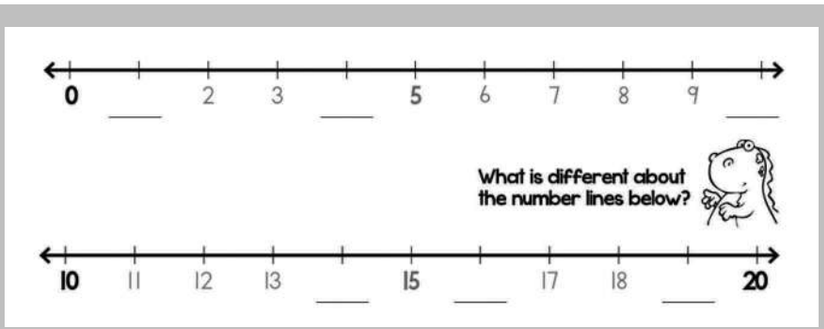
It takes time for children to learn to use number lines. Children slowly develop their **number lines** to be less structured.

Diagram




You should carefully choose the numbers to add and subtract, as well as the representation that they use. At first children start adding with numbers between 0 and 10. Then this shifts to 0 to 20. They use a structured number line, and get used to completing a number line by putting the numbers in order – with smallest on the left and biggest on the right.

Activity for learners




Children need to learn that the number line can start at any number. They imagine where 0 is.

In grade 1, the number that is added on is very small (add on 1 then add on 2 etc). They learn that:

- **One more than** is the same as **adding one** – hop forwards.
- **One less than** is the same as **subtracting one** – hop backwards

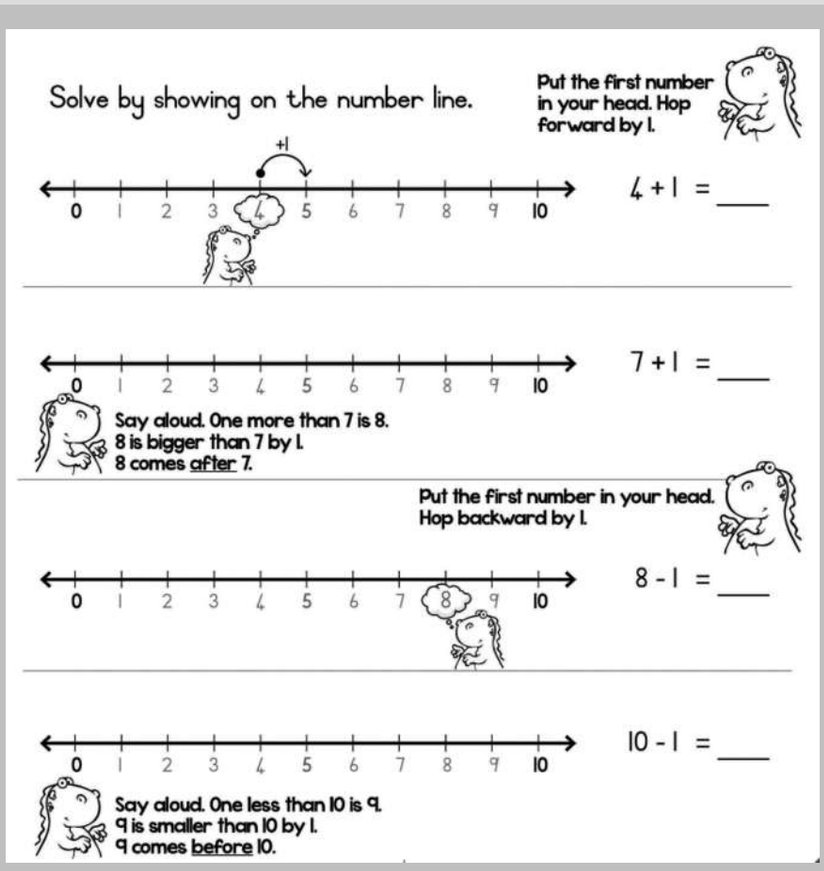
**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

**NA 8**

**Overarching ideas** for school arithmetic

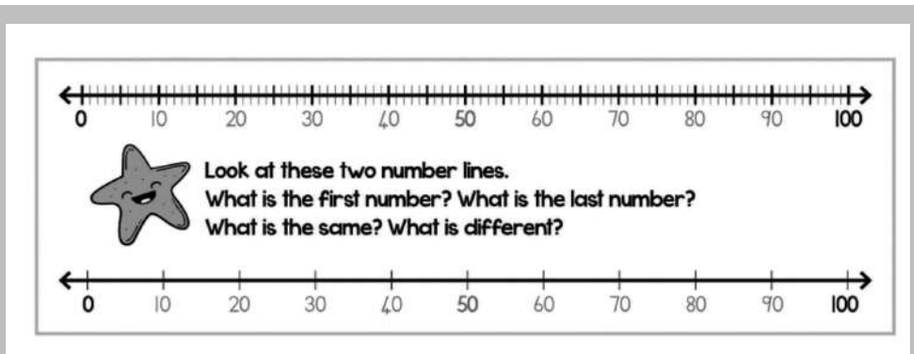
Activity for learners




Notice that by using number lines they are ordering the numbers. They learn what comes before and after. They must count backwards and forwards in 1s (the small number song). This is the **ordinal** meaning of number.

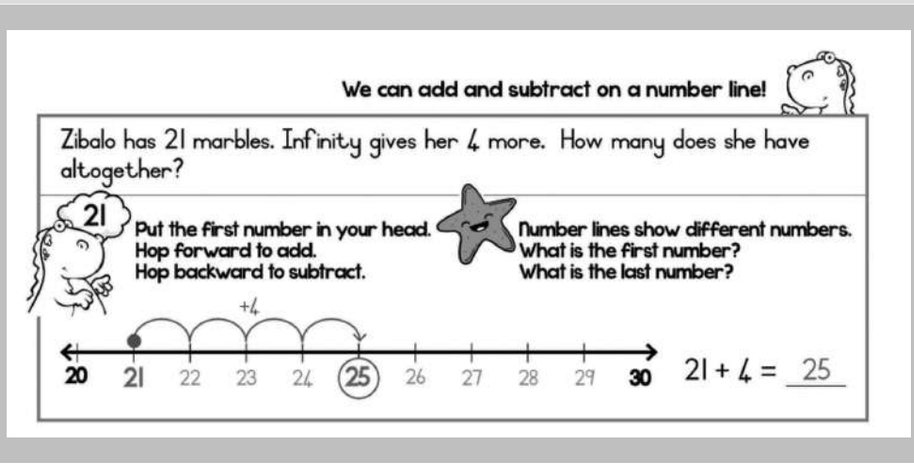
Later (Grade 2) children start working with the medium number song in 10s. Their structured number line is in 10s.

Examples



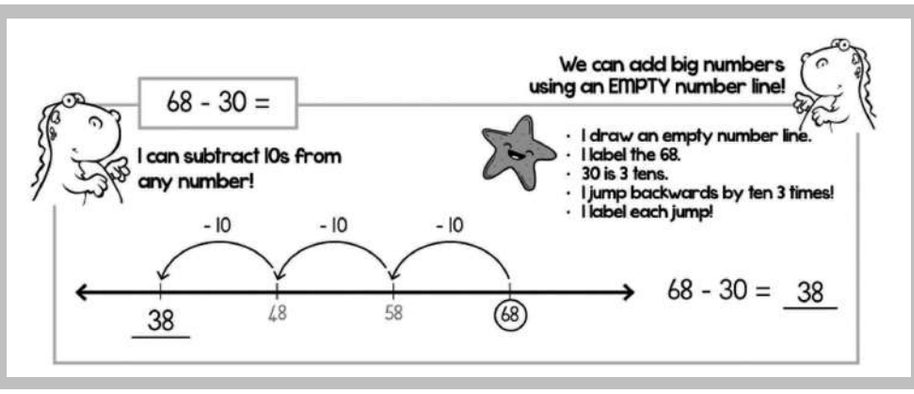

At first all the 1s are marked on the number line. Later the marks for the 1s are removed. They can add and subtract on number lines where only the tens are shown.

Examples




As children get used to number lines that start at any number, they start working on an empty number line (usually by about Grade 3).

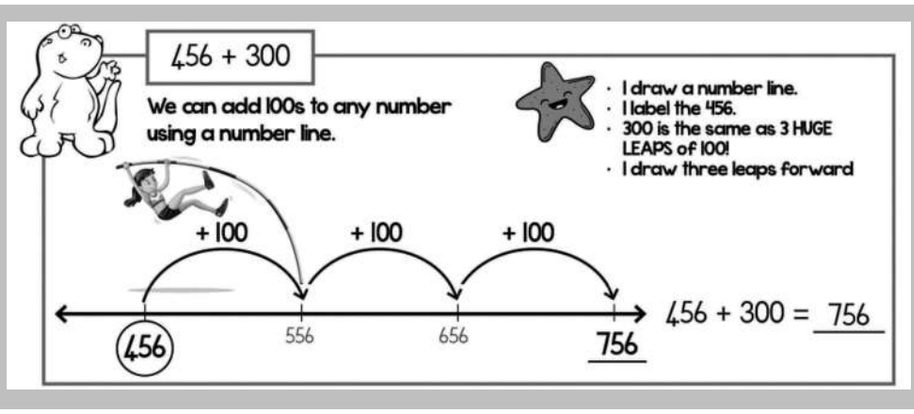
Examples




Notice that subtracting 30 is the same as subtracting 3 tens. This is counting back using the medium number song (in tens). Subtract 30 = subtract 10, subtract 10, subtract 10 = subtract 3 tens

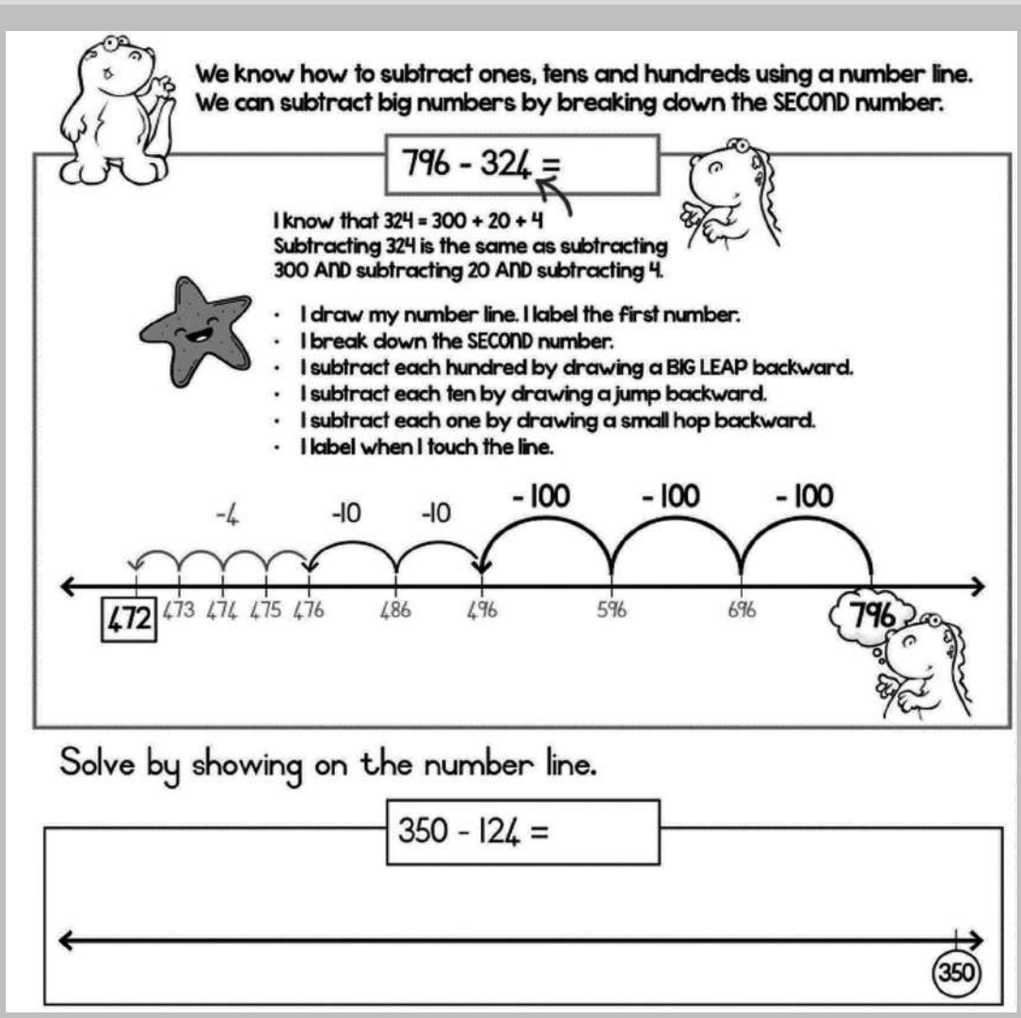
Later (Grade 3), children can use a number line for adding or subtracting hundreds.

Examples



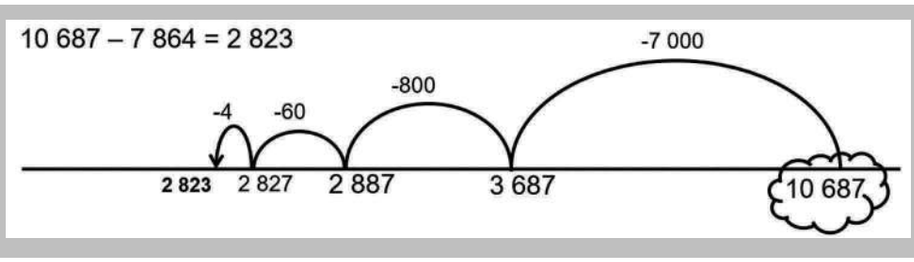

Now children can make HUGE LEAPS of 100s, BIG JUMPS of 10s and SMALL HOPS of 1s.

Activity for learners




By intermediate phase children who have developed their mental map of a number lines can leap in millions, 100s and 10s of thousands, thousands, hundreds, tens and ones. They don't need to jump one hundred at a time, but they keep track of their mental calculating using the empty number line.

Examples




The **jump strategy** - where you **break down the second number** - is always the same:

- **For adding:** Put the first number in your head. Break down the second number (using expanded notation) and add each part. This means you **count on o**r **jump forwards.**
- **For subtracting:** Put the first number in your head. Break down the second number (using expanded notation) and subtract each part. This means you **count backwards or jump backwards.**

Lets talk maths

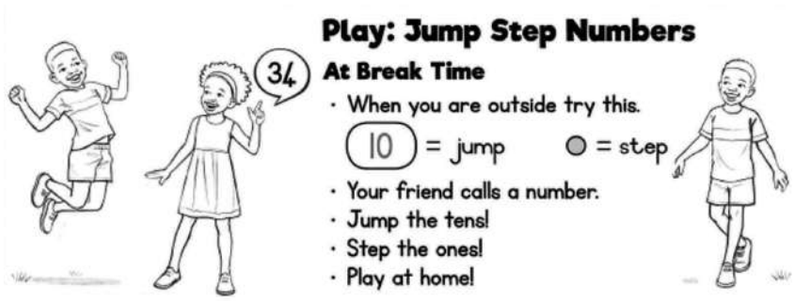
Let's Talk Math in English!

| In isiXhosa we say: | In English we say: |
|---|---|
| | Solve by breaking down the SECOND number. |
| | Solve by drawing pictures. Draw (10) to show 10. |
| | Solve by showing on the number line. |
| | I know that **4** + **3** = **7**. Therefore I know that **40** + **30** = **70**. I know that **9** - **4** = **5**. Therefore I know that **90** - **40** = **50**. |
| | I know that **30** + **40** = **70**. Therefore I know that **35** + **40** = **75**. I know that **70** - **30** = **40**. Therefore I know that **76** - **30** = **46**. |

DAY 5

| | |
|---|---|
| | Draw the number. Draw (10) to show 10. |
| | Place value. |
| | The value of 3 in 34 is 30. |
| | The value of 4 in 34 is 4. |

## SOME IDEAS FOR YOUR CLASSROOM




### Play: Jump Step Numbers

**At Break Time**

- When you are outside try this.

(10) = jump ○ = step

- Your friend calls a number.
- Jump the tens!
- Step the ones!
- Play at home!

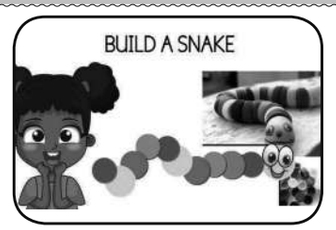
**GPM 3**

**Engage learners productively** in mathematics

**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

Game




SCAN ME

**Build a snake**

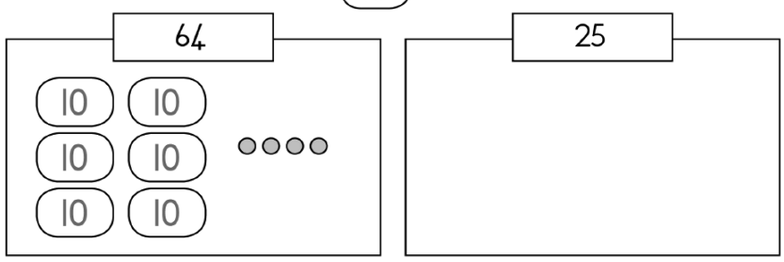
Use blocks or bottle tops to build a snake. This is for addition and subtraction. Download or copy the instruction cards

## Task: 2.2.1

Break down the second number, using number pictures.

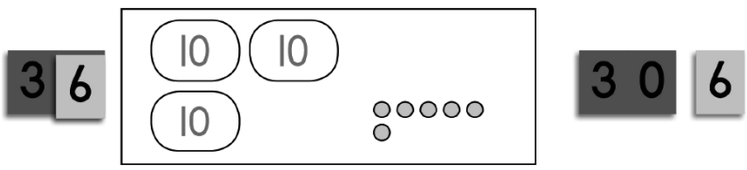
1

Draw the number. Draw (10) to show 10.




2

Draw the number. Write the place value cards.
Complete the number sentences.

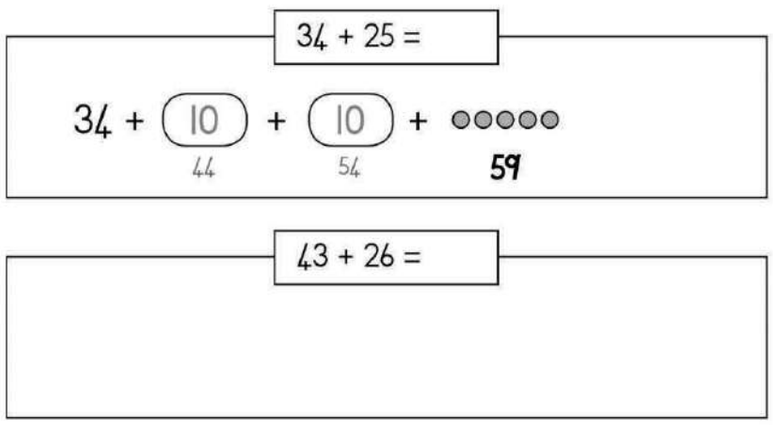



36 = 10 + 10 + 10 + 6 36 = 30 + 6


32 = _____ 32 = _____

3

Solve by drawing a picture. **Bandile drove 34 kms to shop and then 25 kms to visit his mother. How many kms did he drive altogether?**



4.


Usenathi had R75. She bought a book for R30. How much Rand does she have now?

R75 - R30 = R45

75 - 10 - 10 - 10

65 55 45

I write the first number. I put it in my head.

I DRAW the second number. 30 is 3 tens! I TAKE AWAY each 10!

63 - 20 =

5. Solve by drawing a picture:

78 - 37 =

78 - 10 - 10 - 10 - ●●●●●●●

Now you draw the second number!

68 - 32 =

68 -

6. Solve by breaking down the second number.

78 - 45 = 78 - 10 - 10 - 10 - 10 - 5 = _____

57 - 24 = 57 - _____ = _____

**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

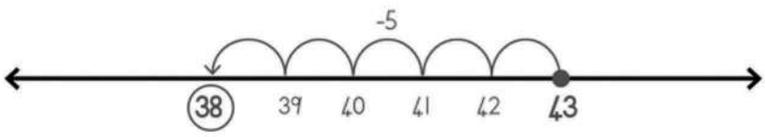
**NA 3.2**

Explain and identify **sets**, set membership, set relations, set operations and set partitions

## Task: 2.2.2

**Practice drawing and explaining the break down the second number strategy**

1. Complete these calculations on a number line.




43 - 5 = ____

57 + 4 = ____

65 - 6 = ____

77 + 5 = ____

2. Now generalise. Explain how you subtract using the **counting on strategy** on a **number line.**

| |
|---|
| |

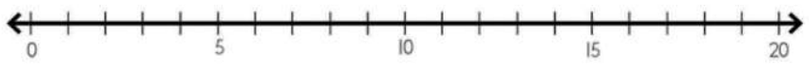
3. Show how you would use number pictures to solve using the **count on strategy.**

| | |
|---|---|
| | 57 + 4 = |
| | 65 - 6 = |

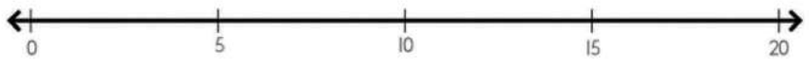
Few student teachers have had an opportunity to develop their mental map for number lines. In this task we give you lots of practice working on structured and empty number lines.

4. Draw a dot and write the number on the line.

12




8

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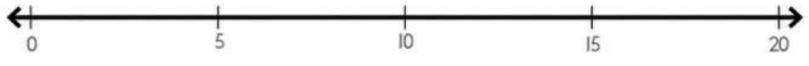


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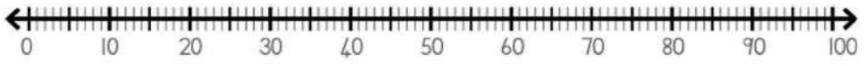


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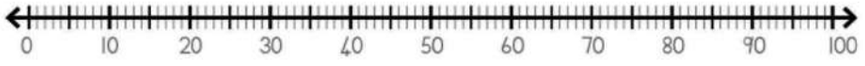


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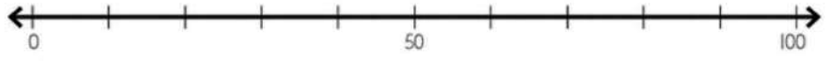
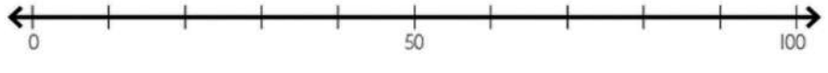


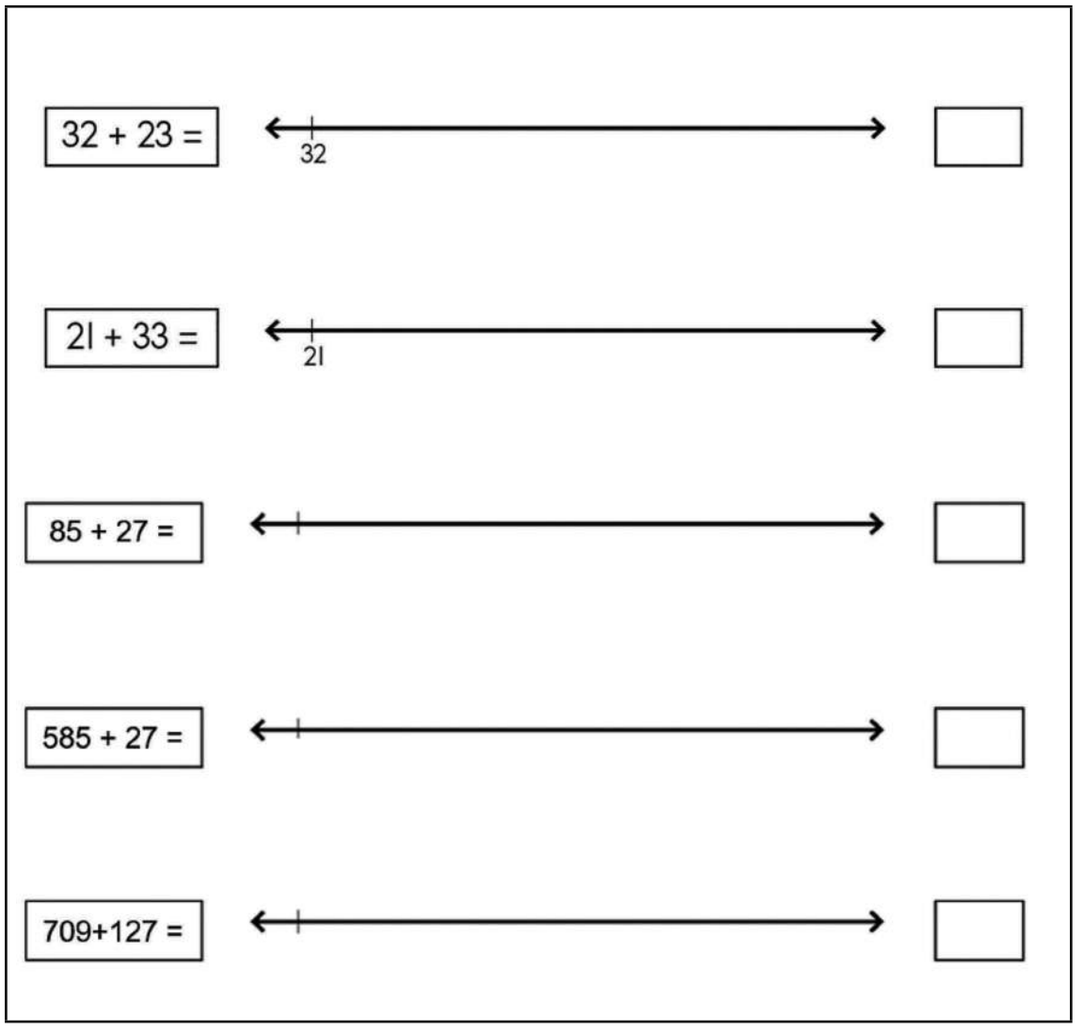

75




5. Show how you would use **number pictures** to solve

| | |
|---|---|
| | 112 - 36 = |
| | 104 - 38 = |

6. Add by showing the **break down the second number strategy on the number line.** Label your jumps. Label where you land.




7. Write down what you would say to explain how to add using **break down the second number on a number line.** This is also known as the **jump strategy.**

**MAT 4**

**Reflect** for action

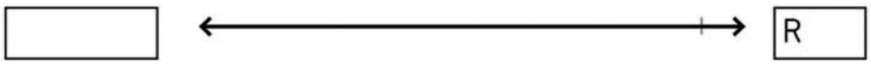
8. Subtract by showing on the **empty number line**. Label your jumps. Label where you land.

62 - 34 = ____

762 - 34 = ____

762 - 134 = ____

Bev has R55. She buys a magazine for R23. How much money does she have now?

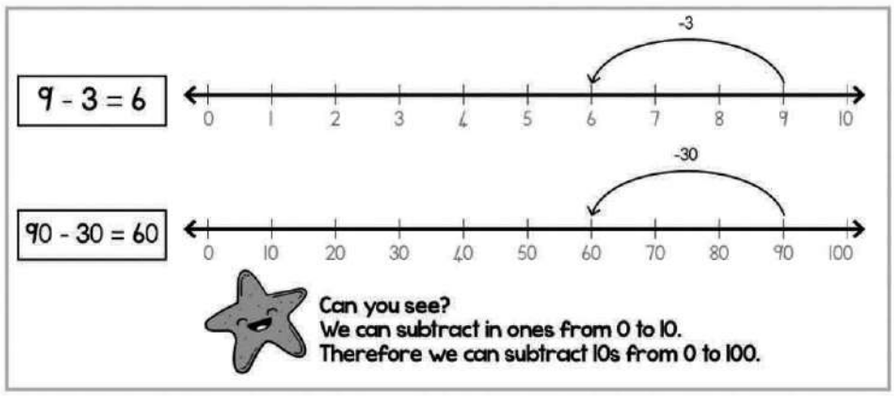



9. Write down what you would say to explain how to subtract using **break down the second number** on a number line. This is also known as the **jump strategy.**

10. Solve 62 - 34 = _____ using break down the second number with number pictures.

## Task: 2.2.3

Adding and subtracting bigger numbers




1. Solve by showing on a semi-structured number line. Label your jumps. Label where you land.

80 – 30 =

800 – 300 =

80 000 – 30 000 =

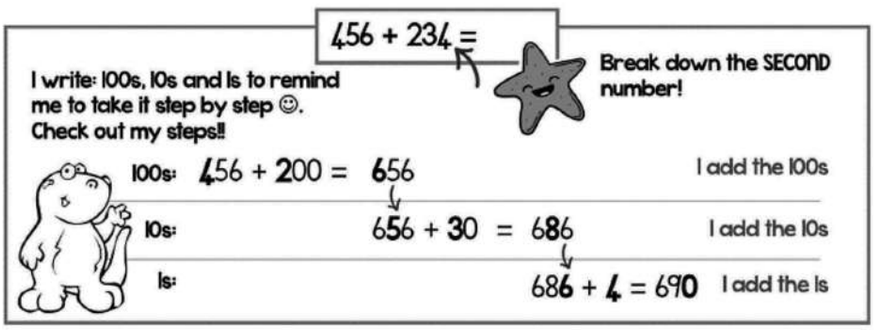
2. Solve by showing on an empty number line. Label your jumps. Label where you land.

281 – 34 =

8 281 + 234 =

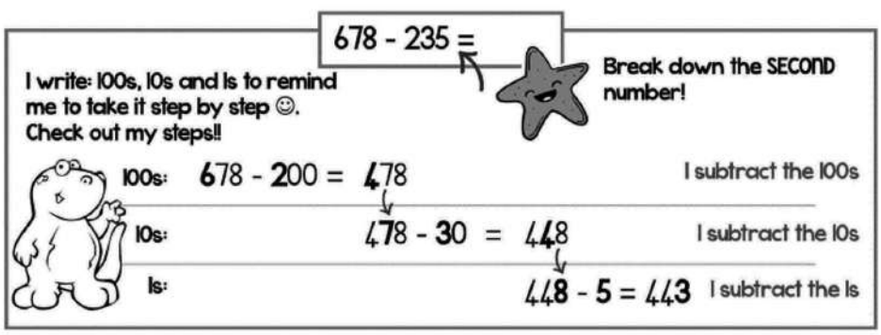
8 281 - 234 =

3. You can also show the breaking down the second number strategy using number sentences. (This is only encouraged from the second half of grade 3).
   Complete addition with the **break down the second number strategy using number sentences.**




Solve by breaking down the SECOND number. Show each step of your work.

562 + 325 =

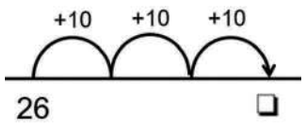
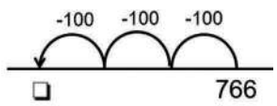
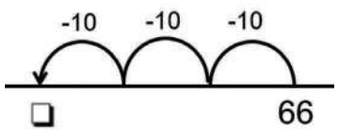
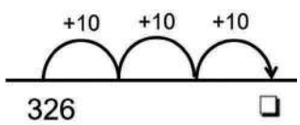



Solve by breaking down the SECOND number. Show each step of your work.

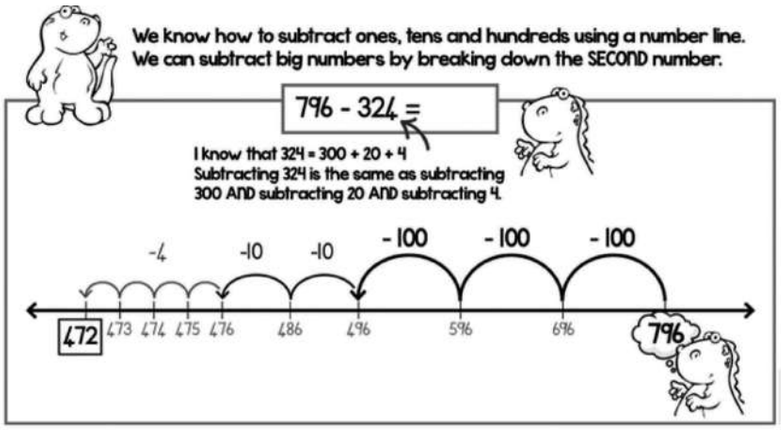
897 - 325 =

## Task: 2.2.4

Two minute tango – small, medium, large songs

| | | | |
|---|---|---|---|
| **1** | 33; 23; 33; 43; _____ | **11** | 100 more than 23 is _____ |
| **2** | 67; 57; 47; 37; _____ | **12** | 100 less than 787 is _____ |
| **3** | 8 + 10 = _____ | **13** | One less than 300 is _____ |
| **4** | 8 + 40 = _____ | **14** | 642; 742, 842, _____ , |
| **5** | 34 – 10 = _____ | **15** | _____ = 37 + 40 |
| **6** | 34 – 20 = _____ | **16** | _____ = 37 + 43 |
| **7** |   | **17** |   |
| **8** |   | **18** |   |
| **9** | Subtract 10 from 83 | **19** | 76 – 30 = _____ |
| **10** | If I increase 69 by 10, I get _____ | **20** | 76 – 34 = _____ |

The **jump strategy** – break down the second number, in hundreds, tens and ones.




**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

**NA 3.2**

Describe the range of **representations** and increasingly sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## SUMMARY 2.2

- You are teaching a class to subtract 3000 – 148 = ...
  Show what you would write on the board to guide children how to use the **jump strategy** – where you **break down the second number:**
  - using number pictures; and then
  - using number lines.

**Break down the second number (using number pictures)**
3000 – 148 = ...

| BLACKBOARD (What do you write/draw?) |
| --- |
| TEACHER TALK (What do you say?) |

**Break down the second number (using number lines)**
3000 – 148 = ...

| BLACKBOARD (What do you write/draw?) |
| --- |
| TEACHER TALK (What do you say?) |

## VISIT THE TENS

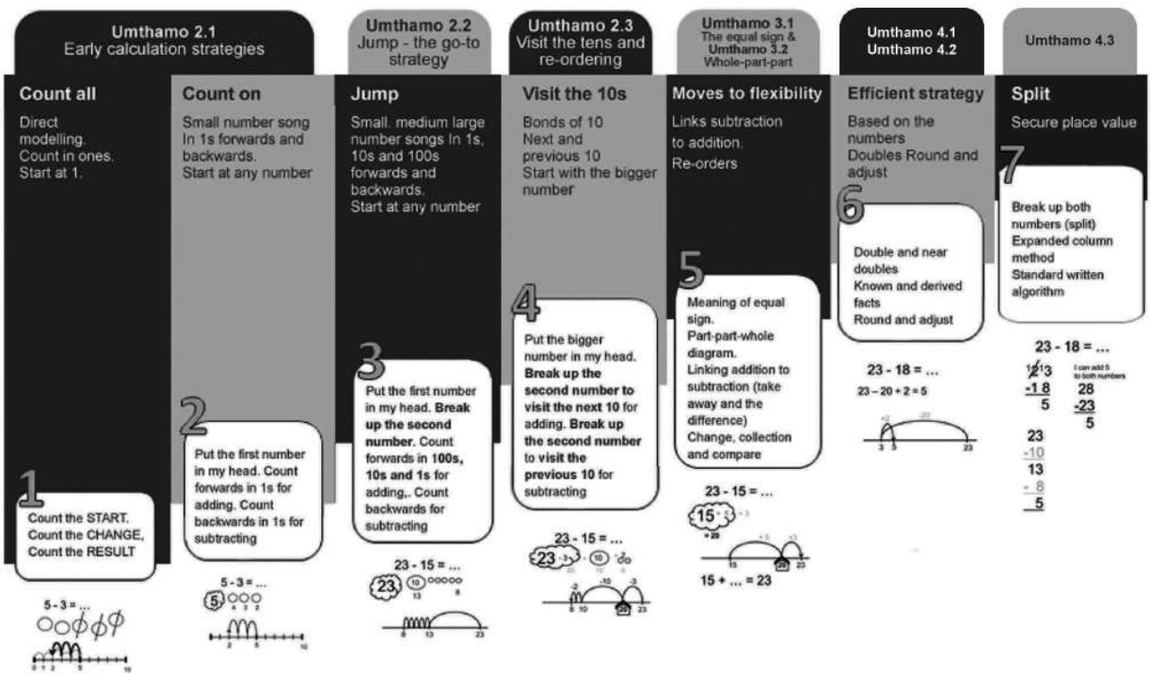
Key Ideas

By the end of this umthamo you should be able to:

1. Explain and use the break down the second number strategy **to visit the tens** for adding and subtracting, using:
- Number pictures; and
- Number lines

## LEVEL 4 VISIT THE 10S

Children **count all** and then **count on** in 1s. From the about the middle of Grade 1 they should be encouraged to **count on.** Towards the end of Grade 1, they start to use known facts (bonds of ten) to **visit the next and previous ten.** Once they know the small, medium and large counting songs (in 1s, 10s and hundreds) they can **count to calculate.**




In this umthamo we focus on **Level 4. Break down the second number to visit the ten.** For addition, you fill up or visit the next ten. For subtraction you takeaway to visit the previous ten.

To use the **visit the ten strategy,** learners need the following fluencies:

- Know their bonds of 10 (rapid call).
- Know all bonds (rapid call) to break down the second number to make a ten

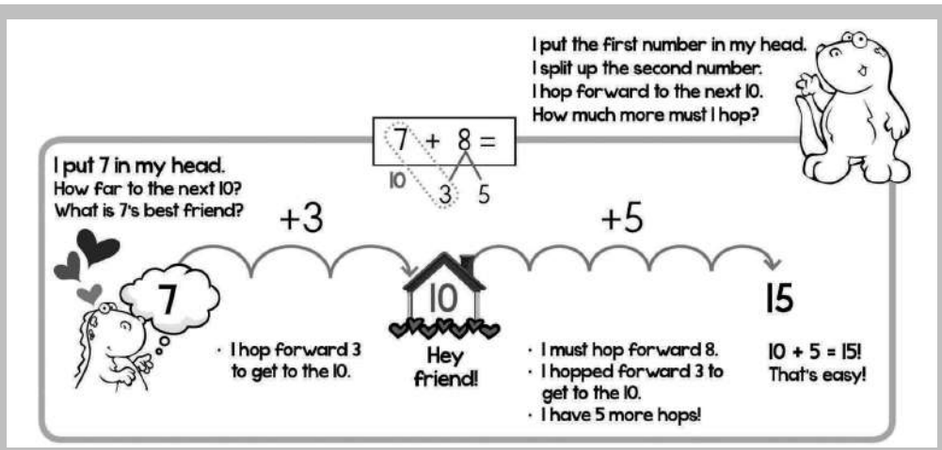
**NA 3.2**

Describe the range of **representations** and increasingly sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## Visit the next ten to add

**Visit-the-tens** is also called **jump-via-ten** or **bridge-through-tens**. The first number is kept whole, and the second number is broken up flexibly so that a jump can be made to a friendly ten (or multiple of ten) to make the calculation easier.

Activity for learners




Notice that the 10 is a friendly house to visit.

Lets talk maths

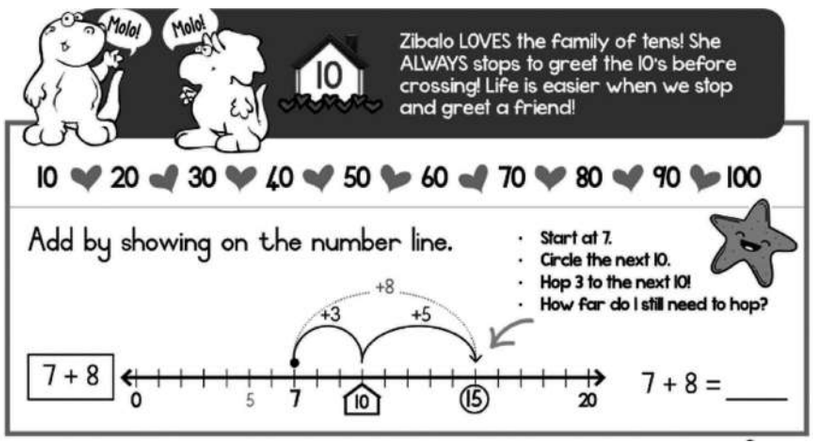
Notice also that they need to know that: 10 add 5 is 15.
In English, this is not so easy. 'Ten add five' = 'fif-teen' *[5 and 10].*
This may sounds like *fif-ty [5 tens].*
In isiXhosa, this is easier: 'ishumi dibanisa isihlanu' = 'ishumi elinesihlanu' *[10 and 5]*
*When the counting unit (tens) are explicit, then the language naming system is transparent.*

*Number names in isiXhosa are transparent. English number names are not transparent.*
*You can't hear the counting unit for 'eleven' [10 and 1] or 'twelve' [10 and 2] or twenty [2 tens].*

*How do counting names work in other languages? Are they transparent?*

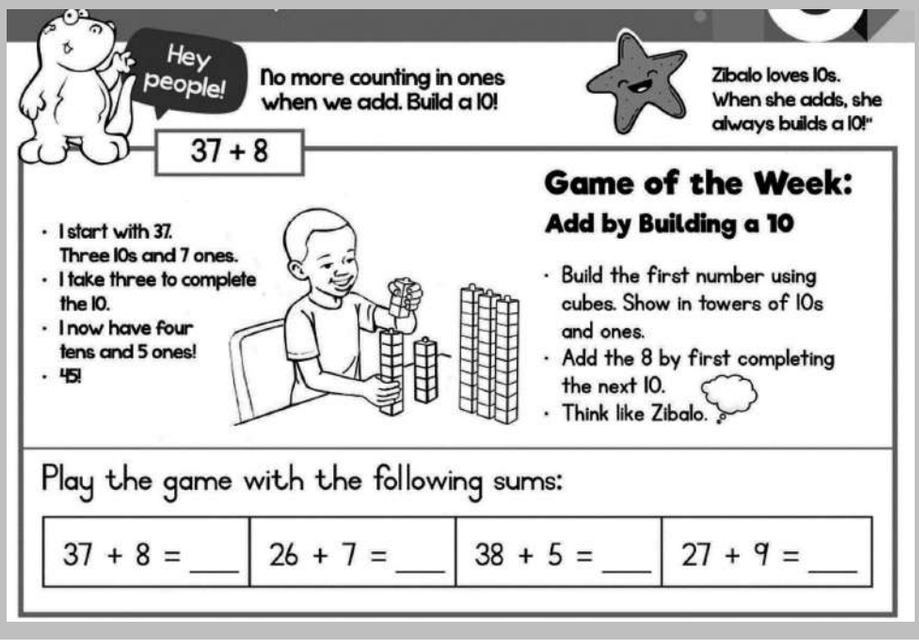
Visit the tens can be done using an empty number line. Learners must know "how far to the next ten".

One of the ways to **break down the second number** is to break it up to **visit the ten**.

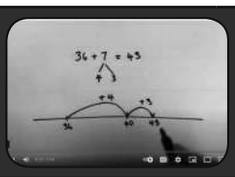


Activity for learners

A game of building towers of ten can help build the idea of visiting or making ten.




These videos are about **visiting the ten** as a calculation strategy using an empty number line. You can get more materials from the mental starters assessment project (MSAP) for Grade 3 in all South African languages – see page 152 of this book.



SCAN ME

**VISIT THE NEXT TEN FOR ADDITION**

| Visit the tens - 1 | Visit the tens - 2 | Visit the tens - 3 | Visit the tens - 4 | |
|---|---|---|---|---|
| | | | | Lesson 4 homework is Worksheet 1 |

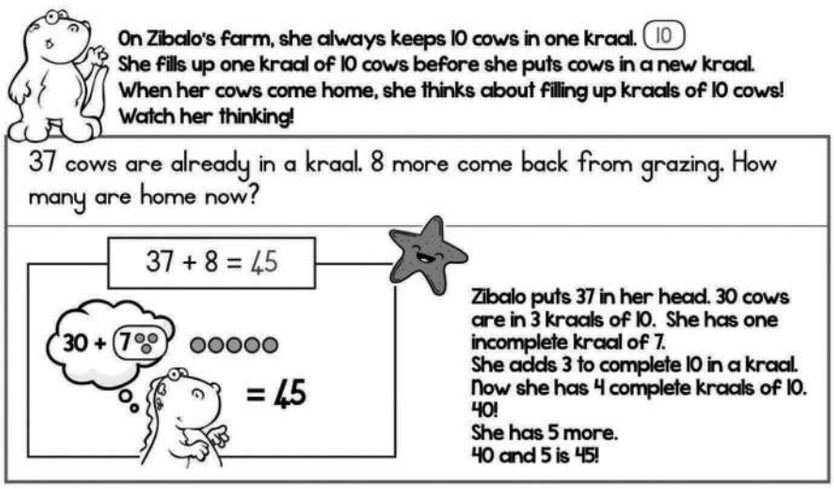
SCAN ME

**VISIT THE PREVIOUS TEN FOR SUBTRACTION**

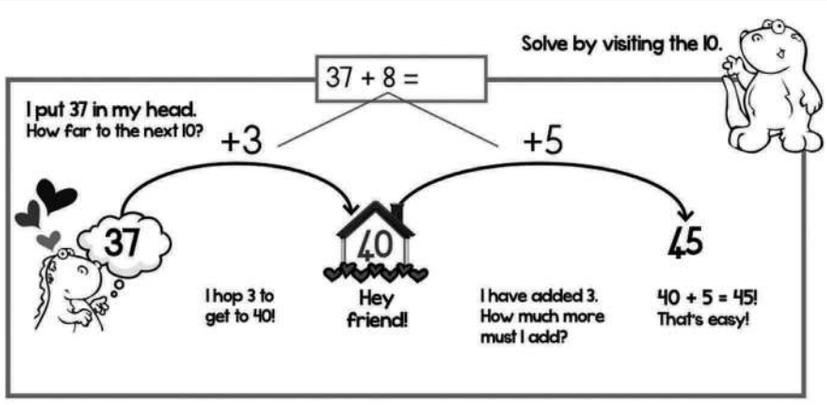
| Visit the tens - 5 | Visit the tens - 6 | Visit the tens - 7 | Visit the tens - 8 | |
|---|---|---|---|---|
| | | | | Lesson 8 Homework is Worksheet 2 |

You can encourage children to **visit the tens** using **number pictures** and **number lines** and **number symbols**

**Visit the tens – number pictures**




**Visit the tens – number lines**




**Visit the tens - number symbols.**

$37 + 8$

$= 37 + 3 + 5$

$= 40 + 5$

$= 45$

Put 37 in my head
The next multiple of 10 is 40.
I need 3 more to get to 40.
I break down 8 into 3 and 5.
37 and 3 get me to 40, and
40 and another 5 gets me to 45

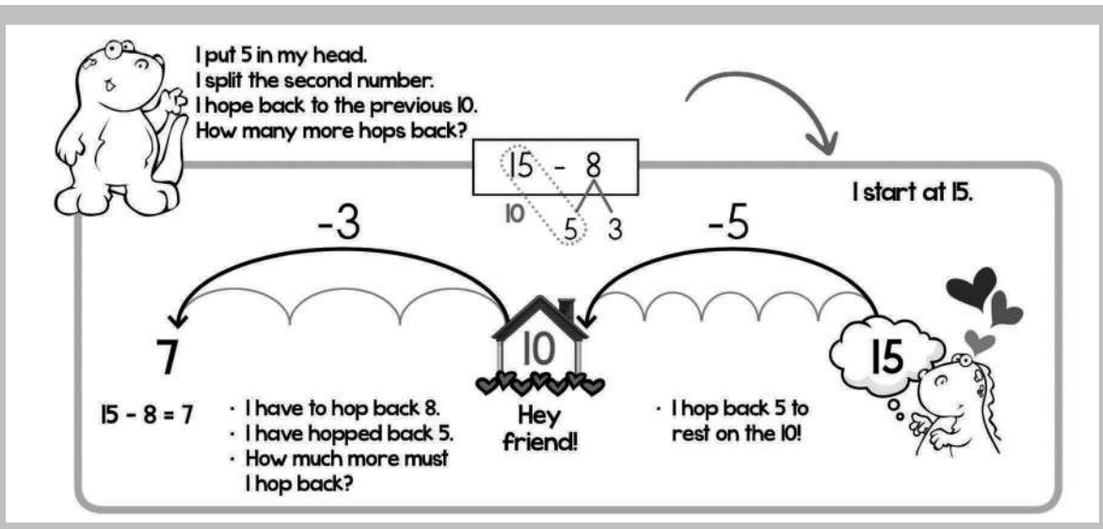
The **multiples of 10** (the medium number song) are **friendly numbers**. Stop and visit. Greet the ten then move on.

Molo 10! Sawubona 10! Dumela 10!
Xewani 10! Hello 10! Ndaa 10!
More 10! Sharp-sharp 10!

# VISIT THE PREVIOUS TEN TO SUBTRACT

## Visit the tens - subtraction

Examples



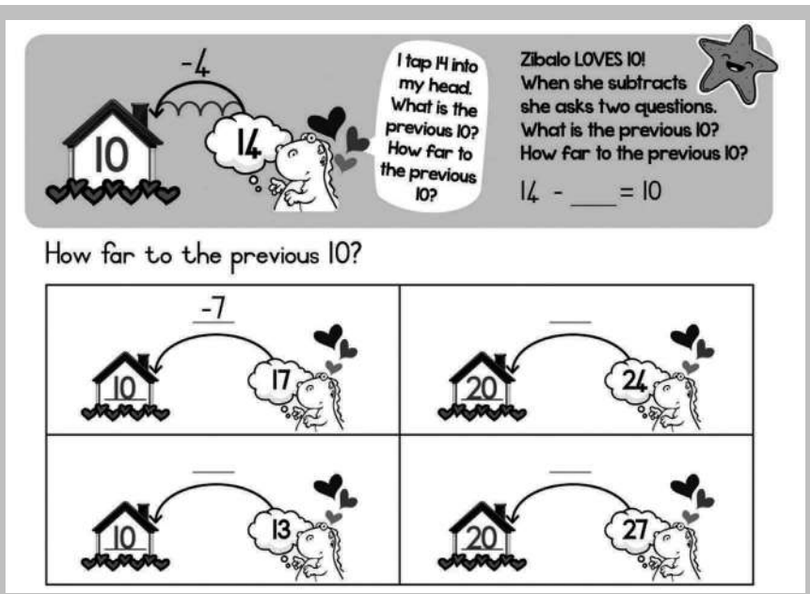

This is an example from Grade 2. Learners are encouraged to **hop** in ones. Later they **jump** in bigger numbers.

## Visit the tens - subtraction

Visit the tens can be done using "how far to the previous ten?" and the small counting sequence.

Visit the tens can be done using an **empty number line**. Learners must know "how far to the **previous ten**".

Activity for learners




**NA 8**

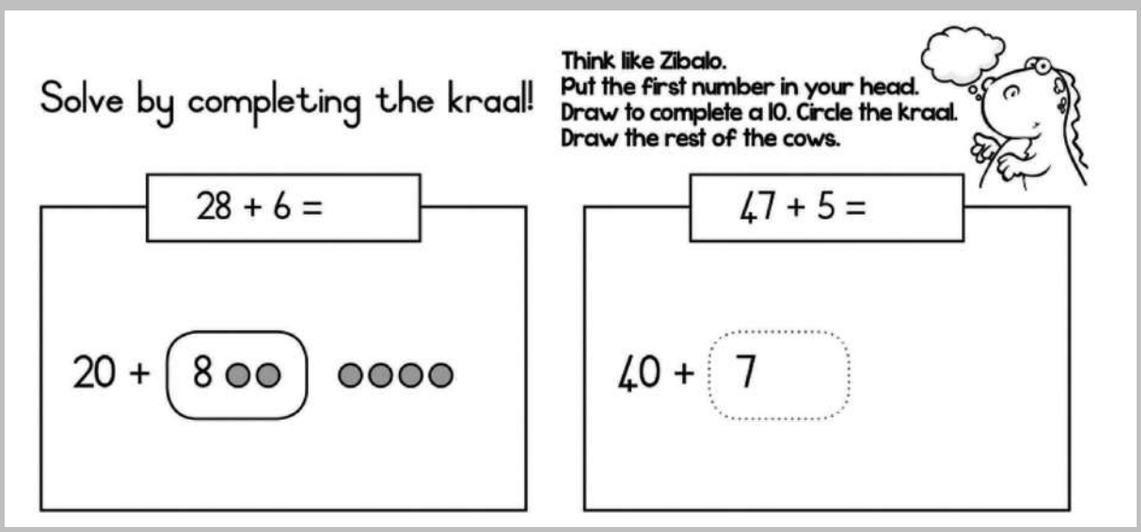
**Overarching ideas** for school arithmetic

## SOME IDEAS FOR YOUR CLASSROOM

Help your learners use **the counting pathway to visit the ten.** They use discrete objects to make a group of ten (like ten cows in one kraal).

Activity for learners

Use **number pictures** – 10 cows in a kraal – to help children "fill up the ten" in the **counting pathway**.




Notice that both examples start with the bigger number. You should include some examples that start with the smaller number (for example 6 + 28 = ...). Encourage learners to be flexible, and reorder. They can start with the bigger number as this is more efficient.

Help your learners use **the measurement pathway to visit the ten.** They use a continuous quantity that they must measure. (Like the length of a pathway measured in steps). Every tenth step is a safe or friendly house to visit. This is the medium number song, starting at 0: 0; 10; 20; 30; 40; etc....:

Lets talk maths

Masithethe Izibalo ngesiNgesi!

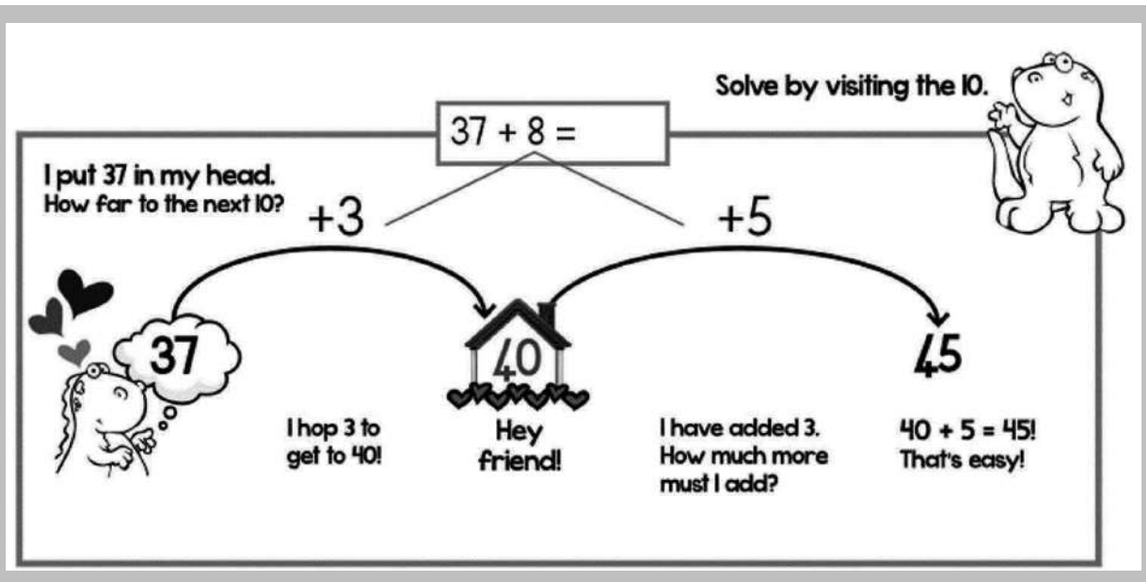
| NgesiXhosa sithi: | In English we say: |
|---|---|
| Singacinga ngamanani njengamanyathelo endleleni. | We can think about numbers as steps along a path. |
| unye ngaphezu | one more |
| unye ngaphantsi | one less |
| Sombulula. Bonisa umsebenzi wakho kumgca manani. | Solve. Show your work on the number line. |
| Ndizoba ichaphaza ndize ndibhale inani kumgca manani. | I draw a dot and write the number on the number line. |
| NDIDIBANISA imivo ngokutsibela PHAMBILI ngoo-l. | I ADD ones by hopping FORWARDS by 1s. |
| NDITHABATHA imivo ngokutsibela NGASEMVA ngoo-l. | I SUBTRACT ones by hopping BACKWARDS by 1s. |

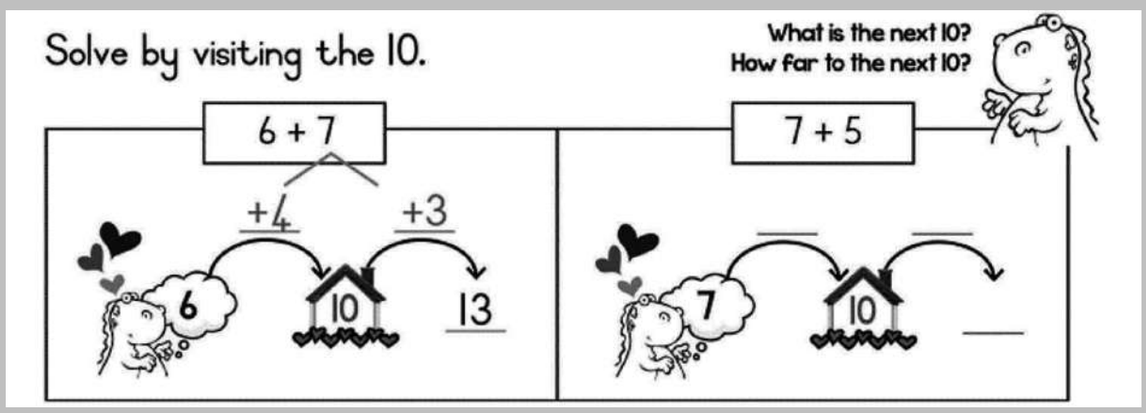
**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

Activity for learners

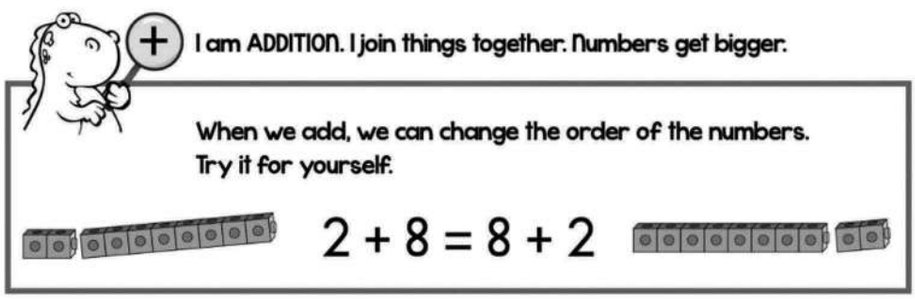
In the **measurement pathway**, use **number lines** to visit the friendly tens.




Remind learners to be flexible. Give them problems like 8 + 37 = ...
You can expect them to start with the bigger number, because 8 + 37 = 37 + 8.

Children need to be flexible and think about the numbers involved. It is more efficient to start with the bigger number. They can re-order for addition.



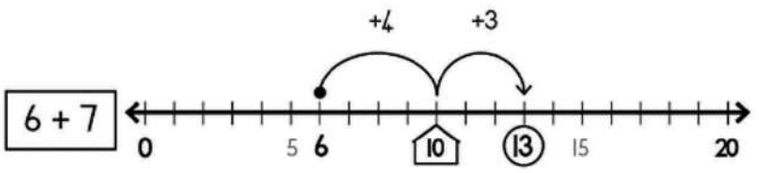

NA 7

Early algebra

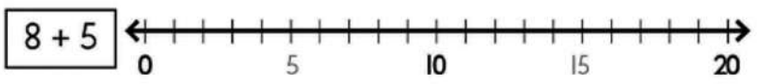
Activity for learners

Draw a house around the next 10.
Add by visiting the next 10.

**Hop to the next 10! How far do I still need to hop?**




6 + 7 = 13




8 + 5 = ____

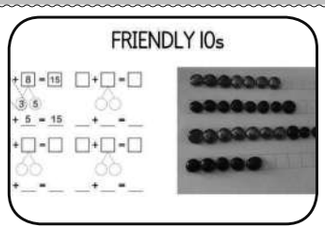
Lets talk maths

DAY 5

**Let's Talk Math in English!**

| NgesiXhosa sithi: | In English we say: |
|---|---|
| Uzibalo uyawathanda ama-10. | Zibalo loves the 10s! |
| UZibalo usoloko ewabulisa ama-10 phambi kokuba awele. | Zibalo always greets the 10s before crossing! |
| 37 + 8 | 37 + 8 |
| Gcina ama-37 entloko. | Put 37 in your head. |
| Ngubani i-10 elilandelayo? | What is the next 10? 40! |
| Ndixhuma ka-3 ukuze ndiphumle kuma-40 | I hop 3 to rest at 40. |
| Uxhumela phambili kangakanani? | How much further do you hop forward? |

Game




SCAN ME

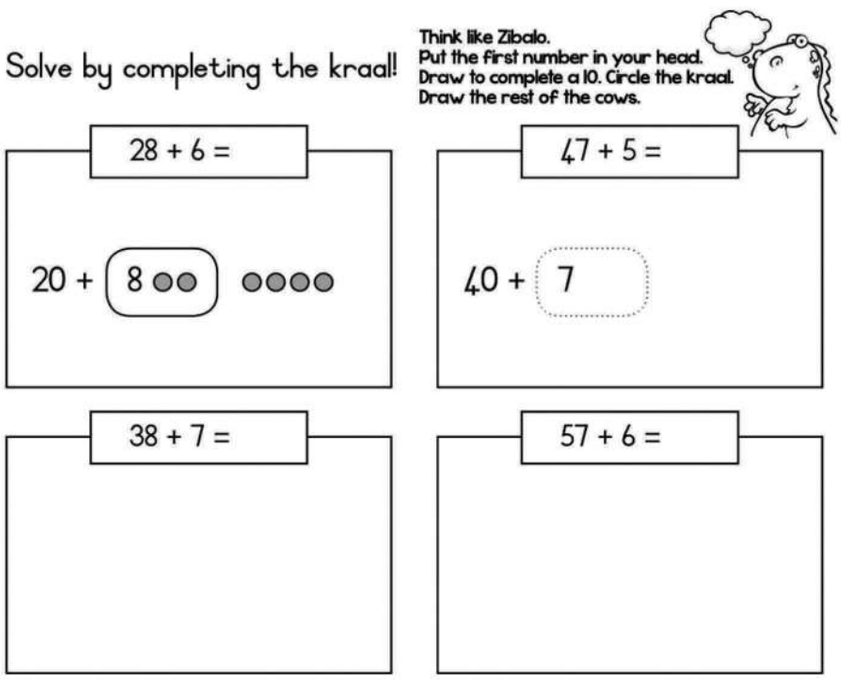
**Visit the tens**

Learners practice breaking down the second number to visit the tens. https://mathsclubs.co.za/activities/friendly-tens/

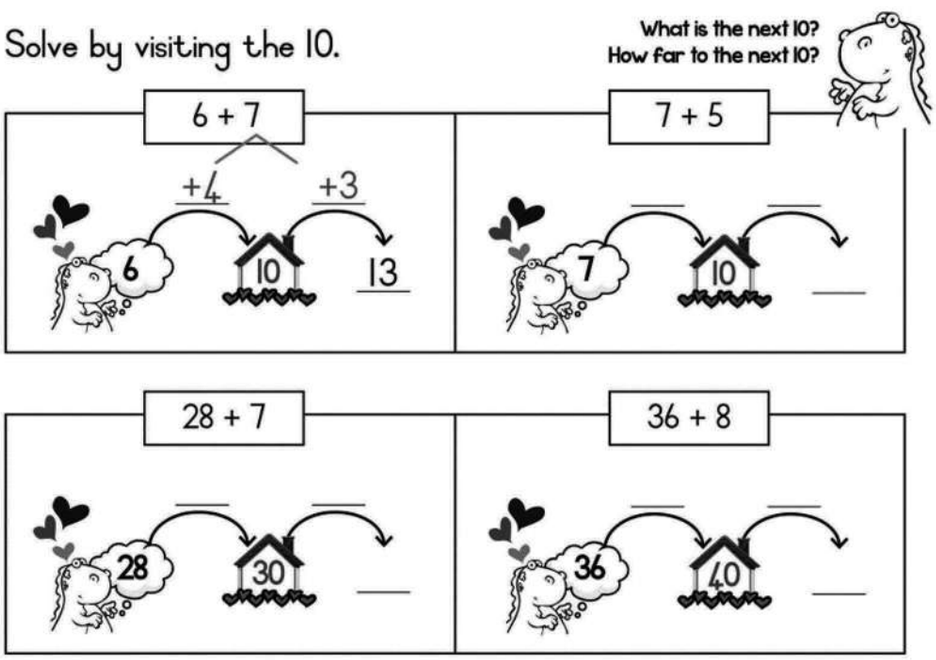
## Task: 2.3.1

### Visiting the next or previous ten

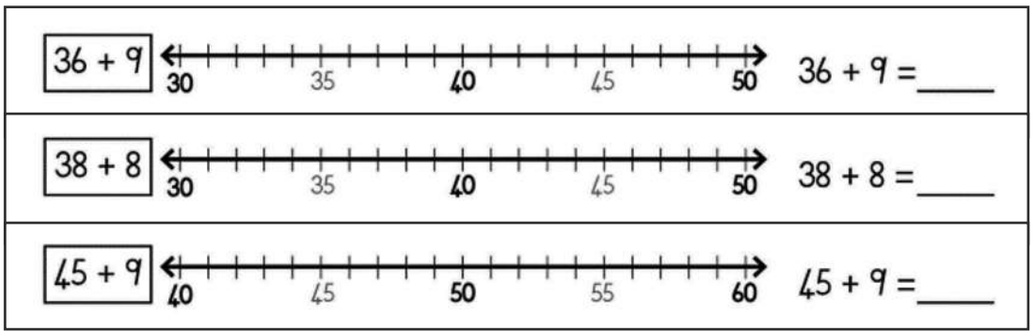
1. Solve by **visiting the ten** – using **number pictures**



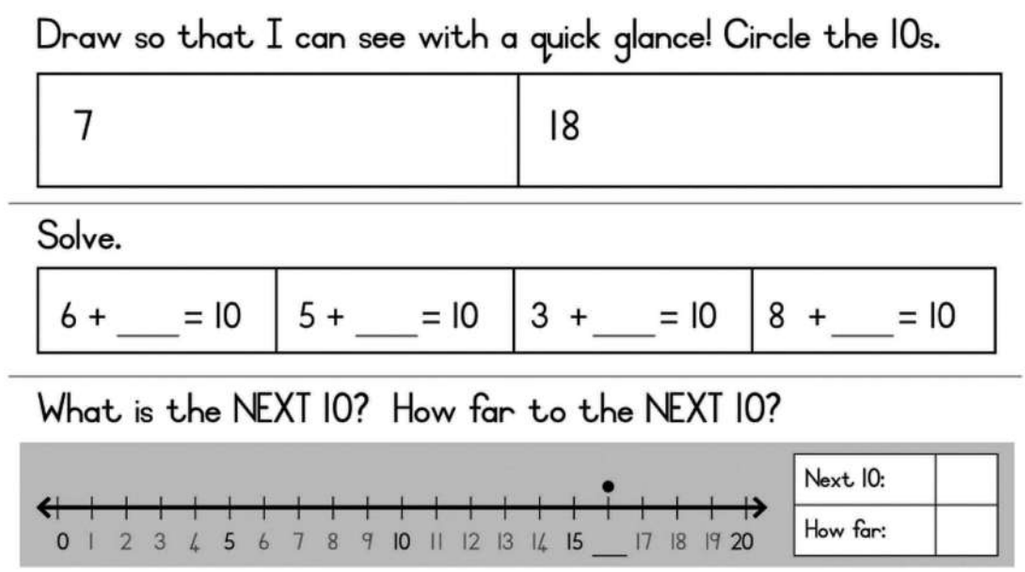

2. Solve by **visiting the ten** – using **number lines**



3. Solve by **visiting the ten** - using **number lines**




4. Complete these tasks that help develop the visit the tens strategy




5. How will you break down the second number to visit the ten?

**Add:**
Break down the second number to visit the **next ten** and **add each part**.

8 + 7

8 + 6

5 + 6

**Subtract:**
Break down the second number to visit the **previous ten** and **subtract each part.**

12 – 5

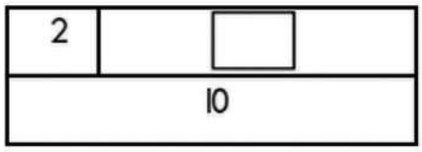
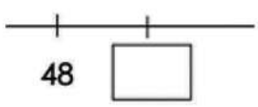
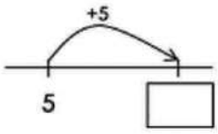
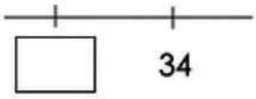
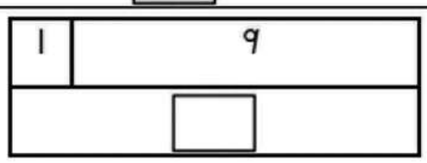
12 – 7

11 – 7

6. Practice visiting the ten. These should be easy - rapid recall calculations. Time how many you do in 2 minutes.

**Bridging Through Ten: Pre-Test**

**PART I** — 2 minutes for this page

| | |
|---|---|
| 1. $7 + 3 =$ ☐ | 11. $50 + 6 =$ ☐ |
| 2. $2 + 8 =$ ☐ | 12. $3 + 60 =$ ☐ |
| 3. $10 = 7 +$ ☐ | 13. $40 - 7 =$ ☐ |
| 4. 8 less than 10 is ☐ | 14. $40 + 8 =$ ☐ |
| 5.   | 15. What is the next multiple of 10?   |
| 6.   | 16. $100 + 27 =$ ☐ |
| 7. $10 - 5 =$ ☐ | 17. What is the multiple of 10 before 34?   |
| 8. $10 - 4 =$ ☐ | 18. ☐ $+ 7 = 50$ |
| 9.   | 19. $30 -$ ☐ $= 27$ |
| 10. ☐ $+ 10 = 10$ | 20. $87 = 80 +$ ☐ |
| Total out of 20 | |

**MAT 2**

**Represent and use** mathematics

**MAT 3**

**Reason mathematically**

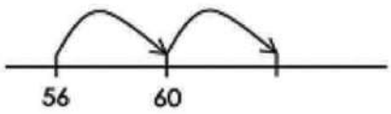
**GPM 4**

Teach a **balanced mathematics curriculum**

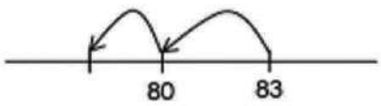
7. Now use the bridging the ten strategy for slightly more difficult calculations. Work quickly. Time yourself. You should finish in 3 mins.

**Bridging Through Ten: Pre-Test**
**PART 2** — 3 minutes for this page

1. $56 + 8 = \square$




2. $83 - 4 = \square$




3. $93 - 7 = \square$

4. $67 + \square = 73$

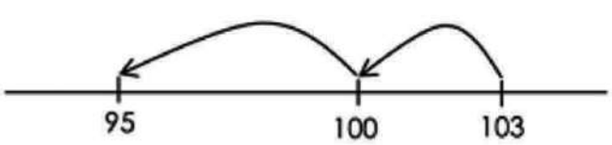
5. $\square + 7 = 82$

6. $67 + 5 = 67 + 3 + \square$

7. $94 - \square = 94 - 4 - 2$

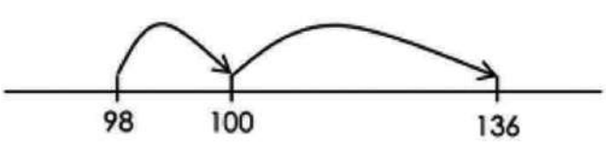
8. $98 + 56 = 98 + 2 + \square$

9.




$103 - \square = 95$

10.




$98 + \square = 136$

Total out of 10

## Task: 2.3.2

**NA 7**

**Early algebra**

**MAT 1**

**Playful engagement** to search for and develop mathematical insight

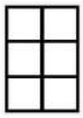
### Investigate squares

The counting unit is small squares.

Shape 1...

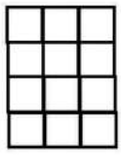
... has 2 squares

Shape 2...



...has 6 squares

Shape 3...

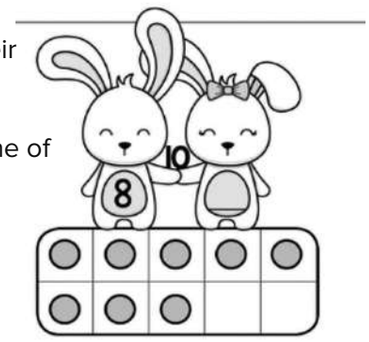


...has 12 squares

What about shape 4, shape 5, shape 10? And shape 100?
What is the relationship to get from the shape number to the number of squares?
If the input is the shape number, and the output is the number of squares, what is the function?

## Task: 2.3.3

### Bonds of 10




To be able to visit the tens, children must know their bonds of 10 very well.

This should be rapid recall. Try this song (to the tune of row-row-row your boat)

*9 and 1 are bonds of 10*
*8 and 2 are friends*
*7 and 3*
*6 and 4*
*5 and 5 are twins!*

When adding you can change the order. So look for the bonds of ten, and add those together first.

Solve.

| | | | |
|---|---|---|---|
| 5 + ____ = 10 | 7 + ____ = 10 | 6 + ____ = 10 | 2 + ____ = 10 |
| 3 + ____ = 10 | 8 + ____ = 10 | 10 + ____ = 10 | 4 + ____ = 10 |

Circle the 10s. Can you find the total?

| | | |
|---|---|---|
| 6 9 4 1 → 20 | 5 9 3 1 5 → ____ | 6 7 3 2 4 → ____ |
| 6 8 4 4 2 → ____ | 5 5 7 5 3 → ____ | 8 7 2 3 7 → ____ |

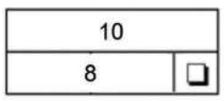
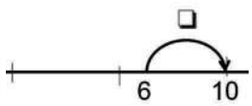
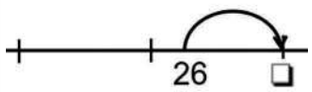
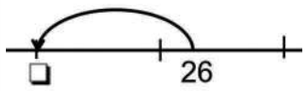
Find the 10. Then add.

Draw a line to find each 10!

| | | |
|---|---|---|
| 10 6 + 7 + 4 = 17 | 7 + 8 + 2 = ____ | 7 + 3 + 6 = ____ |
| 5 + 5 + 8 = ____ | 6 + 8 + 4 = ____ | 5 + 9 + 1 = ____ |
| 7 + 6 + 4 = ____ | 7 + 2 + 1 + 5 = ____ | 7 + 5 + 2 + 3 = ____ |

## Task: 2.34

**Two minute tango – Bonds of 10, next and previous ten**

| | | | |
|---|---|---|---|
| **1** | 7 + 3 = _____ | **11** | _____ = 37 – 7 |
| **2** | 10 = 7 + _____ | **12** | 60 + _____ = 64 |
| **3** |   | **13** | 100 – 80 = _____ |
| **4** |   | **14** | 267 + _____ = 270 |
| **5** | 10 – 8 = _____ | **15** | 7 + _____ = 12 |
| **6** | 50 + 4 = _____ | **16** | 37 + _____ = 42 |
| **7** | 70 + _____ = 76 | **17** | 13 – _____ = 9 |
| **8** | The next ten is..,   | **18** | 23 – _____ = 19 |
| **9** | The previous ten is..   | **19** | 323 – _____ = 319 |
| **10** | 48 – _____ = 40 | **20** | 230 – _____ = 190 |

To **add by visiting the ten**, you need to know (1) the **bonds of ten**, (2) the **next ten**, and (3) **how far to the next ten**?

To **subtract by visiting the ten**, you need to know (1) the **bonds of ten**, (2) the **previous ten,** and (3) **how far to the previous ten**?

## SUMMARY 2.3

- What are the key ideas from this umthamo?
- Which fluencies must children know – as rapid recall – to be able to visit the tens?.

# UNIT 3: BIG IDEAS FOR FLEXIBILITY

In this unit we discuss and reflect on additive relations which are situations and calculations involving adding or subtracting. This spans level 2 to 5 and covers key ideas needed to start to **count to calculate flexibly.** The go-to **jump strategy** – where you **break down the second number** – is reliable and accessible to most children. You should make sure that all the children in your class can use jump (or break down the second number) as a go-to strategy. But it is not always the most efficient strategy.

**Key Ideas**

In this unit we focus on the **concepts needed to start to calculate flexibly and reason additively.** We focus on how learners develop their mathematical thinking about additive relations over time.

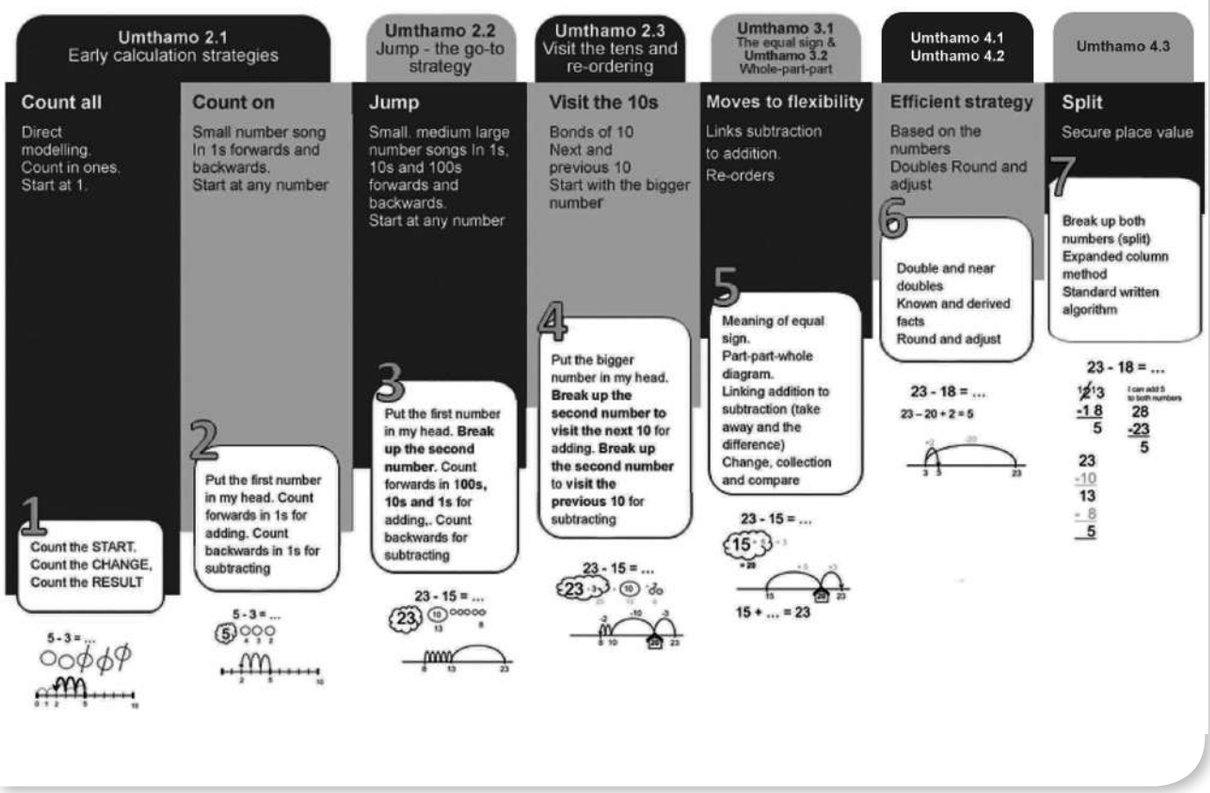
There are three imithamo:

**Umthamo 3.1: Equals means "is the same as"**

**Umthamo 3.2: Part-part-whole structure**

**Umthamo 3.3: Additive relations word problem types**

## Level 5: Moves to flexibility



# UMTHAMO 3.1

## EQUALS MEANS "IS THE SAME AS"

Key Ideas

In this umthamo we focus on supporting learners to use the equals sign: =.
By the end of this umthamo you should be able to:

- Explain that **equals means "is the same as"**; and
- Solve **missing number problems;**
- Participate in and facilitate **number talks.**

### The equals sign

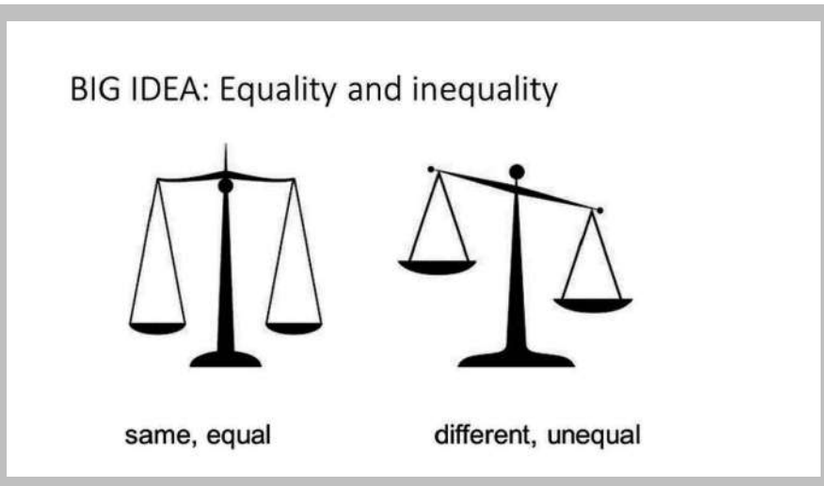
Knowing the meaning of the equals sign is a key idea that is needed to link addition to subtraction.

In English, we say "equals" or "is equal to". In isiXhosa, we might say iyalingana or yenza.

But the equals sign is a symbol where there is a lot of confusion. It is very important to be clear about what equals means.

Equals does not mean "Find the answer". It does not mean "makes"
Equals means "is the same as".

Diagram




**NA 7**

**Early algebra**

What is on the left hand side of = must be the same as the right hand side of =.

Diagram

Source: unicef.org

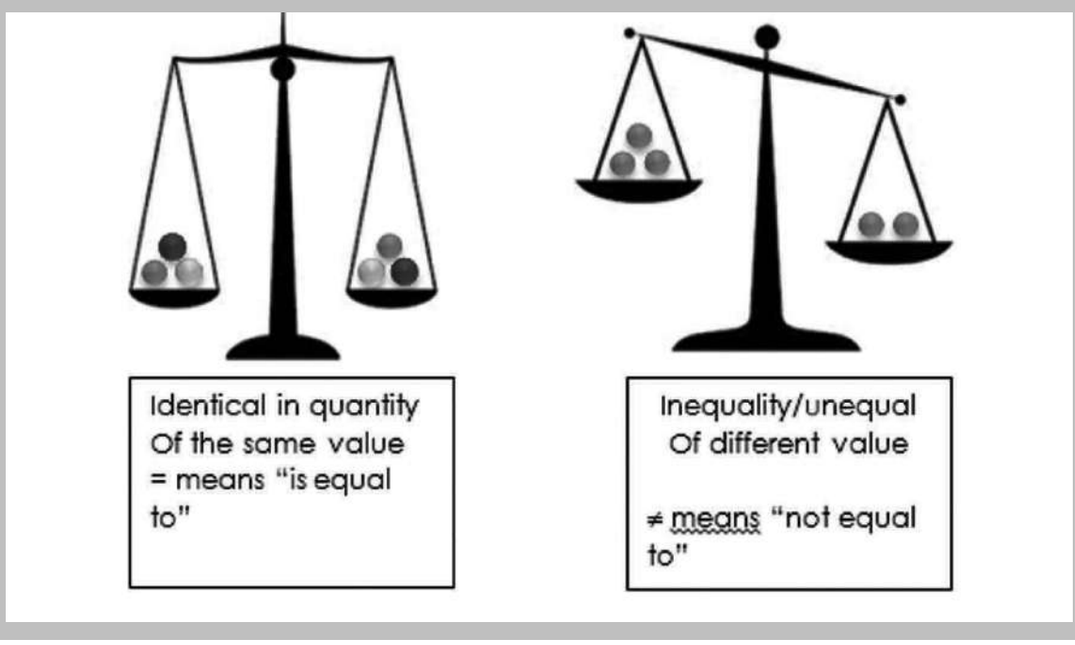
**NA 8**

**Overarching ideas** for school arithmetic

A balance beam is like a see saw. To balance, the weight must be the same on each side. Mostly when children play on a seesaw they move up and down. To balance a see-saw it must be still, with the same weight on each side.

The beam balance is an important metaphor for the equals sign = which means "is equal to". This is often misunderstood at primary school level.

In Diagram




'Is equal to' does not mean 'Find the answer'. It means 'is the same value as'.

The values on both sides (left and right) of the = sign should be equal, the same in quantity, or have the same value.

## 'Is equal to' or equality is represented as a balance

Examples

$6 + 2 = \square + 4$

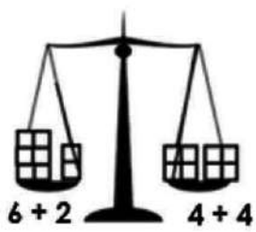
6 plus 2 is 'the same as' **a number** plus 4.

We need a number to **balance** the scales.

$6 + 2 = 8$, so $\square + 4$ must be equal to 8.

$\mathbf{4} + 4 = 8$, so the unknown number is **4.**

So, $6 + 2 = \mathbf{4} + 4$

## Missing number problems

In high school we call a **number sentence** that states a relationship of equality an **equation.** So an equation always has an equal sign. The verb in the sentence is “is equal to” which means “is the same as”.

If the **number sentence** has a **missing number**, we can find the missing number that makes the number sentence true.
A very simple missing number problem is something like:

Examples

### An easy missing number problem

$7 + \square = 10$

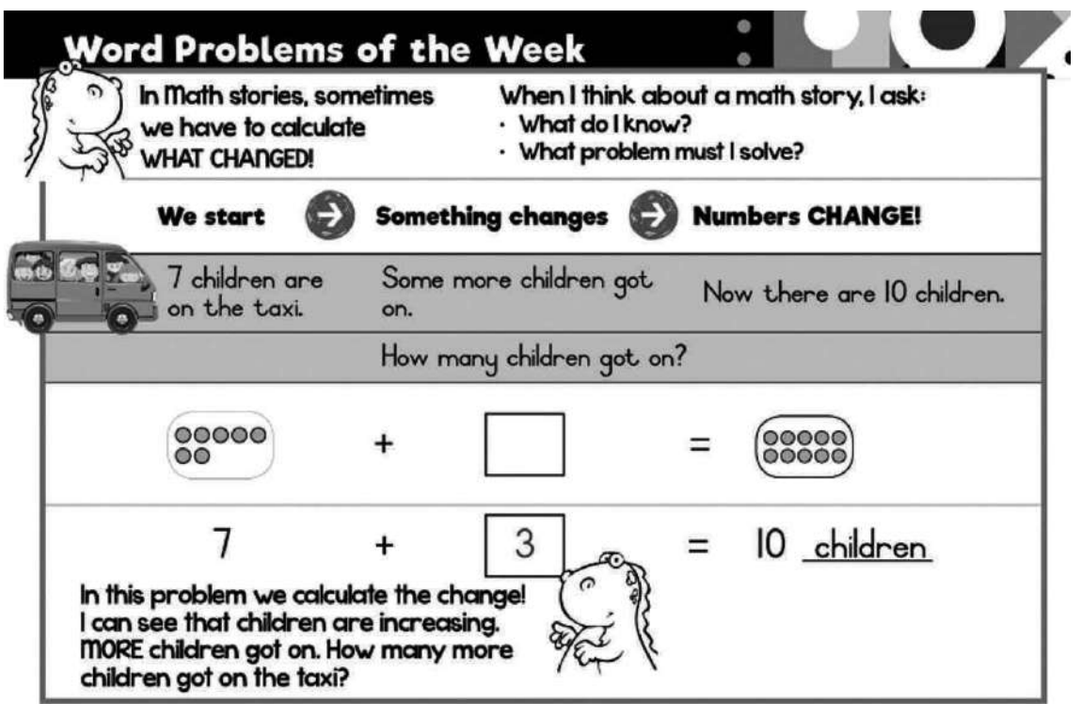
**NA 7**

**Early algebra**

As a teacher you could ask your class:
I am thinking of a number.
7 plus “my number” is 10.
What is “my number”?

**→ Solution**

The missing number is 3. Why? Because: 7 + 3 = 10




As the expressions get more complicated, solving missing number problems becomes more difficult.

Examples

### Missing number problem – "I am thinking of a number"

I am thinking of a number. 8 plus 2 **is the same as** 'my number' plus 3.
What is my number?

**NA 7**

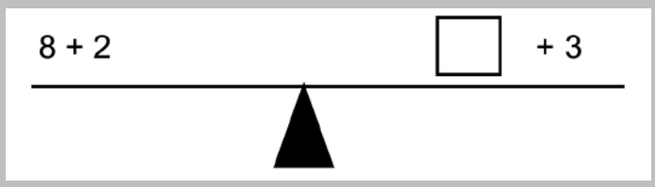
**Early algebra**

**NA 8**

**Overarching ideas** for school arithmetic

**→ Solution**

We need to find when '8 plus 2' **is the same as** 'my number plus 3'
8 + 2 = 'my number' + 3.




Left hand side: 8 + 2 = 10
Right hand side: 'my number' + 3.

| Left hand side | **is the same as** | Right hand side. |
|---|---|---|
| 10 | = | 'my number' + 3. |

So, my number is 7

**Why?**

Because

| | | |
|---|---|---|
| 10 | = | 7 + 3. |

You can imagine a number sentence as a balance beam.
The left hand side **is the same as** the right hand side.

We use missing number problems from grade 1 all the way to high school. In the next examples we use the same problem, but change the way that learners write down the problem.

Examples

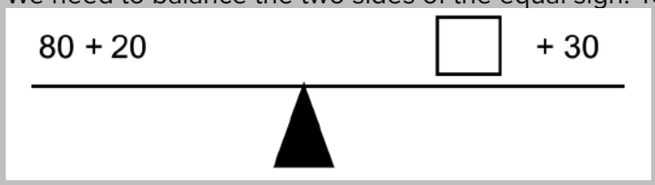
### Missing number problem – "What goes in the box?"

Here is a number sentence:

80 + 20 = ☐ + 30
What goes in the box?
Fill in the missing number, to make the number sentence true.

**→ Solution**

We need to balance the two sides of the equal sign. You can use a balance scale drawing:




| Left hand side | **is the same as** | Right hand side. |
|---|---|---|
| 80 + 20 | = | ☐ + 30 |

100 = ☐ + 30
My number is 70

Why? Because
100 = 70 + 30

These balance beams looks neat. But rough drawings are fine.

This same problem is something presented to learners by the end of primary school, as they move into high school and start working with algebra. If an **equation** has an **unknown** we can solve the equation to find the unknown that makes the equation true.

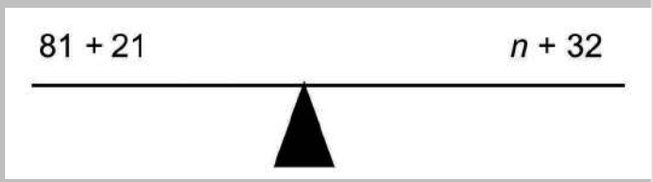
Examples

**Example: Missing number problem – “Solve an equation”**

Here is an equation:
81 + 21 = *n* + 32.
Solve for *n*. For which value(s) of *n* is the equation true?

**→ Solution**

We need to balance the two sides of the equation.




LHS = RHS
So,
102 = *n* + 32
I subtract 32 from both sides of the equation:
102 – 32 = *n*
*n* = 70

Why? Because
81 + 21 = 70 + 32

In this case the **unknown** is *n*. And you find that *n* must be 70 for the equation to be true. The skills you teach at primary school, are needed for high school algebra. How you teach primary school to make the shift from numbers to letters, either difficult or easy.

The above examples all used the **similar missing number problems**:

8 + 2 = ☐ + 3

80 + 20 = ☐ + 30
81 + 21 = *n* + 32

The numbers got bigger. We have also shown how we talk and write about this problem as learners get older.

Lets talk maths

In Foundation Phase we say:
- I am thinking of a number
- What goes in the box?

In Intermediate Phase we say:
- What's the missing number?
- Find the value that makes this true?

By Senior Phase we say:
- Solve the equation, for *n*.
- Find *n* or find *x*.

Another way to make the same problem more difficult it to replace the **whole numbers**, with **fractions decimals or percentages**. It may also be more difficult if it refers to **measurements such as length, mass, capacity, time or temperature.**

## Number talks

This reading is required for written task: Task 1.2.1: Reading – Reed (2005)

A **number talk** is an maths activity that you give to the class, where they show you their different **representations** and **calculation strategies**. To **represent**/show their working children could use **number pictures, number lines or number sentences.**

To **calculate** they use different strategies: **count all, count on, break down the second number, visit the ten.** For subtraction they could use a **'take away'** or **'find the difference'** strategy, or they could use a **part-part-whole diagram** to rewrite the subtraction number sentence, as an addition number sentence.

Number talks encourage children to be flexible. They talk about their representations and their strategies. They learn from eachother. As the teacher you should encourage efficient strategies.

Reading

### Reading 1

SCAN ME

**Replace Jumptrak with Reed (2005)**

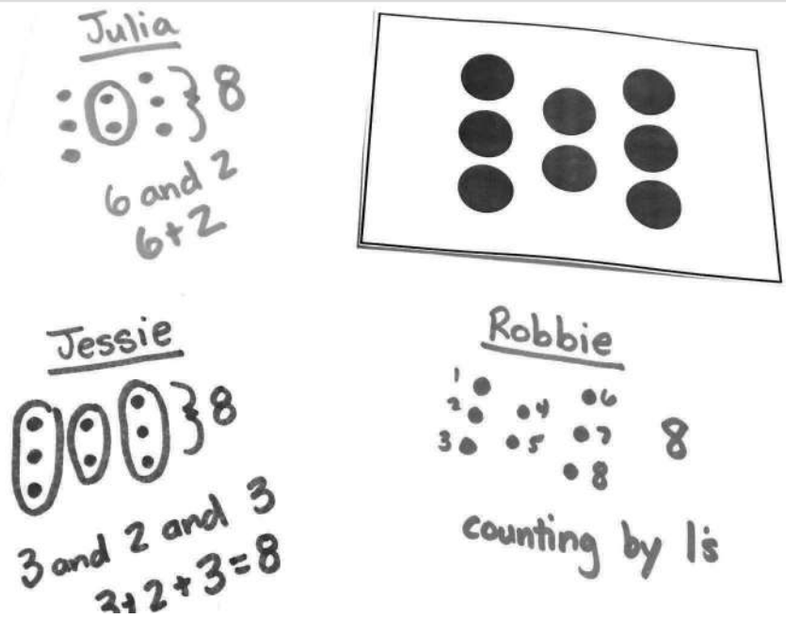
Reed, C. (2005) Number talks: How and Why?, The Brownbag teacher. https://brownbagteacher.com/number-talks-how-and-why/, accessed on 11 September 2021

**MAT 1**

**Playful engagement** to search for and develop mathematical insight

### NUMBER TALKS: HOW AND WHY?

Number talks offer a bridge between conceptual understanding and mental math. During mini-lessons and Guided Math, I am ALL about manipulatives. Base 10 pieces, unifix cubes, cuisenaire rods – give me all the math things. Especially in 1st grade, manipulatives offer a concrete way to represent a number and their thinking. At some point, this concrete thinking has to begin to transition into abstract and mental math processes. Number Talks offer a daily, short, structured way for students to talk about math with their peers.




## WHAT IS A NUMBER TALK?

Number Talks are short (10ish minutes), daily exercises aimed at building number sense. Number sense is the ability to play with numbers meaning students can visualize problem solving, perform calculations quickly, and are flexible in their mathematical strategy. Students who have strong number sense solve problems in more than one way and check that their answers make sense.

As a part of this routine, students are thinking, asking their peers questions, and explaining their own thinking all while the teacher records the thinking.

## WHAT DO NUMBER TALKS LOOK LIKE?

When possible, I love slipping a number talk (Kindergarten, 1st Grade & 2nd Grade) in at the beginning of our Guided Math block. If we’re running late, I’ll slip a Number Talk in during an awkward transition or at the very end of the day. A traditional talk follows this routine –

**1. Present the class with a problem.** Students are seated in a common area with no math tools. Number Talks are mental exercises. I store my number talks (Kindergarten, 1st Grade & 2nd Grade) on binder rings next to my document Camera for easy access.

**2**. **Give think time.** (When a student raises their hand, it tells other students they were not fast or good enough in their thinking. To account for this, students give me a sly thumbs-up on their chest to signal that they have thinking they want to share. Then, they put up multiple fingers (on the same hand) to indicate they have multiple ways to answer/solve the problem.

**3. Call on a student – “**_____, what are you thinking?” Have the student explain their thinking and then give their answer. My focus is always on the process not the answer.

**4. Offer the strategy to the class**. Student will elaborate (if correct) or ask questions (if misguided). If students aren’t sure they understand a friend, they will say “I’m a little confused. Could you tell me in a different way?” If students agree with the student sharing they will do sign-language for

'me too' (thumb near chest and pinky pointing at friend). This builds the student's confidence and also offers the student sharing positive affirmations.

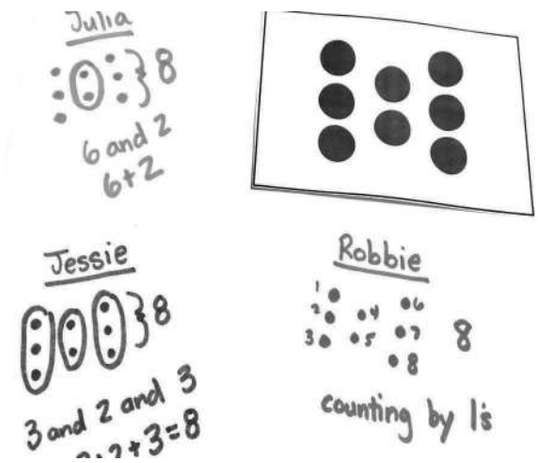
## SCAFFOLDING STUDENTS

If students have never been expected to explain their mathematical thinking, students may be very resistant to share their own strategies. So, how do I encourage students to share their own thinking/strategies?

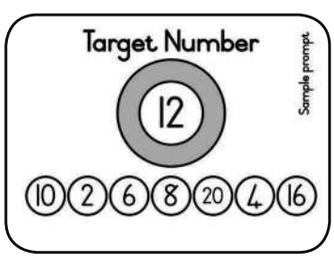
- **Model, Model, Model** – The first 2 days of Number Talks, I do a lot of modeling. After this, I completely release the Number Talks to the students.
- **Sentence Stems** – "In my head I saw..." "My first step was..." "At first, I decided to try..." "I needed to..." "Since I wasn't sure what to do, but I noticed..."
- **Force It** – Awkward silence is your friend. Sometimes I will wait (for extended periods of time) for students to share their thinking. Even if a friend is STRUGGLING with a problem or sharing their thinking, I do not come to the rescue. Growth comes when friends are able to sort out their own thinking and my 1st grade friends are able to do this...even if it is a little painful for me.

## SUBITIZING QUICK IMAGES

Subitizing is the ability to quickly identify the number in a group of objects without individually counting the objects. It is a concrete way to explore composing and decomposing numbers. Students will see dot patterns in different ways, and that is a great thing. When showing students 6, made from a pyramid students might see 3 and 3, 5 and 1, or 3 and 2 and 1. These are all ways to build 6 and lead students to understand that numbers can be composed and decomposed in many different ways.




**You can find out more about number talks here:**




SCAN ME

**Rhodes numeracy**

South African Numeracy Chair (SANC) at Rhodes university has many resources on its website.

SCAN ME

**You cubed**

SCAN ME

**Kentucky Centre for Maths**

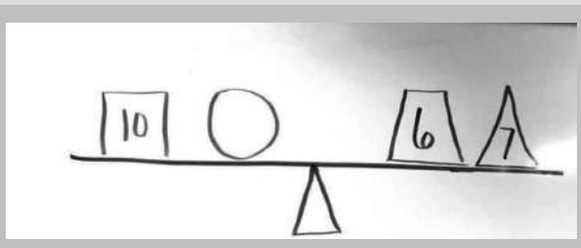
These websites have ideas for Number Talks in Foundation and Intermediate Phase.

Examples

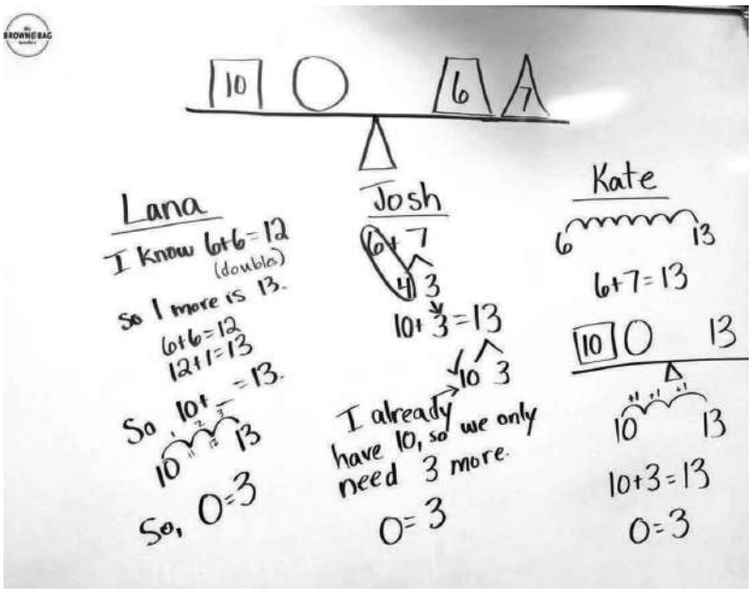
### A balance beam number talk

A teacher draws this on the board:

She asks: What value is the circle? The teacher writes done the strategies that learners offer. Here are her notes for Lana, Josh and Kate




Source: brownbagteacher.com/number-talks-how-and-why/




Reed (2015) draws a balance-beam to represent the idea of equality. There is the same total value on each side. She gives each number a different shape. The learners first work out the right hand side of the balance beam.
They calculate: 6 + 7 =

- Lana uses **doubling**. 6 + 6 = 12 and then adds 1 more to get 13.
- Josh **breaks down the second number and visits the 10,** for 6 + 7. He breaks up 7 into 4 and 3 (because he knows that 6 + 4 is 10.
- Kate puts the first number in her head, and then **counts on** 7 in ones to reach 13.

Then they have to find the left hand side of the number sentence or equation.
The right hand side is 13. So the left hand side must also be 13.

They calculate: 10 + ☐ = 13

- Lana puts the first number in her head. Then she **counts on** 3 to reach the target of 13.
- Josh knows that he needs to add 3 more to 10 to get to the target of 13. He uses a **known fact** 10 = 3 = 10)
- Kate puts the first number in her head. On a number line, she **hops on in ones** to reach the target of 13.

The phrases shown in bold are **calculation strategies**. An equation – just like a balance beam - will stay balanced if you:

- Subtract the same amount from both sides
- Add the same about to both sides

(you can also multiply or divide both sides by the same amount, and it stays balanced).

**NA 7**

**Early algebra**

**NA 8**

**Overarching ideas** for school arithmetic

## Sentences and expressions

**A sentence** is a complete idea. It has a subject, a verb, and an object.

This is an example of a simple sentence:

**Example**

A simple sentence in English

| | | |
|---|---|---|
| The tall girl | **kicks** | the ball. |
| subject | verb | object |

**A phrase or expression** is not a complete idea. It is only part of a sentence.
This is an example of a simple expression:

**Example**

An expression in English

The tall girl
Phrase/expression

The phrase "the tall girl" has a definite article (the), an adjective (tall), and a noun (girl). It is not a complete idea.

A subject or object in a sentence is usually a noun.
An adjective describes a noun
A verb is a "doing word".

**An expression** in maths is not a complete idea. It is has no verb.
An expression is just a piece of maths. It does not state a relationship.
This is an example of a simple mathematical expression:

**Example**

An expression in maths

Six and four
6 + 4
expression

**A number sentence** communicates a complete idea. It shows a relationship between two mathematical expressions. This is an example of a simple number sentence:

**Example**

A number sentence in maths

| | | |
|---|---|---|
| Six and four | **is the same as** | ten. |
| 6 + 4 | = | 10 |
| expression | verb | expression |

In maths we state relationships between expressions with **verbs** like:

- ‘Is equal to’ or  ‘is the same as’ or ‘has the same value as’
- ‘is not equal to’ or ‘is not the same as’;
- ‘is bigger than’ or ‘is more than’ or ‘is greater than’
- ‘is smaller than’ or ‘is less than’ or ‘is fewer than’

Lets talk maths

In maths we use these verb phrases to state relationships about measurement:

| | | |
|---|---|---|
| Length<br>Height<br>Width | Is longer than<br>Is taller than<br>Is wider than | Is shorter than<br>Is shorter than<br>Is narrower than |
| Mass | is heavier than | is lighter than |
| Time (duration)<br>Time (event) | Is longer than<br>Is later than | Is shorter than<br>Is sooner than |
| Capacity | Is fuller than | Is emptier than |

**GPM 2**

Knowledge of **measurement**

How do you say these phrases for maths relationships in an African language?

Activity for learners

**Game 1:**

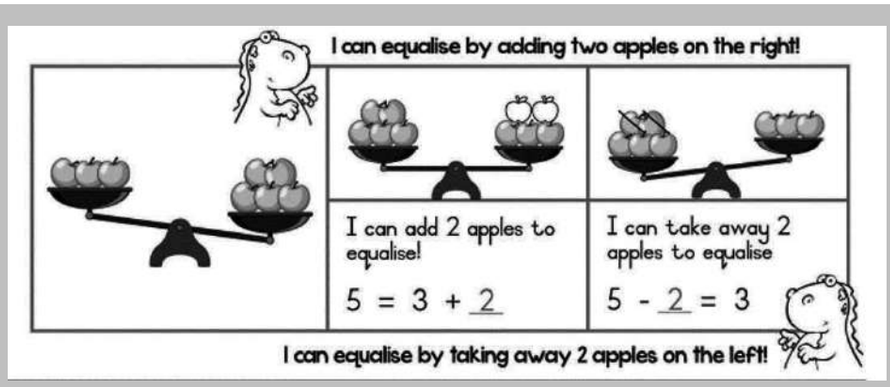
**1-2-3 VEZA: COMPARE!**

- You and your child put both hands behind your back.
- Together with your child say, “1, 2, 3, Veza!”
- Show fingers on one or two hands. Compare.
- You say, “I have 8 fingers, you have 6. 8 is 2 more than 6!”
- Your child says, “I have 6 fingers, you have 8. 6 is 2 less than 8!”
- When your child is ready, let your child start!


### Equalise as balance

The equal is means “is the same as”. Equals is like a balanced beam.
Both sides of the blance must be same.

Activity for learners



## Take away and difference

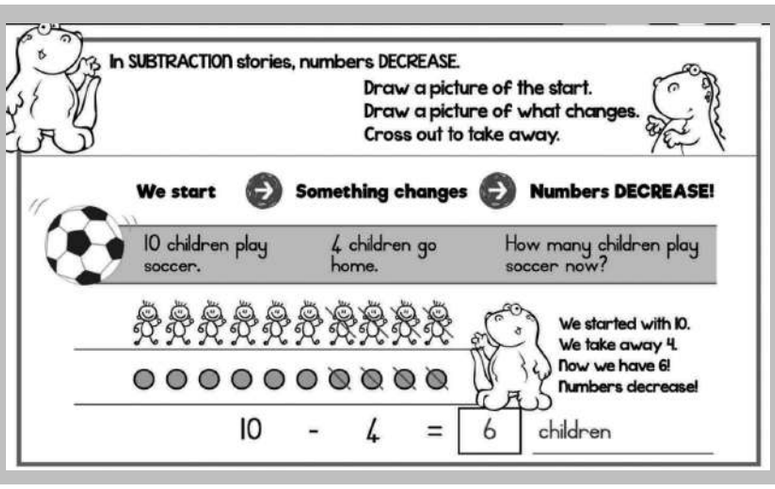
Subtraction is used for two different contexts:

**Take away:** I have 9 beads. I take away 7 beads. I am left with 2 beads.
9 – 7 = ...

**Compare/difference:** I have 9 beads. You have 7 beads. What is the difference? I have 2 more than you.
7 + ... = 9

Look at this **take away situation**. An iconic picture of children is given. An indexical picture using dots is also given to show the action of taking away or removing.

Examples

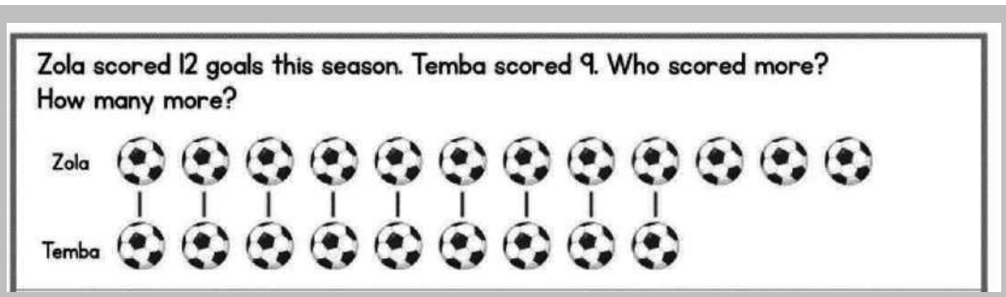


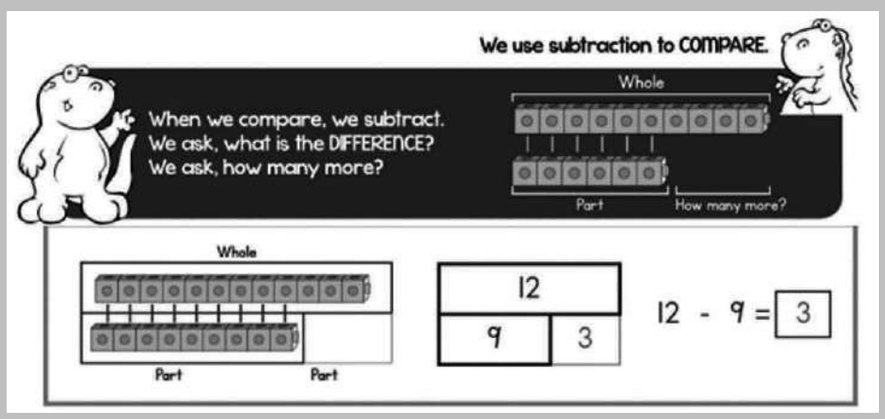

**NA 3.1**

Demonstrate an understanding of the **part-part-whole relationship** and link between the operations of addition and subtraction

Look at this **compare situation.** An iconic picture of balls are matched one to one. The problem is also modelled using concrete materials (manipulatives).

Examples

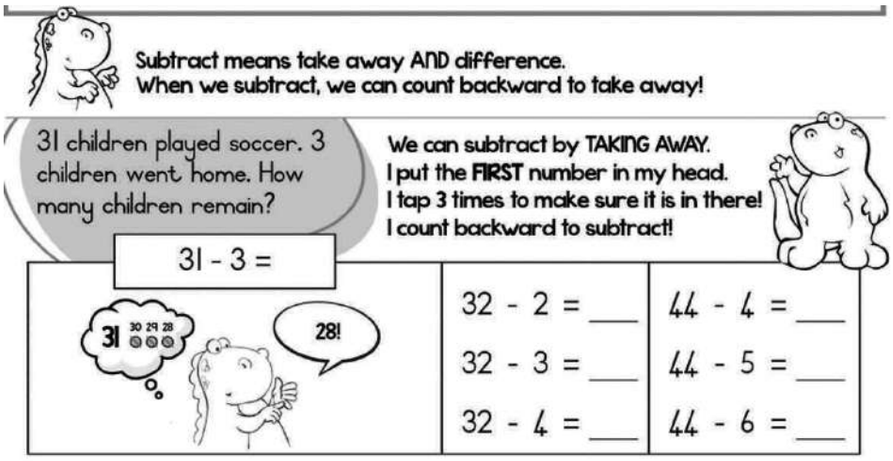


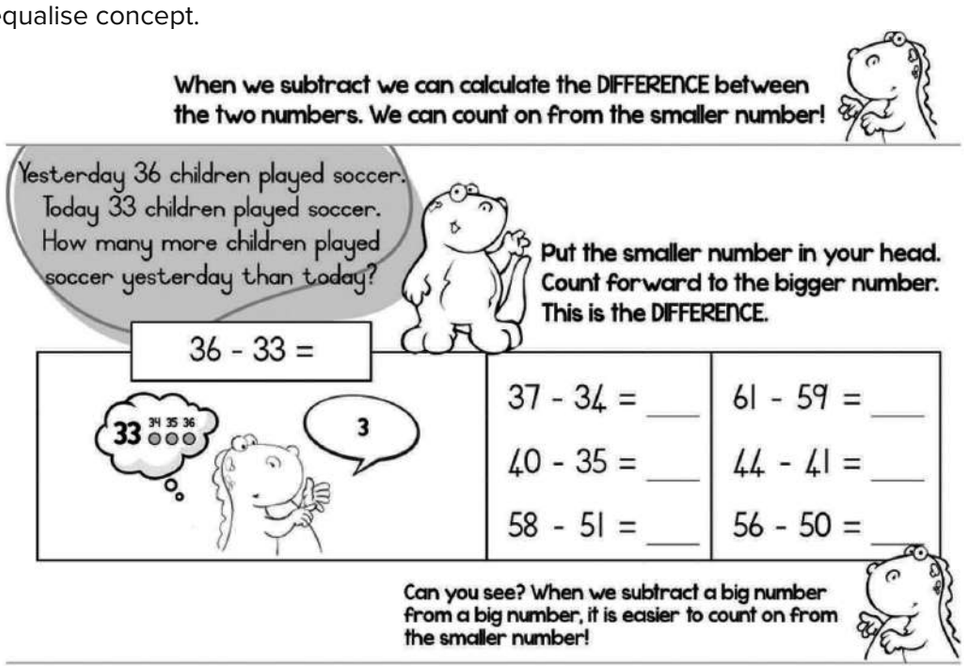
So there are two different **problem situations** for subtraction – **take away** and **compare/ difference**. As a teacher you must make sure that children solve both types of subtraction problems.

If learners understand that subtraction is not always take away – but is also used for comparing, then they can start to be **flexible** with their **calculation strategies**.

The **count back calculation** strategy uses **subtraction as “take away”** concept.




The **count forward to reach a target** strategy uses **subtraction as “find the difference”** or equalise concept.




To subtract you can either

- take-away (put the first number in your head and break down the second number to jump backwards) or
- find the difference (put the first number in your head and count fowards to reach the target). Which strategy you choose depends on the numbers.

**NA 7**

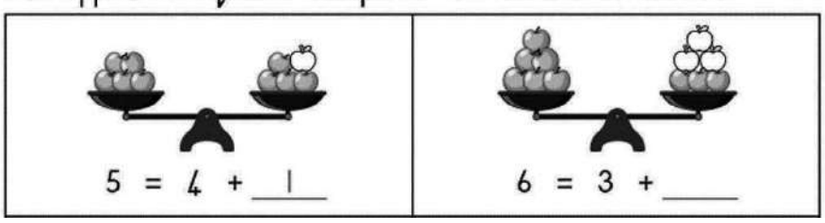
**Early algebra**

**NA 8**

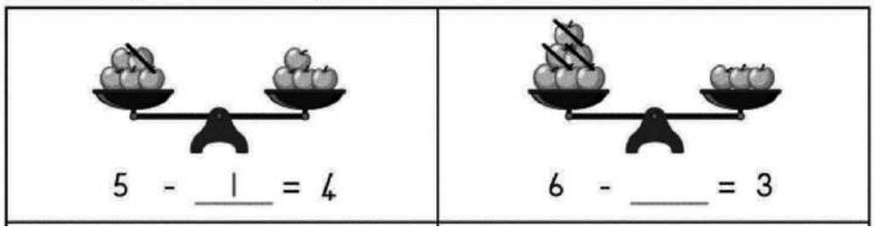
**Overarching ideas** for school arithmetic

Activity for learners

Add apples to equalise. Complete the number sentence.




Take away apples to equalise. Complete the number sentence.




## Moving the position of the 'missing number' (the unknown)

Children mainly see calculations presented like those on the left. To remind children that equals means "is the same as", you should also give them problems like those on the right.

Activity for learners

| | |
|---|---|
| 7 + 3 = _____ | _____ = 7 + 3 |
| 10 - 3 = _____ | _____ = 10 - 3 |

To help children make sense of the different syntactical representations (number sentences, and missing number problems it is helpful to **link the number sentence to a word problem** or story sum.

Here are **examples of addition word problems** where the position of the unknown (the missing number) is moved.

Examples

7 + 3 = _____

| | |
|---|---|
| 7 | 3 |

**Example:** There are 7 children in the taxi. 3 more get on. How many are children are on the taxi now?

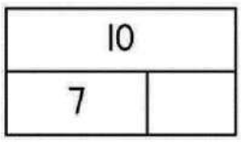
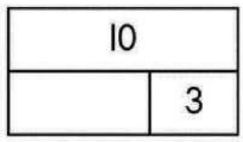
**Using Known Facts:** Put biggest number in our head. I jump forward 3 to 10.

**Counting in Ones:** Put biggest number in our head. Count forward in ones.

**NA 3.2**

Describe and **use addition and subtraction problem types**

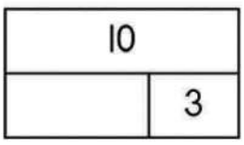
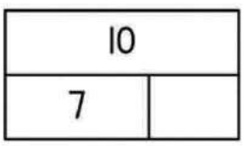
Examples

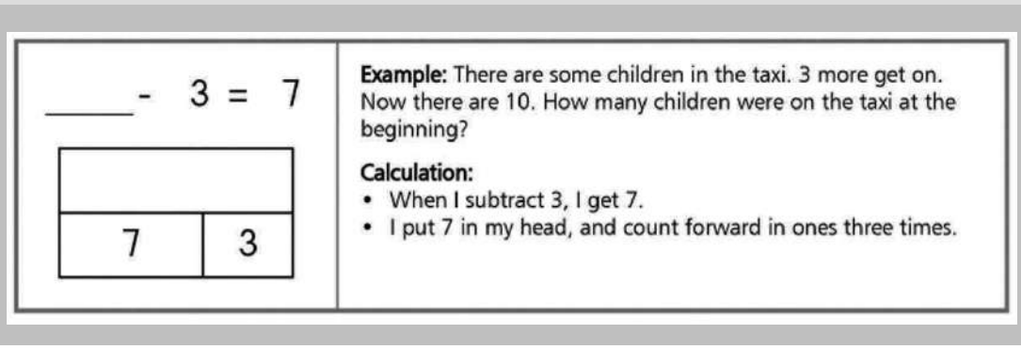
| | |
|---|---|
| 7 + _____ = 10<br>  <br> | **Example:** There are 7 children in the taxi. Some more children get on. Now there are 10. How many children got on?<br>**Calculation:** How many from 7 to get to 10?<br>**Using Known Facts:** I know that 7 + 3 = 10. Therefore, the answer is 3.<br>**Counting in Ones:** What must I add to 7 to get to 10?<br>• Put the 7 in your head. Count forward in ones to 10.<br>• Put the 10 in your head. Count backwards in ones to 7. |
| _____ + 3 = 10<br>  <br> | **Example:** There are some children in the taxi. 3 more get on. Now there are 10. How many children were on the taxi at the beginning?<br>**Calculation:** How much must I add to 3 to get to 10. Or when I add 3 to this number, I get 10.<br>**Using Known Facts:** I know that 7 + 3 = 10. Therefore the answer is 7.<br>**Counting in Ones:**<br>• Put 3 in our head. Count forward to 10.<br>• Put 10 in our head, and count backwards 3.<br>• Try both. Which do you think is a more "efficient" strategy? Why? |

Using the same numbers (10-7-3) and the same context (children getting into a taxi), helps children focus on the main idea: **What happens when the missing number is moved around (**you move the position of the unknown)?.

Here are **examples of subtraction word problems** where the position of the unknown (the missing number) is moved.

Examples

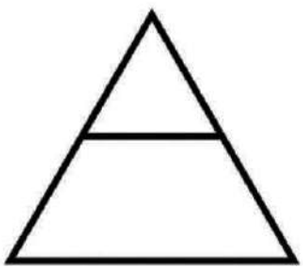
| | |
|---|---|
| 10 - 3 = _____<br>  <br> | **Example:** There are 10 children in the taxi. 3 children get off. How many children are on the taxi now?<br>**Calculating with Known Facts:** I know that 7 + 3 = 10. Therefore, I know: 10 - 3 = 7.<br>**Calculating Counting in Ones:** Put the first number in our head. Count backwards in ones. |
| 10 - _____ = 7<br>  <br> | **Example:** There are 10 children in the taxi. Some children get off. Now there are 7 children in the taxi. How many children got off?<br>**Calculation:**<br>• Put 10 in your head. Count backwards to 7.<br>• Put the number 7 in your head. Count forward to 10. |




## SOME IDEAS FOR YOUR CLASSROOM

Activity for learners

How many triangles? ☐

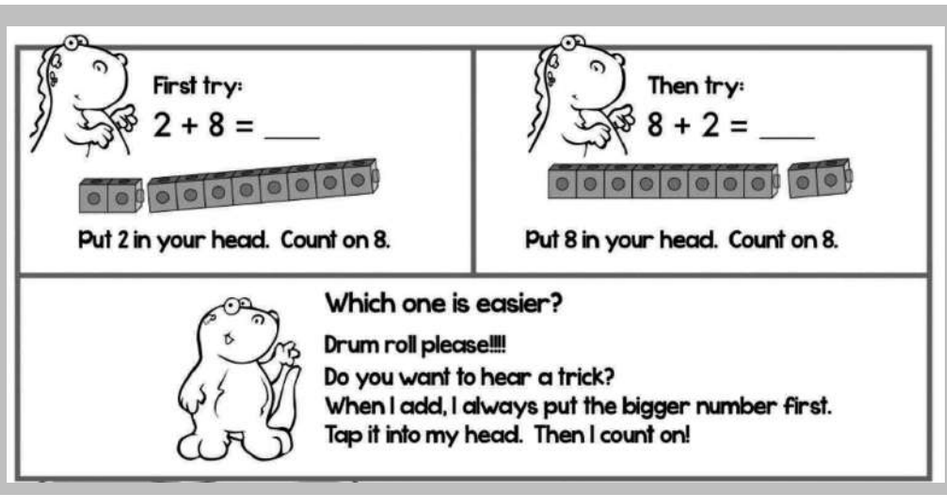


There are different sizes triangles. What is the counting unit?

Draw a line from the apex (top point) the bottom line.

How many triangles?

Activity for learners



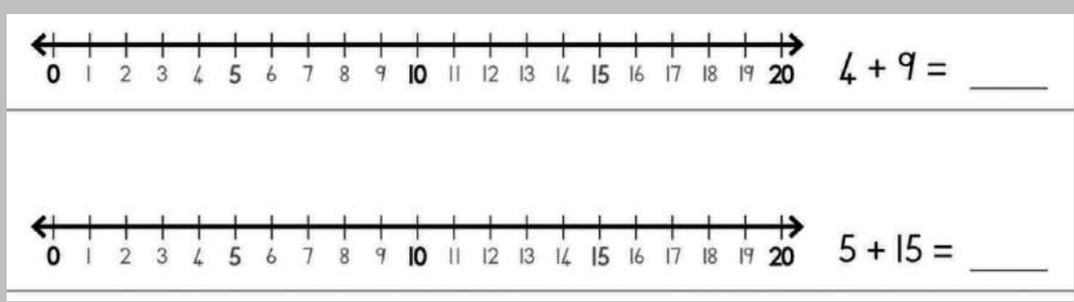

**NA 7**

**Early algebra.**
**The commutative property holds for addition.**

**Does it hold for subtraction too?**

Activity for learners

Solve by showing a number line



## Task: 3.1.1

### Reading – Reed (2005)

Read the article by Reed (2005), on page 92, then answer the questions that follow.

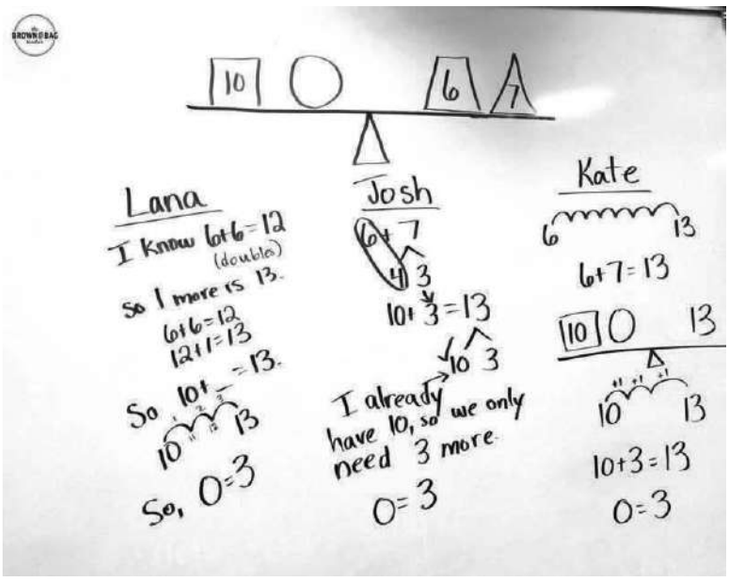



Figure 1: Number talk example from Reed (2015)

SCAN ME

Brownbag teacher: Number talks how and why

**NA 1**

Knowledge and use of **emergent number** awareness of learners

## Task

1. What is a ‘number talk’?

2. What is does ‘subitise’ mean?

3. Reed (2015) has a strategy to stop learners putting up their hands or calling out answers. This gives other learners time to think. She has a strategy for the learners to show her they have the answer, and to show her how many strategies they have found. What does she do?

4. Reed (2015) has a 5 step routine for her Foundation Phase number talks. What are the 5 steps?

5. In the Figure 1, why does each child's solution end with "0 = 3"? Why could this lead to confusion for children?

6. Design a number talk prompt for either grade 2 or grade 5.
   You can get ideas from https://brownbagteacher.com/number-talks-how-and-why/.

7. Show at least 4 different ways that learners could answer the Number Talk prompt.
   Use number pictures, number lines and number sentences.

## Task: 3.1.2

### Take away and difference

1. Show how to solve the take away problem by **counting back in ones, using a number line.** This is a subtraction calculation: 10 – 4 = ...

2. Show how to solve the compare problem by **counting back in ones, using a number line**. This is a subtraction calculation: 12 – 9 = ...

3. Show how to solve the compare problem by hopping **back in ones, using a number picture.** This is a subtraction calculation: 12 – 9 = ...

4. Show how to solve the compare problem by **counting forwards in ones, using a number line.** Now this is an equivalent addition calculation:
   9 + … = 12

5. Show how to the solve the compare problem by **counting forwards in ones, using a number picture.** Now this is an equivalent addition calculation: 9 + … = 12

6. Solve these calculations
   - If it is easier to solve using **take away**, then write **T** next to it.
   - If it is easier to solve using **difference** then write **D** next to it.

| | | |
|---|---|---|
| 31 - 29 = _____ | 48 - 45 = _____ | 97 - 92 = _____ |
| 50 - 47 = _____ | 61 - 59 = _____ | 83 - 79 = _____ |

Look at each problem. Decide whether it is easier to:
- Take Away: Count backward from the biggest number
- Difference: Count forward from the smaller number.

| | | | |
|---|---|---|---|
| 28 - 26 = ___ | 75 - 71 = ___ | 64 - 5 = ___ | 56 - 4 = ___ |
| 37 - 5 = ___ | 71 - 3 = ___ | 68 - 64 = ___ | 72 - 69 = ___ |

**NA 3.2**

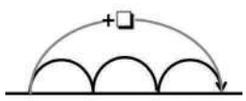
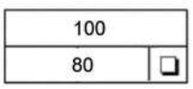
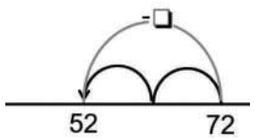
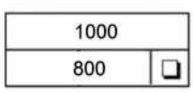
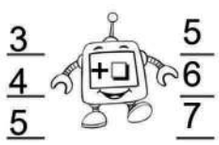
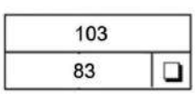
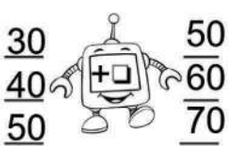
Describe the range of increasing sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## Task: 3.1.3

**NA 8**

**Overarching ideas** for school arithmetic

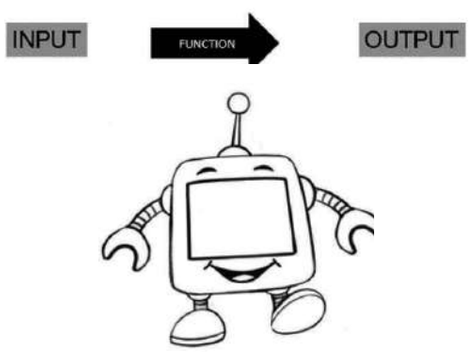
### Two minute tango – missing number problems

| | | | |
|---|---|---|---|
| **1** | 53 + _____ = 60 | **11** | 273 – _____ = 73 |
| **2** | 53 + _____ = 67 | **12** | 273 – _____ = 200 |
| **3** | 253 + _____ = 260 | **13** | 273 – _____ = 100 |
| **4** | 53 – _____ = 50 | **14** | Increase 167 by 20 |
| **5** | 53 – _____ = 30 | **15** | Decrease 167 by 20 |
| **6** | 53 – _____ = 28 | **16** |   _____ |
| **7** |   _____ | **17** |   _____ |
| **8** |   _____ | **18** |   _____ |
| **9** |   _____ | **19** |   _____ |
| **10** | 73 + _____ = 153 | **20** | 45 is _____ more than 40 |

**A function** is special kind of relationship in mathematics. There is are inputs, a rule, and outputs.

For each **input** there is only one **output.**

An example of a function is **add two**: input + 2 = output.



## Summary 3.1

- Explain the **key ideas** in this umthamo.
  Use examples and drawings where appropriate.
- Design a new additive relations **number talk** for Grade 2s or for Grade 5s
- Give some likely responses from children. Include and label **count all, count on, break down the second number (jump)**, and **visit the ten strategies**.

**MAT 4**

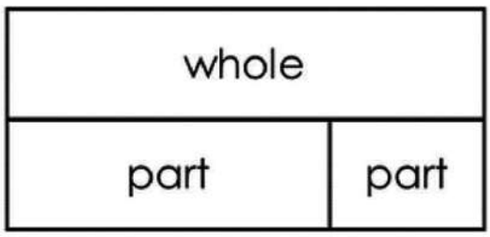
**Reflect** for action

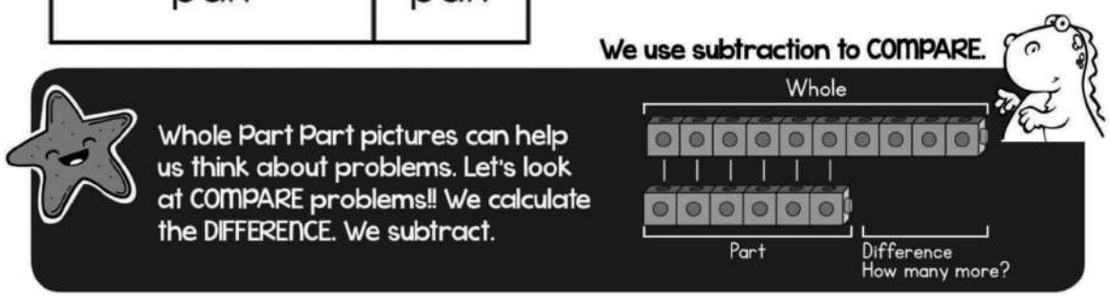
# PART-PART-WHOLE STRUCTURE

**Additive relations (AR)** refers situations and calculations involving adding or subtracting.

## A family of 8 number sentences

Most additive relations problems involve 3 numbers that are all measuring the same unit.
They can be structured using a part-part-whole diagram:




Key Ideas

By the end of this umthamo you should be able to:

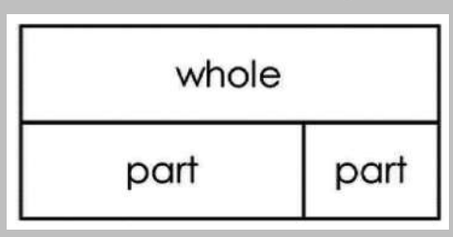
- **Write the family of 8 related facts** for every addition/subtraction fact (so you relate subtraction to addition);
- **Rewrite a subtraction problem as a missing number addition problem**. Solve subtraction using addition.
- **Vary the position of an unknown** to create equivalent missing number problems

Additive relations (AR) refers to situations and calculations involving adding or subtracting.
Adding and subtracting fit together in a part-part-whole structure.
Subtraction is the inverse of addition. Addition is the inverse of subtraction.
There is a family of 8 related additive relations number sentences for each number triple.

**Additive relations** are structured as **whole = part + part**
To understand additive relations it is helpful to know what is true for ALL additive relations.
Most additive relations used in primary school, relate three numbers in a part-part-whole structure.

Diagram

This is a part-part-whole diagram.




**NA 3.1**

Demonstrate an understanding of the **part-part-whole relationship** and link between the operations of addition and subtraction

**Example**

### The number triple 3, 4 and 7

For example, the three numbers (a number triple) are 3, 4 and 7. They are structurally related to each other. The numbers 3 , 4 and 7 fit together.

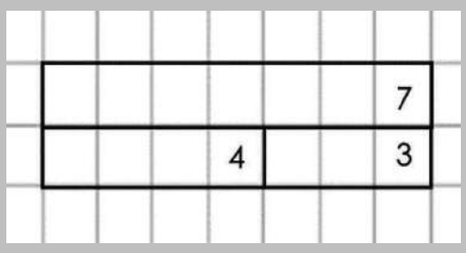
3 add 4 is the same as 7.
3 + 4 = 7

A fact like 3 + 4 = 7 does not exist all on its own.
A part-part-whole diagram can help to see how the numbers 3, 4 and 7 fit together.

**Example**

### Additive relations structure for 3-4-7

**A part-part-whole diagram (3-4-7)**

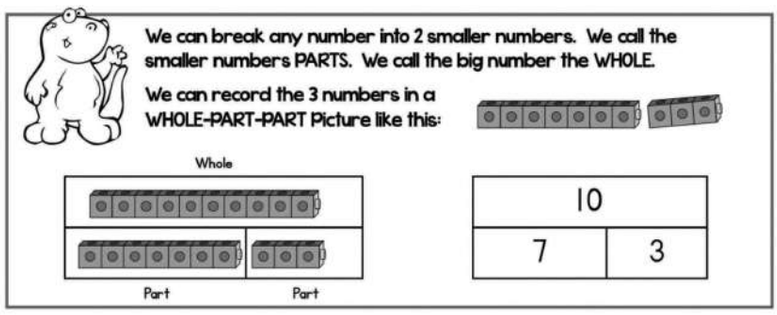



Notice that the top and the bottom of a part-part-whole diagram must be the same length. The two parts, 4 and 3, fit together to make the whole, 7. The part-part-whole diagram uses the **length** of a rectangle to structure an additive relation. This is a helpful shift from numbers as discrete objects to numbers as a measurement (length).

**Example**

### Example: A family of number sentences (3-4-7)

| | |
|---|---|
| 4 + 3 = 7 | 7 = 4 + 3 |
| 3 + 4 = 7 | 7 = 3 + 4 |
| 7 – 3 = 4 | 4 = 7 – 3 |
| 7 – 4 = 3 | 3 = 7 – 4 |




Once you have the part-part-whole diagram, there are then a family of 8 related number sentences.

Knowing that ALL additive relations consist of 3 numbers, that have a part + part = whole structure is very helpful. A part-part-whole diagram helps learners to notice the additive relation structure which shows how the 3 numbers fit together. Any addition fact that learners know, means that they then know 7 other facts too. They can also relate addition to subtraction.

Of course we can have additive relations consisting of more than 3 numbers:
7 = 2 + 2 + 3,

But to check or work with this number sentence we break it up to work with 3 numbers at a time.
7 = 2 + 2 + 3,
7 = 4 + 3. Here we simplified to 3 numbers, using the additive relation: 2 + 2 = 4.

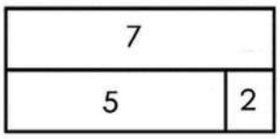
## Linking addition to subtraction

When we know one addition fact like 8 + 3 = 11, then you also know that
11 – 3 = 8, and 11 – 8 = 3. For one additive relation, we actually know 8 related facts.
These related facts all have the same additive relation structure (or part-part-whole diagram).

So, if you know one fact (like 7 – 5 = 2), then you actually know 8 facts

Examples

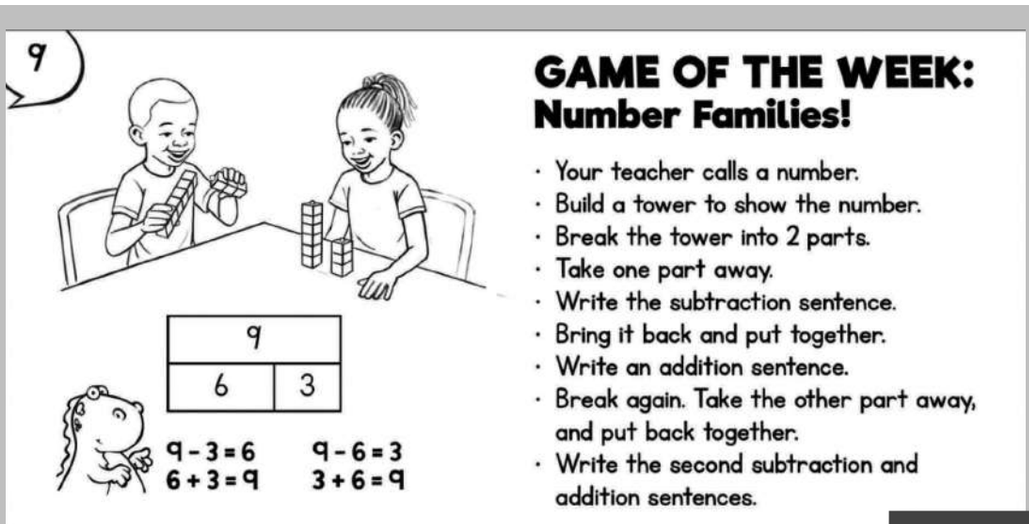
**Part-part-whole structure for 7-5-2**

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|   | 5 + 2 = 7 | 7 = 5 + 2 |
| | 2 + 5 = 7 | 7 = 2 + 5 |
| | 7 − 5 = 2 | 2 = 7 − 5 |
| | 7 − 2 = 5 | 5 = 7 − 2 |

We can structure an additive relation using a part-part-whole diagram.
We call an additive relation such as 7-5-2, a number triple.

If you are trying to calculate: 7 – 5 = ..., you can re-write this as an addition calculation: 5 + ... = 7.

Activity for learners

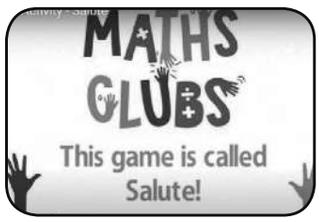



**GPM 3**

**Engage learners productively** in mathematics

## SOME IDEAS FOR YOUR CLASSROOM

**Game**



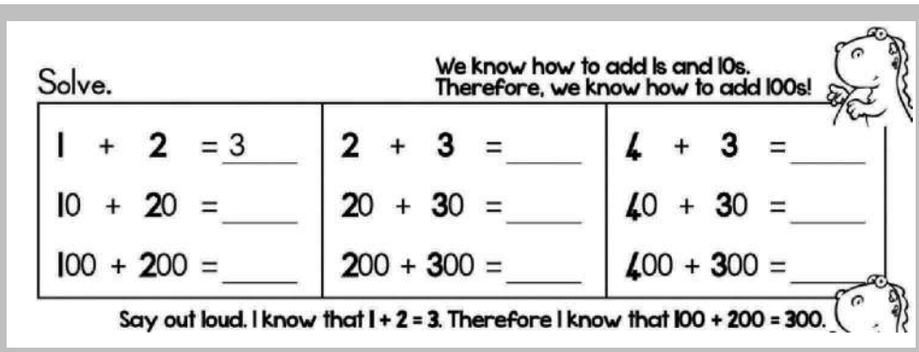

SCAN ME

**Salute!**

See how addition links to subtraction.

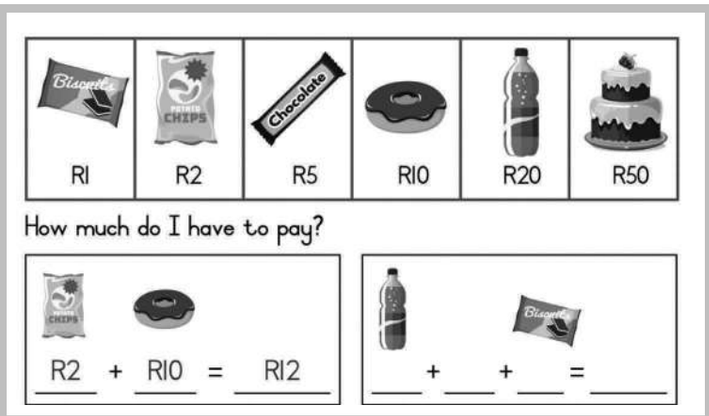
Help your learners make connections between number facts.

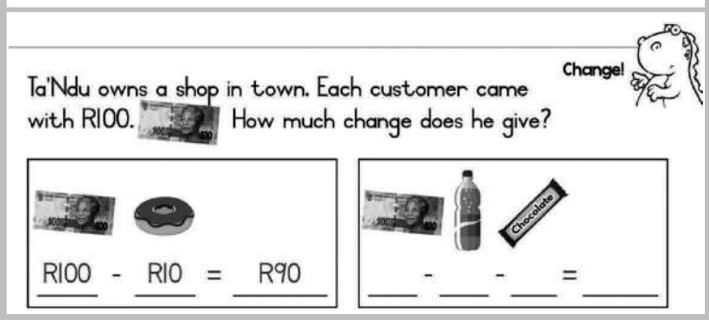
**Activity for learners**




Make sure that when children are working on addition and subtraction, they should work in lots of different contexts. The should have discrete and continuous contexts. Don't forget about money, length, time, capacity and mass!

**Activity for learners**



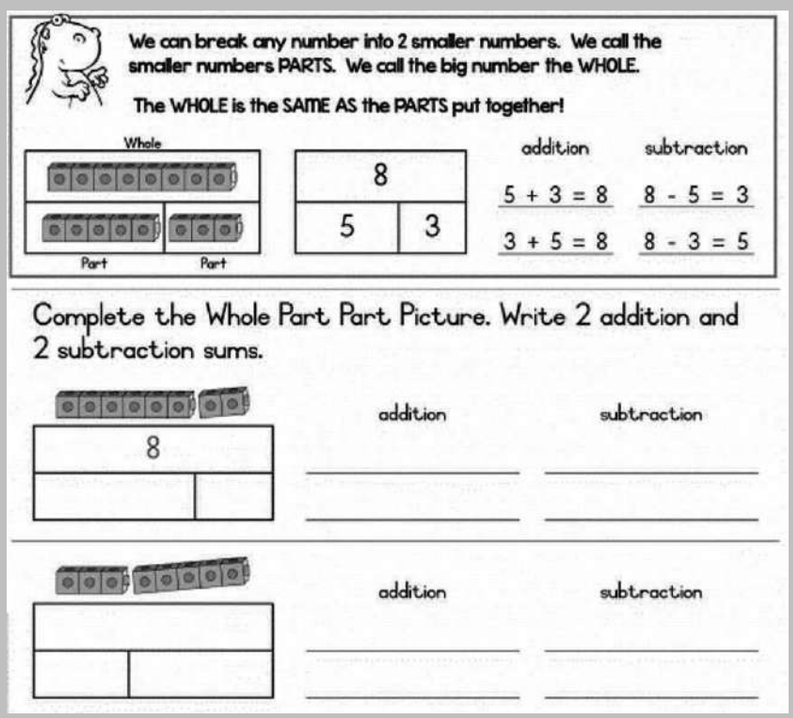




**NA 8**

**Overarching ideas** for school arithmetic. When teaching adding and subtracting, make sure you use discrete objects as well as continuous quantities that can be measured.

Time, length, distance, mass, capacity, volume, money....

Activity for learners



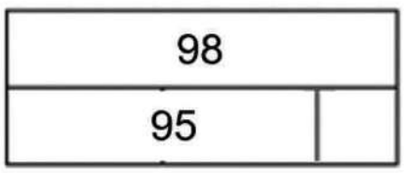

**NA 7**

**Early algebra**

**NA 3.1**

Demonstrate an understanding of the **part-part-whole relationship** and link between the operations of addition and subtraction

When children are secure with the family of 8 number sentences, they can change any subtraction problem into an equivalent addition problem.

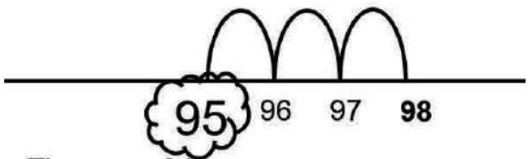



99 – 95 = ...can be re-written as 95 + ... = 99
The addition calculation is much quicker to solve, than the subtraction calculation.

Put 95 in my head.
Count forwards to reach a target of 98.


There are 3 counts.
95 + **3** = 98

Put 95 in my head.
Count forwards to reach a target of 98.



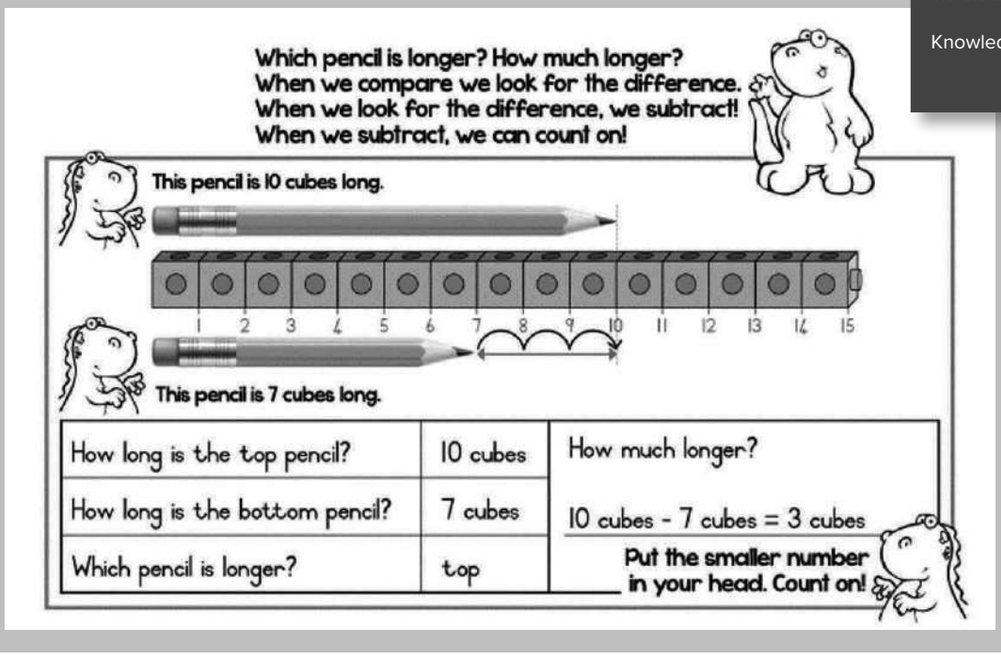

There are 3 hops.
95 + **3** = 98

99 - 15 = ... can be rewritten as 15 + ... = 95.
But which calculation is quicker to solve in this example?

**GPM 2**

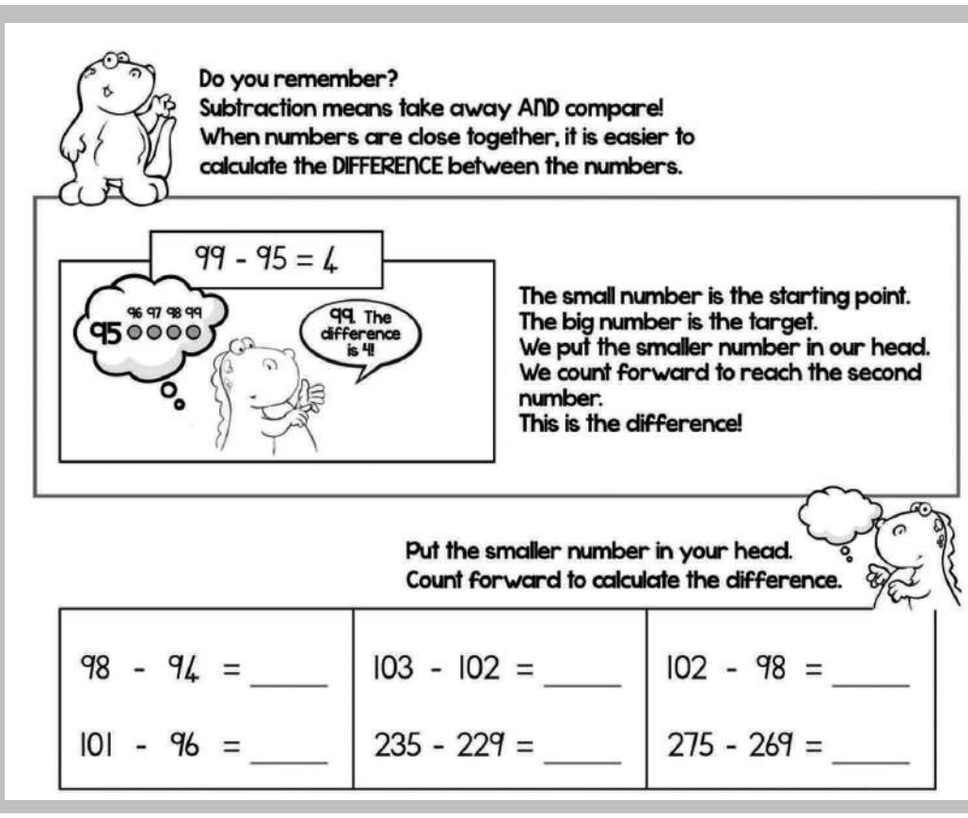
Knowledge of **measurement**

Activity for learners




Remember to use both discrete and continous contexts with your learners

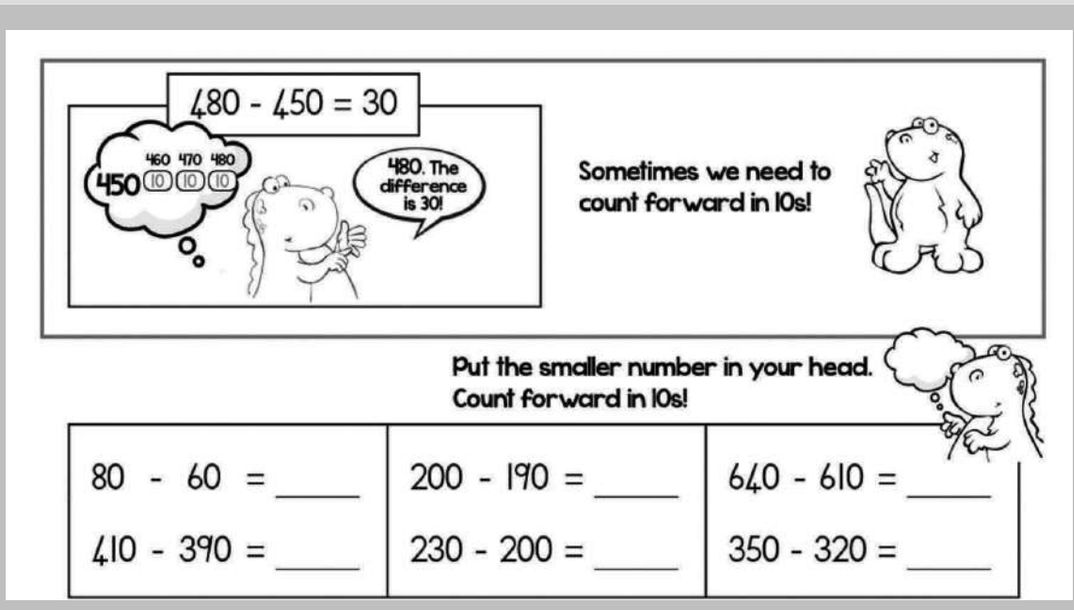
Activity for learners




When numbers are far apart, which strategy is easier for subtraction

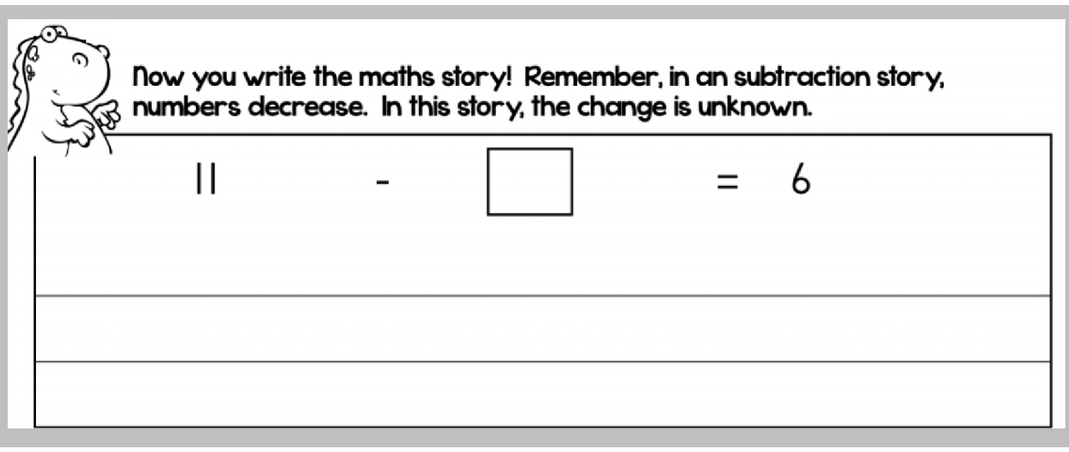
- take-away or
- find the difference?

Activity for learners




It is important for children to tell their own stories. Give them a number sentence, and ask them to make up a word problem or story. Remember their story must have a question, which needs to be answered.

Activity for learners




It is easier for children to draw a part-part-whole diagram and write the family of 8 number sentences for familiar number facts. They should know the bonds of 5 and the bonds of 10 as rapid recall.

This **part-part-whole song** is to the tune of *"He has the world in his hands"*.

*"We have a part-part. And we make it whole.*
*We have a part-part. And we make it whole.*
*We have 7 and 3. And we make it 10.*
*We have 3 and ...? And we make it ...?.*
*We have a part-part. And we make it whole.*
*We have a part-part. And we make it whole.*
*We have 2 and 8 etc*

Act this out using blocks. Show each part and join to make it a whole. Sing this for Bonds of 10 and Bonds of 5.

## Task: 3.2.1

### The number triple 6-4-10

**For all of these questions, use the number triple: 6, 4 and 10**

1. Write you own example of an **additive relation word problem, involving addition.** Use the number triple: 6, 4 and 10

2. Write your own example of an **additive relation word problem, involving subtraction.** Use the number triple: 6, 4 and 10

3. Draw a part-part-whole diagram for 6, 4 and 10.

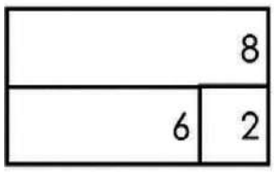
4. Show the calculation 10 - 6 = ... on a **number line**. Use the **take away strategy**. Write down what you say to your class to explain this strategy.

5. Show the calculation 10 - 6 = ... on a **number line.** Use the **difference strategy**. Write down what you say to your class to explain this strategy.

## Task: 3.2.2

### Family-of-8 number sentences

1. Write down the 'family-of-8' number sentences for 8-6-2:

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|  Number triple: 8 and 6 and 2 | 6 + 2 = 8 | 8 = 6 + 2 |
| | | |
| | | |
| | | |

2. Draw a part-part-whole diagram and write down the 'family-of-8' number sentences == for 8 and 2 and 10:

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
| Number triple:<br>8 and 2 and 10 | | |
| | | |
| | | |
| | | |

3. Draw a part-part-whole diagram and write down the 'family-of-8' number sentences for 7 and 11 and 4:

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
| Number triple:<br>7 and 11 and 4 | | |
| | | |
| | | |
| | | |

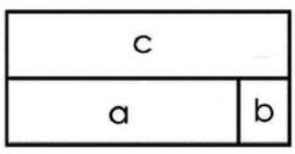
4. Choose your own number triple. Draw a part-part-whole diagram and write down the 'family-of-8' number sentences.

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
| | | |
| | | |
| | | |
| Number triple: | | |

5. Choose your own number triple. Complete the table.

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
| | | |
| | | |
| | | |
| Number triple: | | |

6. Use the number triple **a** and **b** and **c**, and complete the table.

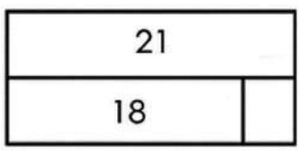
| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|   | | |
| | | |
| | | |
| Number triple: a and b and c | | |

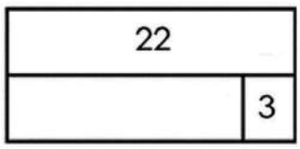
## Task: 3.2.3

### Make a worksheet for missing numbers in 'families-of-8'

1. Imagine you are a teacher for a Foundation Phase class.
   Create a missing number problem worksheet for your learners to complete.

Write down the 8 number sentences where learners must find the missing number.
**Make sure you don't give the learners the answer**. Leave a space, or write a "box"

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|  | 18 + ☐ = 21 | 21 = 18 + ☐ |
| | | |
| | | |
| | | |

| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|  | | |
| | | |
| | | |
| | | |

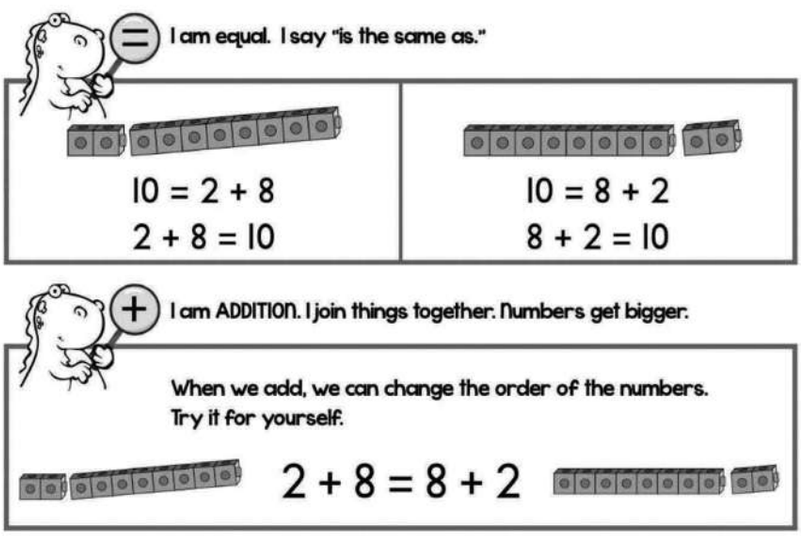
| Part-part-whole diagram | Family-of-8 number sentences | |
|---|---|---|
|  | | |
| | | |
| | | |
| | | |

When you know that there are family of 8 equivalent number sentences, **you can use subtraction to solve an addition problem.**
For example: 4 398 – 4 395 = … can be re-written as 4 395 + … = 4 398.

## Task: 3.2.4

### Two minute tango – re-ordering

| | | | |
|---|---|---|---|
| 1 | Circle two numbers that add up to 10.<br>7 4 2 3 9 | 11 | Circle two numbers that add up to 20.<br>15 4 1 16 8 |
| 2 | Circle two numbers that add up to 10.<br>5 4 1 6 8 | 12 | Circle two numbers that add up to 30<br>18 14 12 7 19 |
| 3 | 7 + ____ = 10 | 13 | Circle two numbers that add up to 30.<br>10 14 9 16 13 |
| 4 | 17 + ____ = 20 | 14 | Double 50 = ____ |
| 5 | 70 + ____ = 200 | 15 | Half of 150 = ____ |
| 6 | 30<br>22 ____ | 16 | 69<br>69 + ____= 100 |
| 7 | 430<br>422 ____ | 17 | 13 + 4 + 7 = ____ |
| 8 | 56 + ____ = 60 | 18 | 2 + 6 + 28 = ____ |
| 9 | 56 + ____ = 80 | 19 | 3 + ____ + 47 = 59 |
| 10 | 256 + ____ = 280 | 20 | 19 + ____+ 11 = 35 |




**GPM 4**

Teach a **balanced mathematics curriculum**

**NA 7**

**Early algebra**

Commutative property:
For which operations (add, subtract, multiply, divide) can you swop the order around, and the answer stays the same?

## Summary 3.2

- Explain the **key ideas** in this umthamo. Use examples and drawings where appropriate.
- Draw a **part-part-whole diagram** for grade 4s (the number range is thousands).
    - First use numbers to show the **family of 8 equivalent number sentences.**
    - Next use one **“missing number”** (call it n or x) to show the family of 8 equivalent number sentences.

# ADDITIVE RELATIONS WORD PROBLEMS

In this unit we discuss the different types of word problems which learners need to solve in primary school. We use problem situations to explain the mathematical operations of add and subtract.

Key Ideas

**By the end of this umthamo you should be able to**

1. Distinguish and use the two models for additive relations situations:
   a. dynamic situations where here is a **change**; and
   b. static situations where there is a **comparison**.

2. Identify and create **dynamic** addition and subtraction word problem types like
   - change (increase) problems like join / combine and
   - change (decrease) problems like separate / take away.
3. Know that the **dynamic (change) problems** are generally easier for learners to solve and have the form **"start +/- change = result".**
4. Identify and create **static** addition and subtraction problem types like collection problems and compare problems
5. Know that the **static** (**collection and compare) problems** are generally harder to solve and have the form **"part + part = whole"**
6. Know how to **introduce compare word problems**, for example compare word problems can be made easier to solve by
   a. making the 1:1 matching action more explicit or
   b. asking the compare question as: 'How many are missing? How many don't get?"
7. **Vary the position of the unknown** in any problem type.

Additive relations describe situations in the real world. You should be able to hear a story sum (word problem), and write down a number sentence. You should also be able to be given a number sentence and tell a story sum (word problem).

**Think about a taxi. Can you make up a story for each problem below?**

| 10 | |
|---|---|
| 8 | 2 |

| | | |
|---|---|---|
| 8 + ____ = 10 | 10 = 2 + ____ | 8 = 10 - ____ |
| ____ = 8 + 2 | ____ = 10 - 8 | 2 = 10 - ____ |
| ____ = 10 - 2 | 2 + ____ = 10 | 10 - ____ = 2 |

## Four additive relation word problems types

Learners learn to make sense of number operations (like add or subtract) from actions and situations that happen in the world around them. The **mathematical operations** describe actions in the real world.

Many maths textbooks start with the mathematical idea (like + or -) and then apply these operations to the real world. So word problems are seen as an application of mathematics that is already known.

But, when working with young learners – they do not know about the mathematical idea of an operation (like + or −) yet. We are teaching them the mathematical idea. **As a teacher you can't start, with what the learners don't know.** You therefore start with what the learners know from the world around them. They have had many experiences and been in situations which they can draw on.

A Grade R child does not know how to read or calculate:

3 + 2 = ☐

But they have experienced a situation in the real world like this:

3 people are in a room. 2 more people enter the room. How many people there are now?

As teachers we introduce the mathematics from what the learners already know. We show them how situations they have experienced, can be thought about mathematically.

To introduce learners to the concepts of adding and subtracting, we need to know **which situations involve additive relations**. We call situations involving adding and subtracting "models". There are two models for additive relations and four word problems types. It is important to know all the word problem types, so that you make sure you learners can work with all four of them.

| Models for additive relations | Additive relations word problem types | |
|---|---|---|
| ACTION / CHANGE<br>like a video<br>**start-change-result** | Change (increase)<br>Combine/join | Change (decrease)<br>Separate/take away |
| NO ACTION/ STATIC<br>Like a photo<br>**part-part-whole** | Collection | Compare |

1. Learners need to work with both **dynamic** and **static** models.
2. Learners should solve all four problem types: **change (increase)** and **change (decrease)**, **collection** and **compare.**
3. You can structure any additive relation using a **part-part-whole diagram.**
4. In this umthamo we discuss the two models for additive relations and the four additive relations word problem types.

**NA 3.2**

Describe and **use addition and subtraction problem types**

There are 4 types of addition and subtraction word problems: change (increase), change (decrease), collection and compare.

Lets talk maths

## Let's talk maths: models and types

A **model** is an example which that is used to follow, or copy an idea.

A fashion model gives an example of what to wear or how to look.


A model car is a small replica (or example) of the actual car.

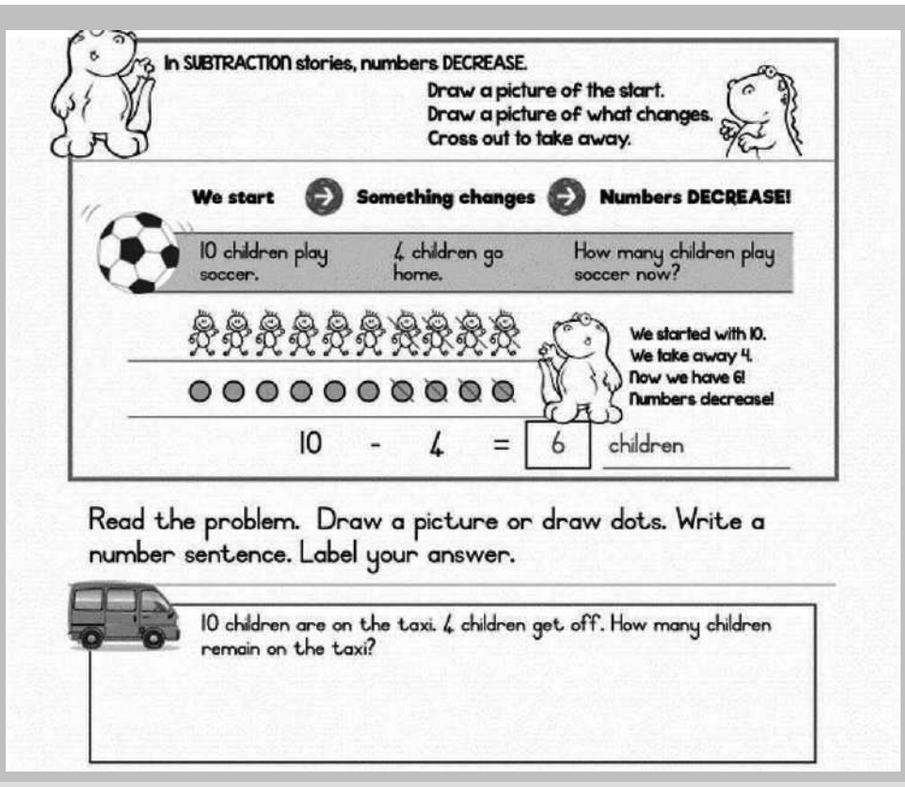
The two models for additive relations situations are:

- Dynamic: Action / change (start-change-result)
- Static: No action (part-part-whole).

A **type** is a category that has particular characteristics.
Word problem types are categories of word problems which have particular actions in the word problem story or situation. The main additive relation word problem types are: change, collection and compare.

This is a change decrease word problem.

Activity for learners




**NA 3.2**

Describe and use **addition and subtraction problem types** and the range of increasing sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## Adding and subtracting as ACTION (start-change-result)

Learners (and many of us) first learn about adding and subtracting as a **change**. For adding there is an action of **joining or combining**. For subtraction there is an action **removing or taking away**.

In both change (increase) and change (decrease) problems, there is an action.
The general form is start-change-result.

## Change (increase) problems

A change (increase) problem is an additive relation word problem type which involves a START, then there is a CHANGE which increases the start to get a bigger RESULT.

Examples

**Example: 5 + 2 = ☐**
Most learners are introduced to adding as an ACTION of joining or combining..

| You have 5 sweets. You get 2 more sweets. How many sweets do you have now?<br><br>5 + 2 = ☐ | | |
|---|---|---|
| **Start**<br>You have 5 sweets. | 5 | ○○○○○ |
| **Change**<br>You get 2 more sweets. | +2 | ○○ |
| **Result**<br>How many sweets do you have now? | = 7 | ○○○○○○○ |

There is an ACTION which shows that something has changed. It is like a MOVIE.
There is a beginning, then a change, and you get a result.

At the **start** there are 5 sweets.
Then something happens. There is a **change.** 2 more sweets are given/bought.
Now we have a new situation and have to find the **result.**

| **Start** | **+/-Change** | **= Result** |
|---|---|---|
| 5 | + 2 | = ☐ |

Two small parts are joined or combined to make a bigger whole.

## Change decrease problems

A change (decrease) problem is an additive relation word problem type which involves a START, then there is a CHANGE which decreases the start to get a smaller RESULT.

Examples

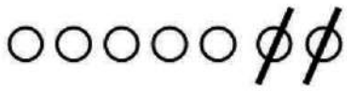
**Example: 7 – 2 = 5**

We often introduce learners to 'subtract' as a take-way ACTION.
A small part is taken away or removed from the whole, and we are left with another small part.

You have 5 sweets. You eat 2 sweets. How many sweets do you have now?

7 – 5 = ?

There is a whole (7). A small part (2) is removed. Now only part (5) is left.

| | | |
|---|---|---|
| **Start**<br>You have 5 sweets. | 7 |  |
| **Change**<br>You eat 2 sweets. | −2 |  |
| **Result**<br>How many sweets do you have now? | = 5 |  |

There is an ACTION which shows that something has changed. It is like a MOVIE.
There is a beginning, then a change, and you get a result.

This is a dynamic story. There is an action, like in a movie/video.
At the **start** there are 7 sweets.
Then something happens. There is a **change.** 2 sweets are removed/taken/eaten.
Now we have a new situation and have to find the **result.**

| **Start** | **+/-Change** | **= Result** |
|---|---|---|
| 7 | - 2 | = ☐ |

One big part is separated into two smaller parts. Something happens to one of the smaller parts.

**NA 2.2**

Show how **counting** is used as fundamental mediating resources to relate **operations on collections** to the basic arithmetic operations.

**NA 3.2**

Describe and use **addition and subtraction problem types** and the range of increasing sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## SOME IDEAS FOR YOUR CLASSROOM

Activity for learners

### Word Problems of the Week

Maths helps us tell stories about change.
In ADDITION stories, numbers INCREASE.

Draw a picture of the start.
Draw a picture of the change.

**We start** → **Something changes** → **Numbers INCREASE!**

| 6 children are on the taxi. | 4 more children get on. | How many children are on the taxi altogether? |
|---|---|---|
| ○○○○○○ | ○○○○ | 6 + 4 = 10 children |

Read the problem. Draw a picture or draw dots. Write a number sentence. Label your answer.

9 children are on the taxi. 2 more children get on. How many children are on the taxi altogether?

Zibalo scored 7 goals this season. Today she scored 3 more. How many goals has she scored altogether?

Mam'Thi baked 10 loaves of bread in the morning. She baked 6 loaves of bread in the afternoon. How many loaves of bread altogether?

Word Problems

## Adding and subtracting as COMPARISON (part-part-whole)

But adding and subtracting situations are not ALWAYS about change. Sometimes there is NO ACTION. The situation is not like a movie. It is more like a photograph.

You have to imagine or introduce an action into the situation. This is the case for **collection** and **compare** problems.

The **static** word problems are more difficult for learners than the **dynamic** word problems.

You can help learners with the **compare problems** types by:

- **Making the 1:1 matching action explicit** in the problem situation. So the stories are about matching spoons to forks, or matching locks to keys;
- **Asking: How many are missing?** Or How many don't get? or How many more to reach the target?. How many are short?

## Collection problems

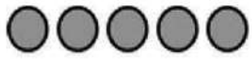
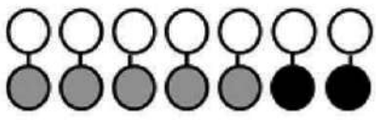
A collection problem is an additive relation word problem type which is static. There are TWO PARTS which together make a WHOLE collection.

Examples

**Example: 7 = 5 + ☐**

Addition and subtraction is also about comparison.
We can compare a big collection (whole), to the smaller parts of that collection.

| |
|---|
| You have 7 sweets. 6 of your sweets are red. The rest are blue. How many blue sweets do you have?<br><br>7 = 5 + ?<br><br>There is a big whole (5). It is made or two part – the red part and the blue part |

| | |
|---|---|
| **Whole**<br>You have 5 sweets. |  |
| **Part**<br>5 of your sweets are red.<br>**Part**<br>The rest are blue. |  |
| How many blue sweets do you have? |  |

There is NO ACTION. Nothing changes. It is like a PHOTO. There is just **one set** or collection (a **whole**).
When you look closer you see that the whole set is actually made of **two different parts**: red and blue.

| | | | | |
|---|---|---|---|---|
| whole | = | part | + | part |

This collection problem could be solved with a missing number calculation:

| | | | | |
|---|---|---|---|---|
| 7 sweets | = | 5 red sweets | + | some blue sweets. |
| 7 | = | 5 | + | ☐ |

If this is modelled, then the child may start with all 7 sweets (the whole). There is only one set of things. But then they realise that the **whole set** is actually made of **two parts**: red and blue. They often don't know what to do now. To work out how many are blue, they have to know to use **1-to-1 matching to compare** the red sweets to the sweets in the whole.

Notice there is nothing in the word problem which tells that child what to do or what the action is. They have to learn that this needs an **action of 1:1 matching** (that is learnt over time, with experience). For this reason, **collection problems** are generally harder for young learners to solve than the change problems.

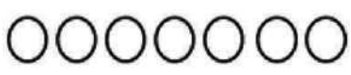
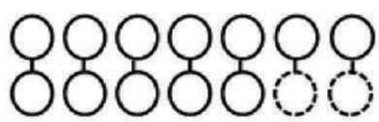
## Compare problems

A compare problem is an additive relation word problem type which is static. A smaller PART is compared to the bigger WHOLE, to find the difference.

Examples

**Example: 7 = 5 + ☐**

Addition and subtraction is also about comparison.
We can have one group or amount which is compared to another group or amount.
This is an example of **compare** problem.

| You have 7 sweets. I have 5 sweets. How many more sweets do you have than me?<br><br>7 = 5 + ☐ | |
|---|---|
| **Whole**<br>You have 7 sweets. |  |
| **Part**<br>I have 5 sweets. |  |
| **Part**<br>How many more sweets do you have than me? |  |

**NA 3.2**

Describe and **use addition and subtraction problem types**

There is NO ACTION. Nothing changes. It is like a PHOTO. This time there are **two sets**: your set and my set.

You have big set (or whole).
Then I have another smaller set (it is only part of your set).

I need to find how many am I missing?, or

This compare problem could be solved with a missing number calculation:

| | | | | | |
|---|---|---|---|---|---|
| 7 your sweets | = | 5 my sweets | + | some more sweets. |
| 7 | = | 5 | + | ☐ |

If this is modelled, then the child may start with the two sets of sweets: Your set of 7 sweets and their set of 5 sweets. They often don't know what to do now. To work out how many are more sweets you have, they have to know to use **1-to-1 matching to compare** the two sets.

Notice there is nothing in the word problem which tells that child what to do or what the action is. They have to learn that this needs an **action of 1:1 matching** (that is learnt over time, with experience). For his reason, **compare problems** are generally harder for young learners to solve than the change problems.

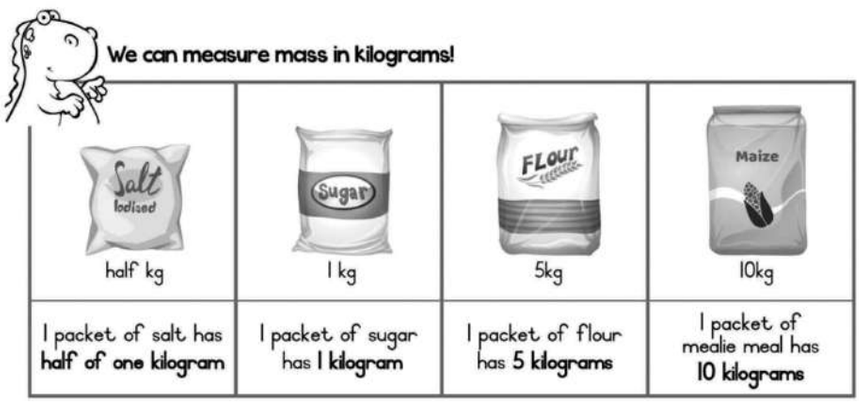
You can help learners learn to introduce a 1:1 matching action by changing the wording of the question:

Like this:

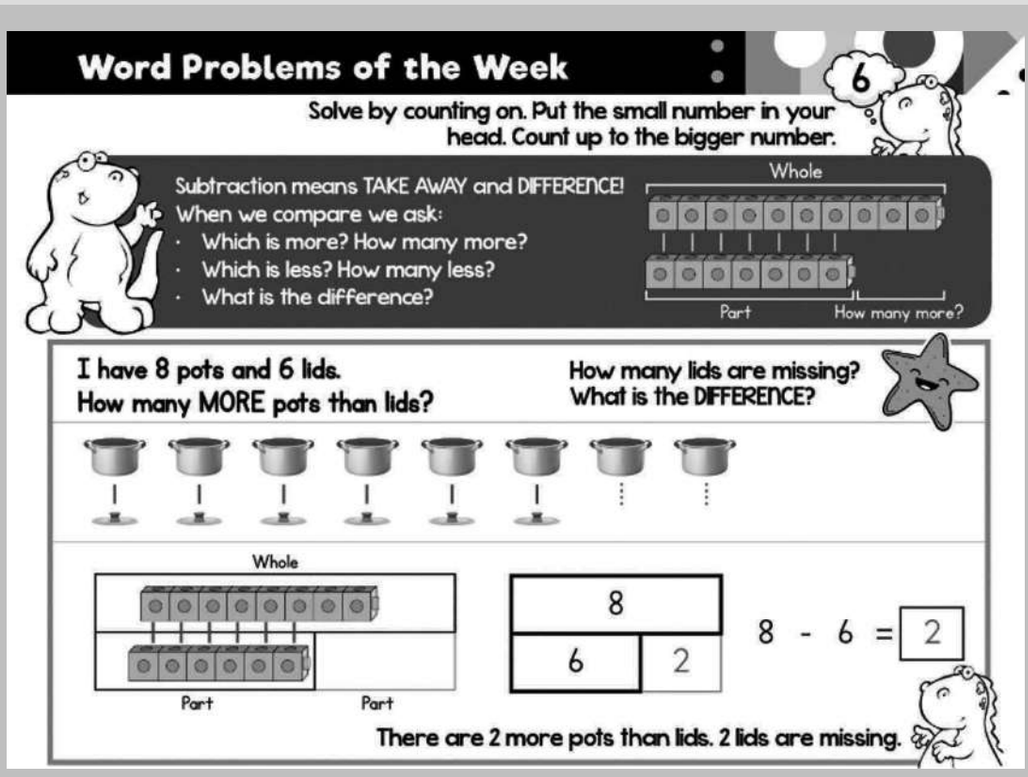
- You have 7 sweets. I have 5 sweets. **How many sweets must I get to have the same as you?**
- You have 7 sweets. I have 5 sweets. **By how many sweets, am I short?**

You can also help learners learn to introduce a 1:1 matching action by having a problem situation where 1:1 action is explicit. For example:

- There are 7 bottles. 5 bottles have lids. How many lids are missing?
- There are 7 locks. There are 5 keys. How many locks don't have keys?
- Dont forget to also compare continuous quantities that can be measured.



Activity for learners




Notice that the situation **requires 1:1 matching**: for each pot there can only be 1 lid. This gives a hint to learners to introduce 1:1 matching action into the problem.

**But don't all additive relations situations have the same structure**?
Every additive relation number triple has a **part-part-whole structure**, right?
Correct.
But when thinking about situations in the real world that involve adding and subtracting, we also focus on the ACTION of the story.

There are two models for the ACTION in the story or word problem.

5. Either, the situation is **dynamic: start - change - result**
6. Or, the situation is **static: whole – part – part**

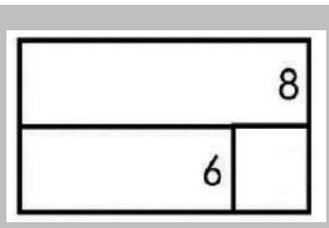
Once you know the "start-change-result" story, you can then imagine or draw the "part-part-whole" diagram for that story.

For example with this **change (decrease) word problem:**

Examples

There are 8 birds in a tree. 6 birds fly away. How birds are left?
You can't see any ACTION in the 8-6-2 part-part-whole diagram.

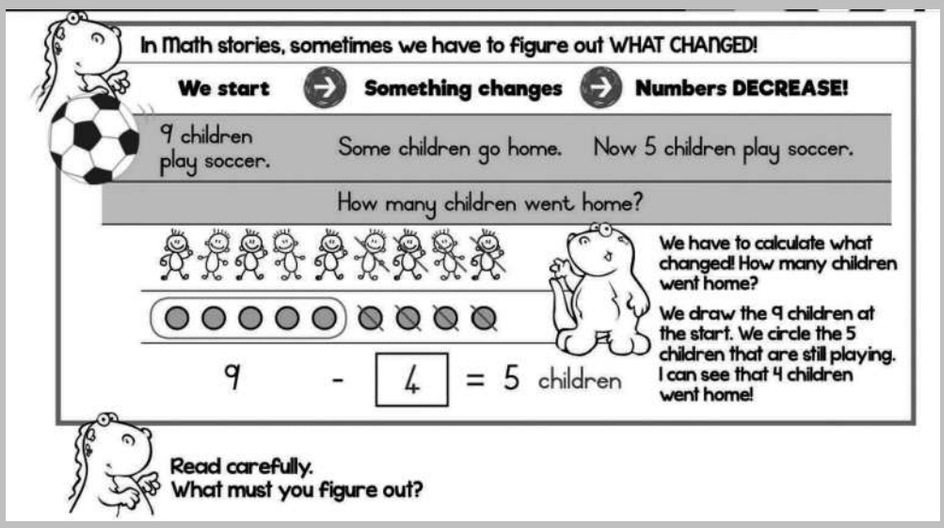
But you can tell or act out the story: You start with 8 birds in a tree, then 6 bird fly away. Now you are left with 2 birds

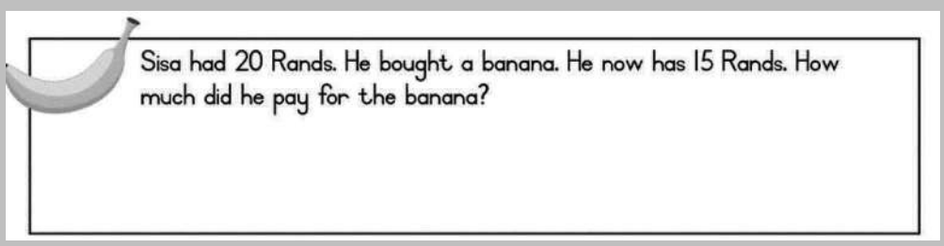


## SOME IDEAS FOR YOUR CLASSROOM

Activity for learners

### Subtraction as take-away or remove

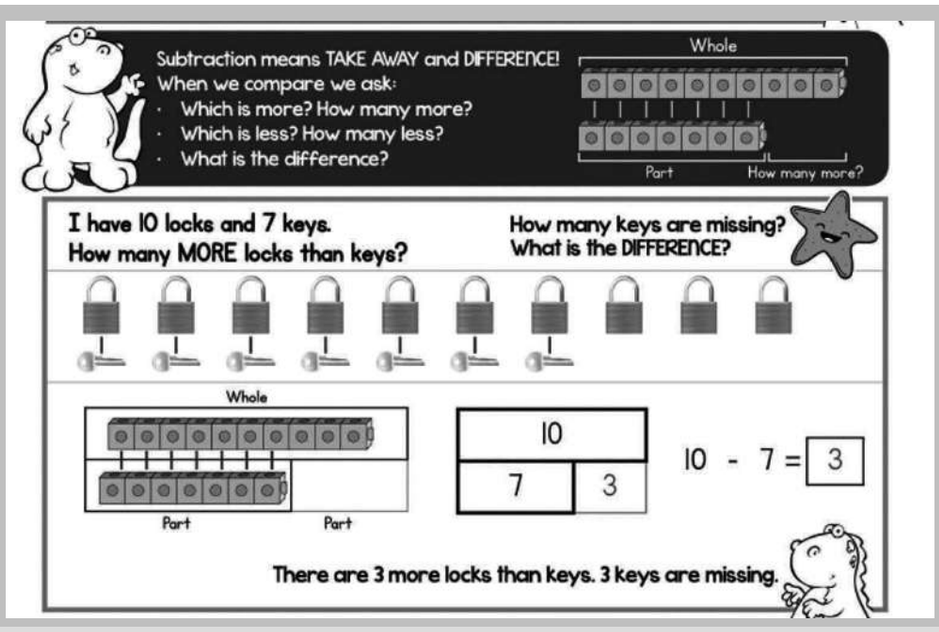



### Subtract as the difference or compare

This compare problem uses **discrete objects** to show subtraction as "**the difference**"

Activity for learners

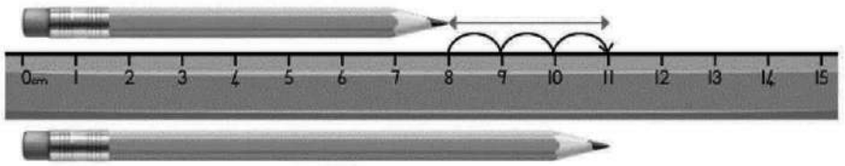



**NA 2.1**

Demonstrate a knowledge of key ways of **representing or modelling numbers,** and moves between these **representations.**

This compare problem uses the context of length, to show subtraction as :the difference

Activity for learners




| | | |
|---|---|---|
| How long is the top pencil? | | How much longer? |
| How long is the bottom pencil? | | cm - cm = cm |
| Which pencil is longer? | | |

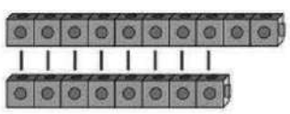
This is a **compare problem**. Locks must match one-to-one with keys. Children focus on "how many are missing?" (the difference)

Activity for learners

Solve by counting on. Put the small number in your head. Count up to the bigger number.

Show the problem using cubes. Complete the Whole-Part-Part Picture. Write 1 subtraction sentence.

There are 10 locks and 8 keys. How many keys are missing?



| | |
|---|---|
| 10 | |
| 8 | |

_____

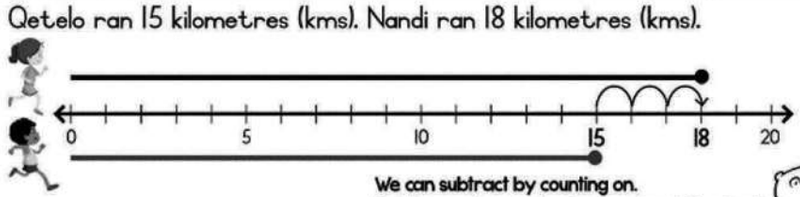
This is a **compare problem**. Kilometres match one to one with kilometres in the continuous quantity of length. Children focus on what is the difference?

Activity for learners

We look for the difference when we subtract! Put the smaller number in your head. Count on!

A kilometre is a unit of measurement. There are 1000 metres in a kilometre!

Qetelo ran 15 kilometres (kms). Nandi ran 18 kilometres (kms).




We can subtract by counting on. Put the smaller number in your head. Count on!

| | |
|---|---|
| Who ran less? _____ | Solve: |
| How much less? _____ | 18 kms - 15 kms = kms |

**GPM 2**

Knowledge of **measurement**

We use metres to measure leangth or distance.

1000 m = 1 km
100 cm = 1 m
1000 mm = 1 m

## Which is the best calculation strategy for subtraction?

To **represent/show their working** children could use **number pictures, number lines or number sentences.** To **calculate** they use different **strategies**: count all, count on, break down the second number, visit the ten.

For subtraction they could use a **'take away'** or **'find the difference'** strategy, or they could use a **part-part-whole diagram** to rewrite the subtraction number sentence, as an addition number sentence.

Activity for learners

1. Which strategy is most efficient for 31 – 3 = ...?

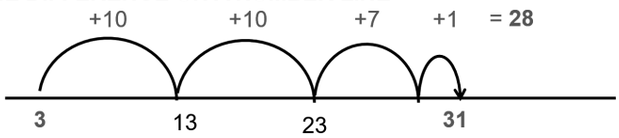
**A. TAKE AWAY ON A NUMBER LINE**


31 – 3 = …

Solving on a number line, using **count back to take away**

Write down the first number. (Put it near the right).
Count or jump back to take-away
Where you land is the answer.

**B. FIND THE DIFFERENCE ON A NUMBER LINE**



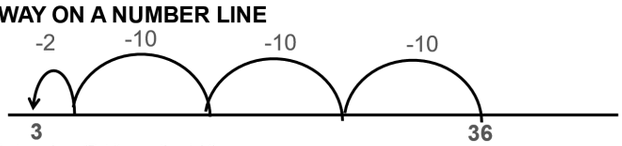

31 – 3 = …

Solving on a number line, using **count forward to reach a target**

Write down the first number
Count or jump back to take-away
Where you land is the answer.

2. Which strategy is most efficient for 36 – 33 = ...?

**A. TAKE AWAY ON A NUMBER LINE**

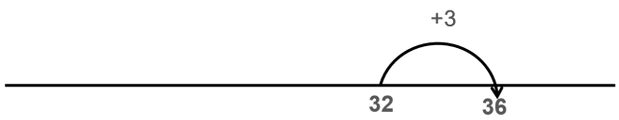



36 – 32 = …

Solving on a number line, using **count back to take away**

Write down the first number. (Put it near the right).
Count or jump back to take-away
Where you land is the answer.

**B. FIND THE DIFFERENCE ON A NUMBER LINE**




36 – 32 = …

Solving on a number line, using **count forward to reach a target**

Write down the first number
Count or jump back to take-away
Where you land is the answer.

Which strategy is more efficient, depends on the numbers involved.

When you **find the difference** on a number line you put both numbers onto the line. Jump from the smaller to the bigger number. The answer is how far you jump.

When you **take-away** on the numbers you write the first number on the number line (towards the right). You jump back using the second number. The answer is where you land.

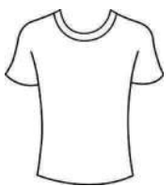
**NA 3.2**

Describe the range of **representations** and increasing sophisticated and flexible **calculation strategies** used to solve addition and subtraction problems.

## Task: 3.3.1

### Additive relations word problems

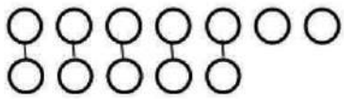
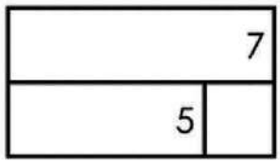
1. Imagine you are teaching grade 2s. Explain how to solve each problem:
   Circle to show whether this situation like a MOVIE (with a change/action) or like a PHOTOGRAPH (no change/action)

| | |
|---|---|
| A.  | I have 7 sweets. I eat 5 of the sweets.<br>How many sweets are left?<br><br>MOVIE / PHOTO?<br>Start-change-result part-part-whole |
| B.  | I have 5 t-shirts. My father buys me some more t-shirts.<br>Now I have 7 t-shirts. How many t-shirts did my father buy?<br><br>MOVIE / PHOTO?<br>Start-change-result part-part-whole |
| C.  | I have 7 bottles. I only have 5 lids.<br>How many bottles do not have a lid?<br><br>MOVIE / PHOTO?<br>Start-change-result part-part-whole |
| D.  | I have 7 pencils. You have 5 pencils. How many more pencils do I have than you?<br><br>MOVIE / PHOTO?<br>Start-change-result part-part-whole |

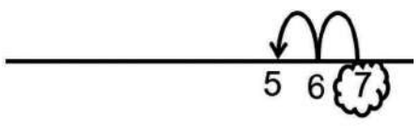
## Task: 3.3.2

**Which diagram or drawing belongs with which word problem?**

1. Discuss in groups. Some diagrams could work for more than 1 problem

| | |
|---|---|
| 1.  | A.<br>I have 7 sweets. I eat 5 of the sweets.<br>How many sweets are left? |
| 2.  | B.<br>I have 5 t-shirts. My father buys me some more t-shirts.<br>Now I have 7 t-shirts. How many t-shirts did my father buy? |
| 3.  | C.<br>I have 7 bottles. I only have 5 lids.<br>How many bottles do not have a lid? |
| 4.  | D.<br>I have 7 pencils. You have 5 pencils. How many more pencils do I have than you? |

2. Solve each word problem, and complete the table**. The first one is done for you.**

| | |
|---|---|
| **Example** | I have 7 grapes. I eat a few of the grapes.<br>I have 5 grapes left. How many grapes did I eat? |
| **Number picture** |  |
| **Number line** |  |
| **Number sentence** | 7 - ☐ = 5 |
| **Answer** | 2 |
| **Unit** | Grapes |
| **Problem type** | Change (decrease) |

| | |
|---|---|
| **A.** | I have 2 tomatoes. I buy some more tomatoes.<br>Now I have 7 tomatoes. How many tomatoes did I have? |
| **Number picture** | |
| **Number line** | |
| **Number sentence** | |
| **Answer** | |
| **Unit** | |
| **Problem type** | |
| **B.** | There are 7 learners. Only 2 learners have hats.<br>How many learners do not have a hat? |
| **Number picture** | |
| **Number line** | |
| **Number sentence** | |
| **Answer** | |
| **Unit** | |
| **Problem type** | |
| **C.** | I have 7 pencils. You have some pencils.<br>I have 2 more pencils than you have.<br>How many pencils do you have? |
| **Number picture** | |
| **Number line** | |
| **Number sentence** | |
| **Answer** | |
| **Unit** | |
| **Problem type** | |

| | |
|---|---|
| **D.** | I have 7 pencils. Only two pencils are sharp. How many pencils are blunt? |
| **Number picture** | |
| **Number line** | |
| **Number sentence** | |
| **Answer** | |
| **Unit** | |
| **Problem type** | |
| **E.** | Sive has 5 points. William has 7 points. How many more points does William have than Sive? |
| **Number picture** | |
| **Number line** | |
| **Number sentence** | |
| **Answer** | |
| **Unit** | |
| **Problem type** | |

**7. Design a worksheet and memo of 4 different additive-relations word problems**.

For Foundation phase design fo**r** Grade 3 (number range is up to hundreds).
For Intermediate Phase design for Grade 5 (number range is up to tens of thousands).
Remember:

Not all questions need to be in the highest number range.

- Include different types of word problems (and label the type in the memo) so your moderator can tell you are covering all types)
- Include some word problems for requiring addition and come requiring subtraction.
- Include different ways that students could answer, and give tick marks to show the marking.

## Task: 3.3.3

### Design a worksheet

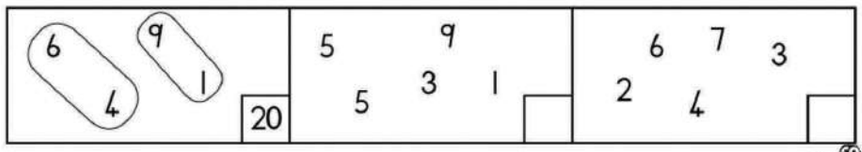
## Task: 3.34

**Two minute tango – re-ordering II**

| | | | |
|---|---|---|---|
| **1** | 7 + 99 = ____ | **11** | 68 – 9 = ____ |
| **2** | 17 + 199 = ____ | **12** | 68 – 29 = ____ |
| **3** | 17 + ____ = ____ | **13** | 96 + 58 + 4 = 100 + ____ |
| **4** | 13 + ____ = 20 | **14** | 99 + 97 + 1 + ____ = 200 |
| **5** | 73 + ____ = 200 | **15** | Double 26 = ____ |
| **6** | 56 + 9 = ____ | **16** | Half of 26 = ____ |
| **7** | 56 + 99 = ____ | **17** | 19 + 13 + 11 = ____ |
| **8** | 45 + ____ = 60 | **18** | 43 + 6 + 27 = ____ |
| **9** | 45 + ____ = 80 | **19** | 13 + ____ + 17 = 49 |
| **10** | 255 + ____ = 280 | **20** | 29 + ____ + 11 = 55 |

You can re-order addition, to **circle or find the tens** ... and the multiples of ten... and hundreds... (the friendly numbers).

Circle the 10s. Can you find the total?




Find the 10. Then add.

Draw a line to find each 10!

| | | |
|---|---|---|
| 10<br>6 + 7 + 4 = 17 | 7 + 8 + 2 = ____ | 7 + 3 + 6 = ____ |
| 5 + 5 + 8 = ____ | 6 + 8 + 4 = ____ | 5 + 9 + 1 = ____ |
| 7 + 6 + 4 = ____ | 7 + 2 + 1 + 5 = ____ | 7 + 5 + 2 + 3 = ____ |

## Summary 3.3

- What are the two models for additive relations?
- Name and give an example for the **four word problem types**.
- Give an example of a **change decrease** word problem with **change unknown**.
- Give an example of **compare problem** with **part unknown**.

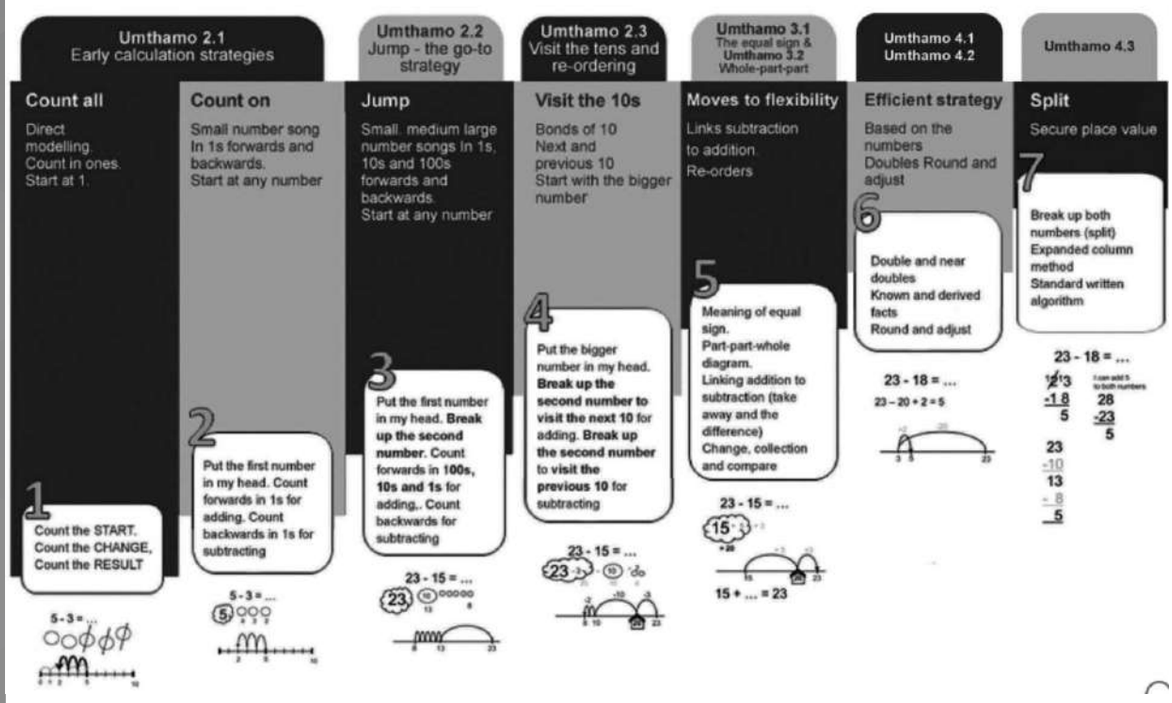
# UNIT 4: CALCULATING FLEXIBILITY

Children start off working **in ones.** They **count all and count on** (count forwards or count backwards) in ones. This is **very inefficient**. Soon we want them to structure in 10s and 1s. As children start to be more flexible in their calculation strategies they **link addition to subtraction (using the whole-part diagram) and finding the difference** rather than just taking away. They break down the second number and:

- use the **go-to strategy or jump strategy** and
- **visit the tens** and **reorder**.

As children start to know their bonds, and understand place value, they can **calculate flexibly** and then use the **break up both numbers (split) strategy**.

## Progression for 7 levels of calculation strategies




At **level 6** learners choose their strategy depending on the numbers involved. They can

- use known and derived addition facts (Umthamo 4.1),
- use doubles and near doubles (Umthamo 4.2); and
- round and adjust (Umthamo 4.3).

At **level 7** children are able to work efficiently with big numbers (100s and 1000s). To do this, they use a split strategy – breaking up both numbers. Children can only fully understand the split strategy when they are very secure in their knowledge of place value.

# UMTHAMO 4.1

In Unit 4 we focus on **calculating flexibly**.

- To do this children must know their **addition number facts and understand place value**.
  - » This could be done mentally, or written down.
  - » The main idea is that children think about the numbers in the calculation and find an efficient way to calculate. Which strategy is used depends on the numbers involved.
  - » Sometimes it is easiest to use a **known or derived fact.**
    I know 3 – 2 = 1. Therefore I know 3 000 – 2 000 = 1 000

## Learning number facts and place value

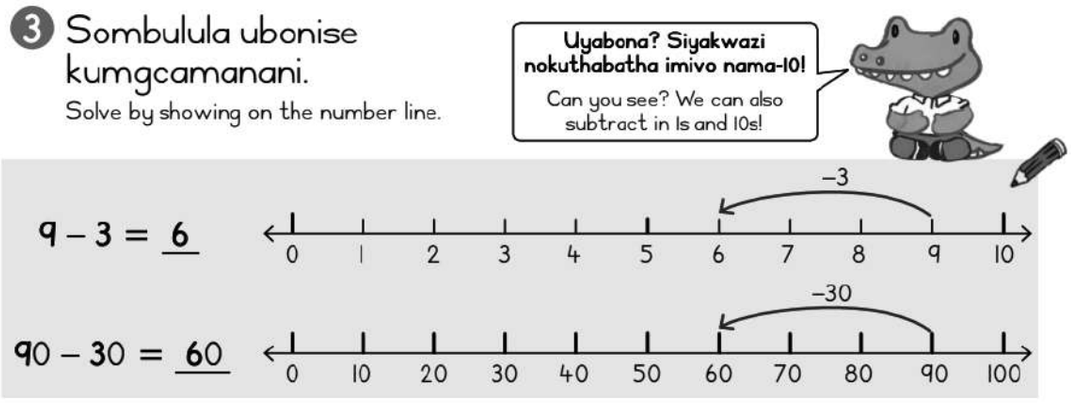
In this umthamo we focus on the **100 addition facts** which children should know instantly (rapid recall) by the end of Foundation Phase.

- If children don't know their number facts, they go back to working in ones counting all and counting on in ones (which is only appropriate in grade 1).
- By Grade 2 they should start count forwards and backwards easily in 10s and 1s.
- At least half the class should know their 100 addition facts well.
- By Grade 3 they should be able to count forwards and backwards easily in 100s, 10s and 1s, as well as in 2s and 5s. Most of the class should know 100 addition facts well.

Children should also learn how to derive facts;

*I know this ...., therefore I know that...*

- I know that 2 + 3 = 5. Therefore I know that 3 + 2 = 5 and 5 – 2 = 3 and 5 – 3 = 2
- I know that 9 – 3 = 6. Therefore I know that 90 – 30 = 60.




To be able to add and subtract big numbers, it is very important that children know most the 100 facts fluently. As a teacher you need to help them make connections and learn the families of facts that belong together.

Key ideas

At the end of this umthamo, you should:

- Quickly complete the **100 addition facts table**
- Name and know all the mental fluencies that children need to add and subtract flexibly
- Derive new addition and subtraction facts using the **whole-part-part diagram** and **families of number sentences**

## 100 Addition Facts

| The first number | + | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | | 4 | 5 | 6 | 7 | | 9 | 10 | 11 |
| | 2 | 3 | 4 | 5 | 6 | | 8 | | | | 12 |
| | 3 | 4 | 5 | 6 | 7 | | 9 | 10 | 11 | 12 | |
| | 4 | | 6 | | | 9 | 10 | 11 | 12 | | |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 | | | | |
| | 6 | 7 | 8 | | 10 | 11 | 12 | | | | |
| | 7 | 8 | 9 | | | | | | | | |
| | 8 | 9 | | | 12 | | | | | | |
| | 9 | 10 | 11 | 12 | | | | | | | |
| | 10 | 11 | 12 | | | | | | | | |

This **addition table** can summarise the 100 facts that that should be known facts by the end of Grade 2.

Start at the + on the top left hand side of the table

Fill in the gaps to complete the rest of the addition table.

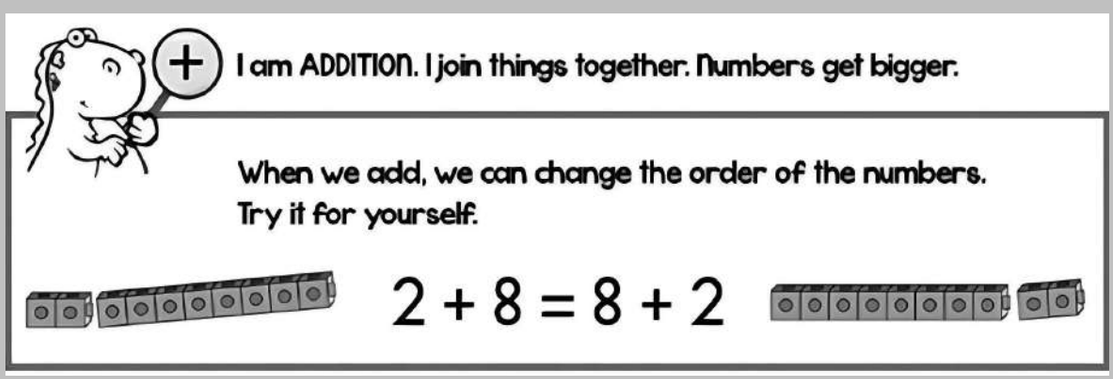
Use this addition table to find the number fact: 2 + 3 = 5.

- 2 is “the first number” in the sum. Move your left finger down the left column to find 2.
- 3 is “the second number” in the sum. Move your right finger across the top row to find 5.
- Move your left finger across. Move your right finger down. You land on 5. So, 2 + 3 = 5.

Luckily children don’t need to learn them all at once:

- The 100 facts connect to each other.
- Because you can re-order for addition, immediately there are only 50 facts to learn. If you know 2 + 3 = 5, then you know 3 + 2 = 5.

Example




- Some facts are easier than others.
  - » Add 1 is just the small number song (start at any number and count on 1 more)
  - » Add 2 is just the small number song (start at any number and count on 2 more)
  - » Add 10 is just the medium number song (start at any number and count on in 10s)
  - » The doubles are quick to learn (3 + 3 = 6). And then near doubles are easier too (3 + 4 = 3 + 3 + 1)

- Some facts are more important than others. Make sure children know these very important facts:
  - » Bonds of 5,
  - » Bonds of 10 and
  - » The doubles; and
  - » Bonds of 20.

## Mental Fluencies

Five strands of mathematical proficiency

Kilpatrick & Swafford (2002) identified 5 strands that are interwoven to form mathematical proficiency:

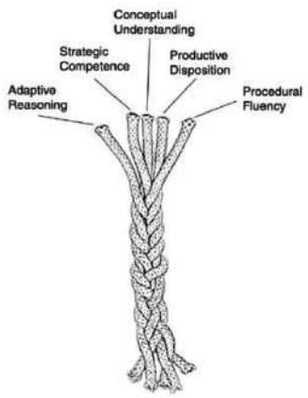
(1) **Conceptual Understanding (Understanding):** Comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean.

(2) **Procedural Fluency (Computing)**: Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.

(3) **Strategic Competence (Applying)**: Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.

(4) **Adaptive Reasoning (Reasoning)**: Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.

(5) **Productive Disposition (Engaging)**: Seeing mathematics as sensible, useful, and doable – if you work at it – and being willing to do the work.




**"The most important feature of mathematical proficiency is that these five strands are interwoven and interdependent."**(page 9, *Helping ChildrenLearn Mathematics*, NRC, 2002)

The National Research Council in the USA has used 5 interwoven strands stands to define what makes someone good at or capable of doing mathematics.[1]

They refer to this as **mathematical proficiency.**

Proficiency means having a high degree of skill or expertise.

1. National Research Council. (2002). Helping Children Learn Mathematics. Mathematics Learning Study Committee, J Kilpatrick and J. Swafford, Editors. Center for Education, Division of Behavioral and Social Sciences and Education.
Washington, DC: National Academy Press.

## Learning 100 number facts by making connections

To add and subtract flexibly, accurately, efficiently, and appropriately children need **procedural fluency.** They should have rapid recall the 100 addition facts. They should be able to quickly derive new facts.

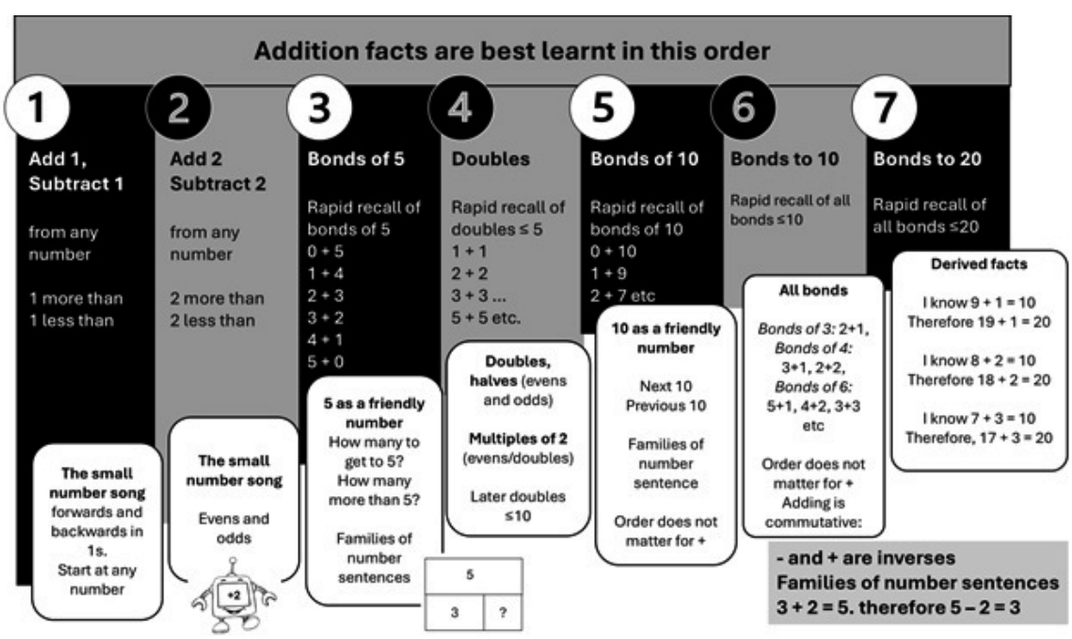
- As a Foundation Phase teacher, you need to help children learn these addition facts in **mental maths**.
- As an Intermediate Phase teacher you will have children who don't know their basic facts – its your job to help them to learn them.

Most children should know at least 100 facts by about the end of Grade 2 beginning of Grade 3.

Luckily learning the facts is not random, or all at once.

| Grade | Mental fluency | Number facts | Representations |
|---|---|---|---|
| Grade R | **Small number song** forwards and backwards. Start at any number | 1 more than my number is...<br>1 less than my number is... | Count all<br>Concrete modelling Number pictures<br>Structured number line 0-10 |
| Grade 1 | **Small number song**. Forwards and backwards. Start at any number.<br>**Medium number song**. Forwards and backwards. Start at 0 or 10....or any multiple of 10 | 1 more than my number is...<br>1 less than my number is...<br>more than my number is...<br>2 less than my number is...<br>Bonds of 5<br>Doubles to 5 | Count all<br>Concrete modelling<br>Structured number pictures (circle the 10)<br>Semi-structured number line 1-10<br>Structured number line 1-20<br>**Count on:**<br>Put the first number in my head count forward for adding<br>Put the first number in my head count backwards for subtracting |

| Grade | Mental fluency | Number facts | Representations |
|---|---|---|---|
| Grade 2 | **Small number song**. Forwards and backwards. Start at any number.<br>**Medium number song**. Forwards and backwards. Start at 0 or 10....or any multiple of 10<br>**Medium number song**. Forwards and backwards. Start at any number | 1 or 2 more than my number is...<br>1 or 2 less than my number is...<br>Bonds of 5<br>Bonds of 10<br>My number plus 10<br>My number minus 10<br>Doubles to 10 | Jump (go-to strategy)<br>Put the first number in my head count forwards/ backwards in 1s<br>Put the first number in my head count forward/ backwards in 10s and 1s<br>.<br>**Visit the tens**<br>**Re-order**<br>**Doubles** |
| Grade 3 | | Doubles to 100<br>My number plus 9<br>My number plus 99<br>Bonds to 20<br>The 100 addition facts | Link subtraction to addition (take-way and the difference<br>Near doubles<br>Round and adjust |




**Tango 1**

Ones, tens and hundreds


**Tango 2**

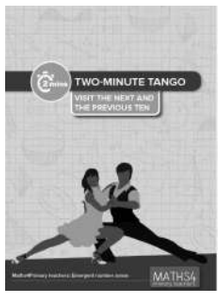
Bonds of 5 and bonds of 10


**Tango 3**

Bonds of 10 and bonds of 20

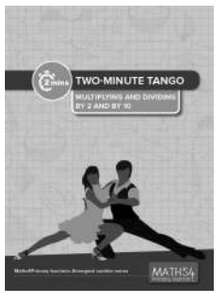

**Tango 4**

Doubling and halving




**Tango 5**

Next and previous 10




**Tango 6**

Multiplying and dividing by 2 and 10

**Tango 7** draws questions from across Tangoes 1-6.
Tango 7 is a good revision to practice the mental fluencies you need to add and subtract flexibly. Every quiz, and the post-test, includes a mental fluency section.


**Jumptrak**

In this course you use jumptrak for:

1. The PRE-TEST and POST-TEST;
2. **Two minute TANGO practice;**
3. PEER MARKER for tasks; and
4. Weekly QUIZZES.

Go online, or go to your app store and download: Jumptrak. Your lecturer will give you your username and password.

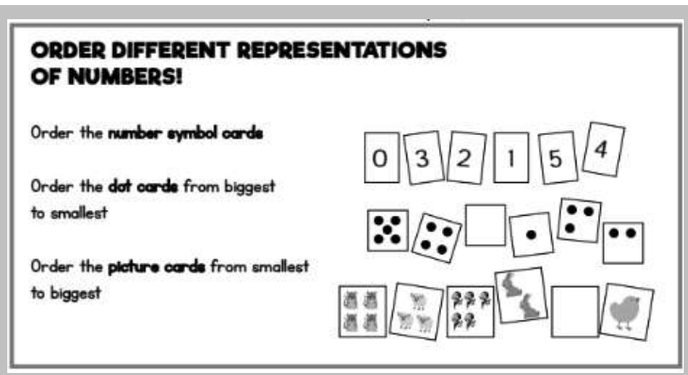
It is your job as (a Foundation or Intermediate Phase) teacher to make sure that children get enough opportunity and practice to know the 100 number facts fluently. Every lesson you have 10 or 15 minutes of mental maths. This is a good time for fluency practice.

**Number facts are usually learnt in this order (from easiest to hardest)**

## 1 Children learn to add 1 to any number. And to subtract 1 from any number

- If children can **count forwards in 1s** from any number (the small number sequence), then they can **add 1** to any number.
  - » They learn that **add 1** is the same as **1 more than**.
  - » They learn that **add 1** is the same as the **next** number. or the number **after.**
- If children can **count backwards in 1s** from any number (the small number sequence), then they can **subtract 1** to any number. They learn that **add 1** is the same as **1 more than**.
  - » They learn that **subtract 1** is the same as **1 less than**.
  - » They learn that **subtract 1** is the same as the **previous** number. or the number **before.**

Example



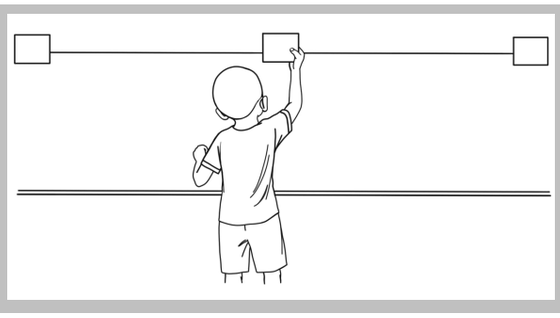

## 2 Children learn to add 2 to any number. And to subtract 2 from any number

This builds on the work done for add 1 and subtract 1. You should use many ways to help children learn these facts:

- Play fizz pop.
- Use strategic counting. Start at a number, Count forwards in 2s. Stop at a number.
- Focus on **odd and even** number songs.
- Use a function machine with +2 and – 2.
- Hop in 2s forwards and backwards on a number line.
- Put a number in the middle of the number line. Which number is 2 smaller? Which number is 2 bigger?
- I am thinking of a number. **'2 more than'** my number is 6. What is my number? _____

**You can use number cards on a number line.** Or draw a number line on the board.

Example

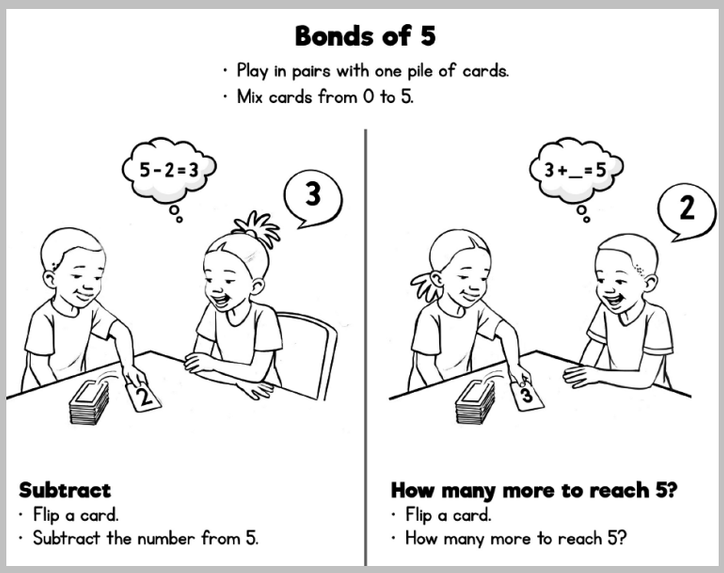


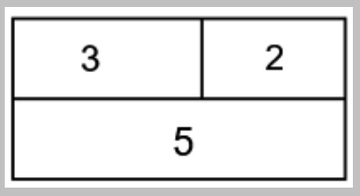
My number is in the middle. **2 more than my number** is to left / right of my number.

## 3 Next, children should learn the bonds of 5 very well

This can be done with one hand using their fingers. They should learn about They should be able to link **subtraction to addition** using a whole-part-part diagram. They should be able to say or write a family of number sentences for each number fact.

Example




**Family of number sentences**

Two addition facts

3 + 2 = 5

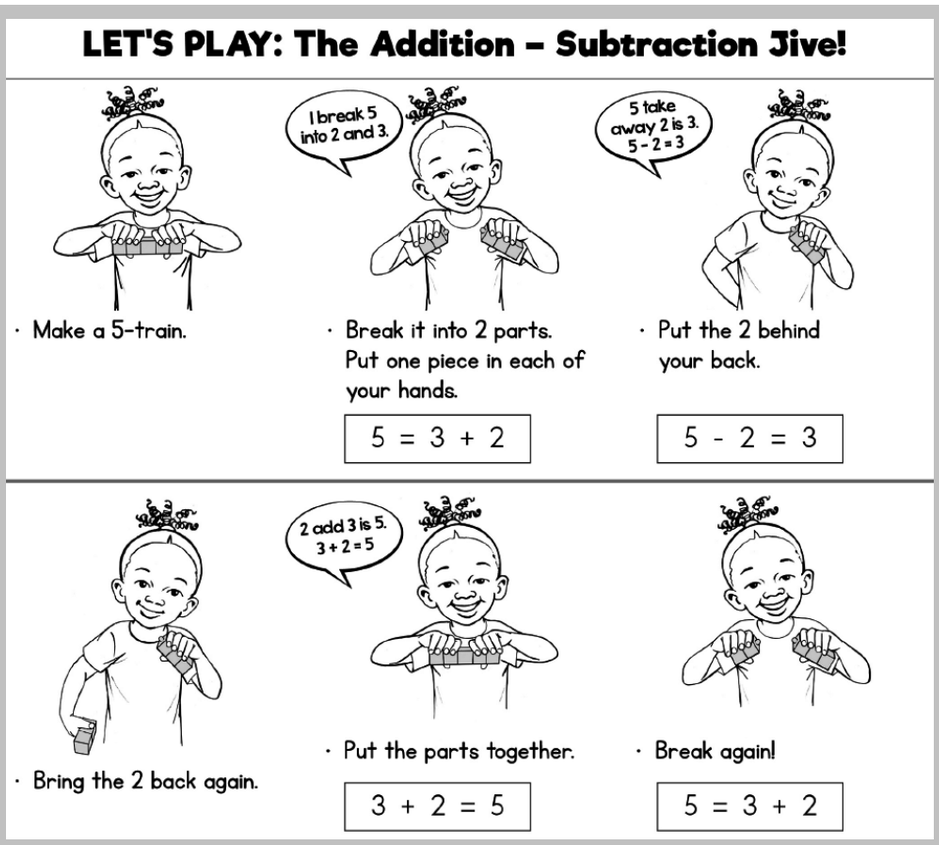
2 + 3 = 5

Two subtraction facts

5 - 3 = 2

5 - 2 = 3

Activity for learners

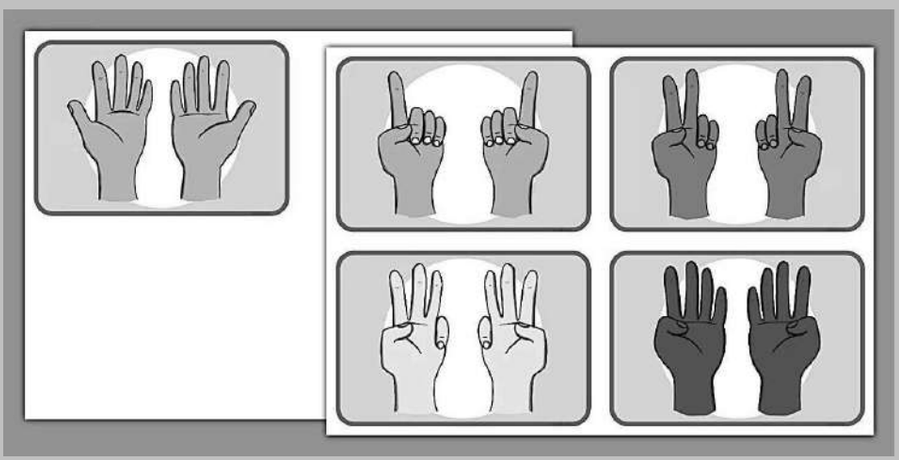


In Grade R and beginning Grade 1, children should start know 1 more, 1 less, 2 more, 2 less and then the bonds to 5.

**4** **Research shows that child learn and remember the doubles easily.**

First they learn the doubles for 1 to 5. Later they learn the doubles for 6 to 10.

Activity for learners



**Game in pairs**

Samp is the leader. They lift their fingers on one hand.

Bean copies samp.

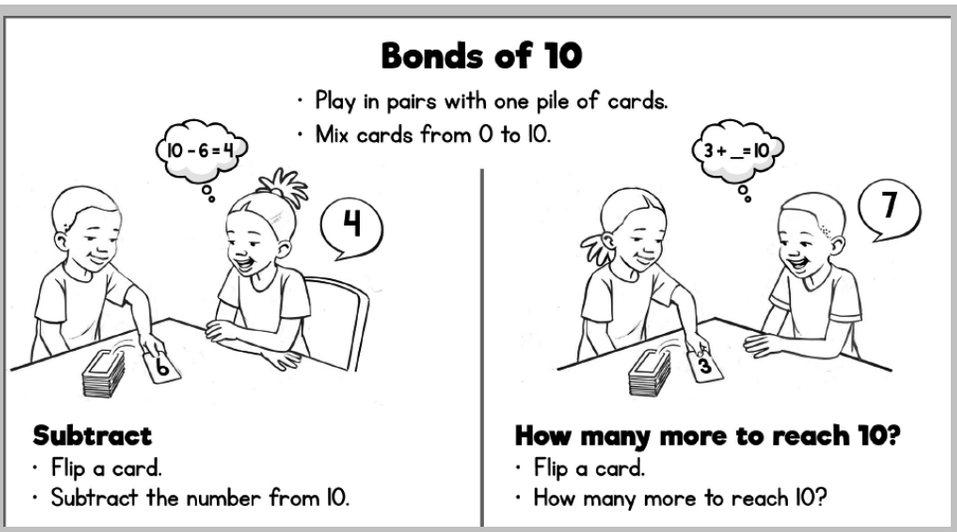
Call out how many fingers together:

Games like 1,2,3 Veza! Doubling

can help children to double and halve.

**5** **By later in Grade 1 and early Grade 2, most of the children should know the bonds of** 10 **very well.**

Activity for learners

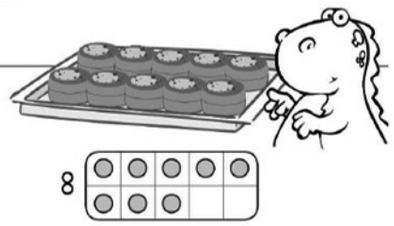


As with bonds of 5, they should be able to use a **whole-part-part diagram** and write a **family of number sentences** for each bond of 10.

Knowing bonds of 5 and bonds of 10 very well, helps children's number sense. They should think about all numbers in relation to a 5 and in relation to a 10. Ten frames – or a tray of 10 scones – as **number pictures** help children develop this number sense.

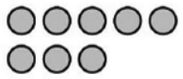
Example

Zibalo loves to bake. She bakes scones to sell.
She ALWAYS bakes scones in rows of 5.
She never places more than 10 scones on one baking tray.



8



| | |
|---|---|
| **STEP 1:** Draw the number. | 8  |
| **STEP 2: Think about 5**<br>• The drawing helps Zibalo think about how the number relates to 5.<br>• Write a number sentence using the number 5. | 8 = 5 + 3 |
| **STEP 3: Think about 10**<br>• The drawing helps Zibalo think about how this number relates to 10. How many more scones to complete the pan?<br>• Write a number sentence using the number 10. | 10 = 8 + 2 |

Children should draw **number pictures,** and think about 5 and think about 10.

Knowing the bonds of 10 well, should help children **link subtraction to addition.** They should realise that they can subtract by finding the difference. The put the smaller number in their head, and count forward to reach the bigger number as a target.

Example

| | |
|---|---|
| 10 - 8 = ___ | 10 - 7 = ___ |
| 8 + ___ = 10 | 7 + ___ = 10 |

I can work out 10 – 7 = _____, because

I find the difference: 7 + _____ = 10.

Put 7 in my head.

Count forwards on my fingers to reach a target of 10.

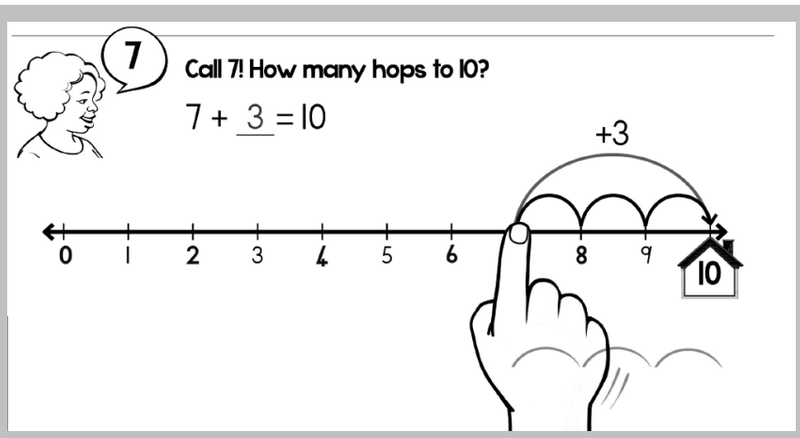
8, 9, 10 ... That's 3 counts. 3 fingers

10 – 7 = 3

In Grade 1 learner should be able to use number sentences with **number symbols.**

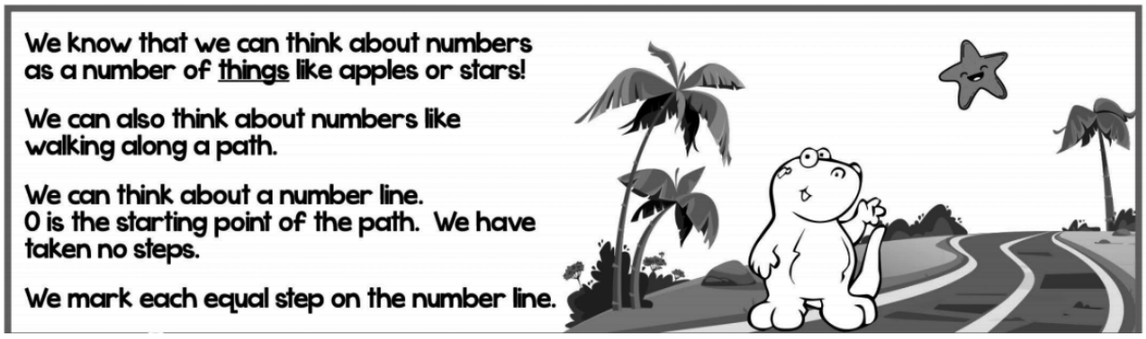
They should be able to use a **number lines**, and think about 5 and thinking about 10.

Example



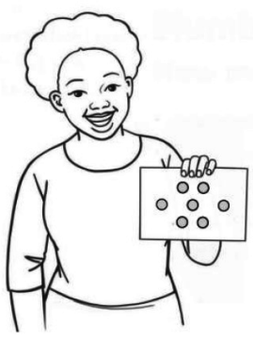

## 6.a Children need to know that 10 is a friendly number, on both pathways to number

They should always structure in 5s and in 10s. So they think about 5s and they think about 10s. They must understand place value. They must add and subtract 10s.




- With discrete objects, they need to know how to bundle 10 ones to make 1 ten.
- With continuous quantities, Zibalo stops and greets the 10 on a number line.

10 is a friendly number in the **counting pathway (discrete objects).** When drawing **number pictures**, children should not draw 10 ones. They can just write 10.



### Number Talk

**How many?**

- Your teacher quickly flashes a dot card.
- Look quickly! How many dots?
- Explain what you see!
- Help your teacher record your model on the board!

## Draw numbers from 10 to 20

In this lesson we will help children to draw number pictures from 10 to 20. The big idea is that from now on, WE DO NOT WANT our learners to draw ten showing the ten ones . We want them to draw the ten as a 10! 10


**Let's put on our 10 glasses! When we wear our 10 glasses, we think in 10s!**


10 is a friendly number in the **measurement pathway (continuous quantities)**. When drawing **number lines**, children show the 10s. Zibalo always stops to visit and greet the 10.

Example

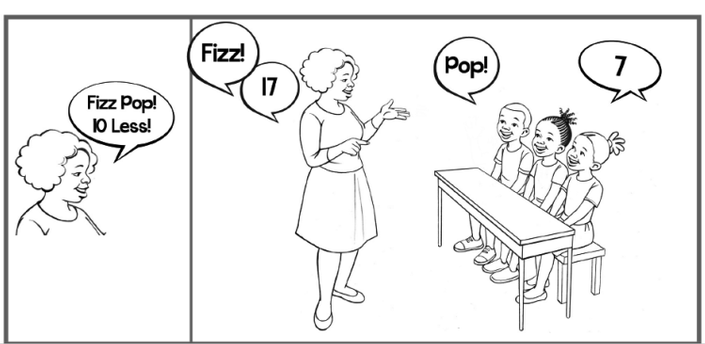

By the end of Grade 2 all children must be able to count forwards and backwards in 1s,10s and 100s.

- We call 1 the small counting number
- We call 10 the medium counting number
- We call 100 the large counting number.

Activity for learners

**10 LESS!**

- Call a number between 10 and 20. Children subtract 10.




**Fizz Pop** can help children learn to

add 10 (10 more)

subtract 10 (10 less)

When they can count in 10s and count in 1s, they can use the **jump strategy.**

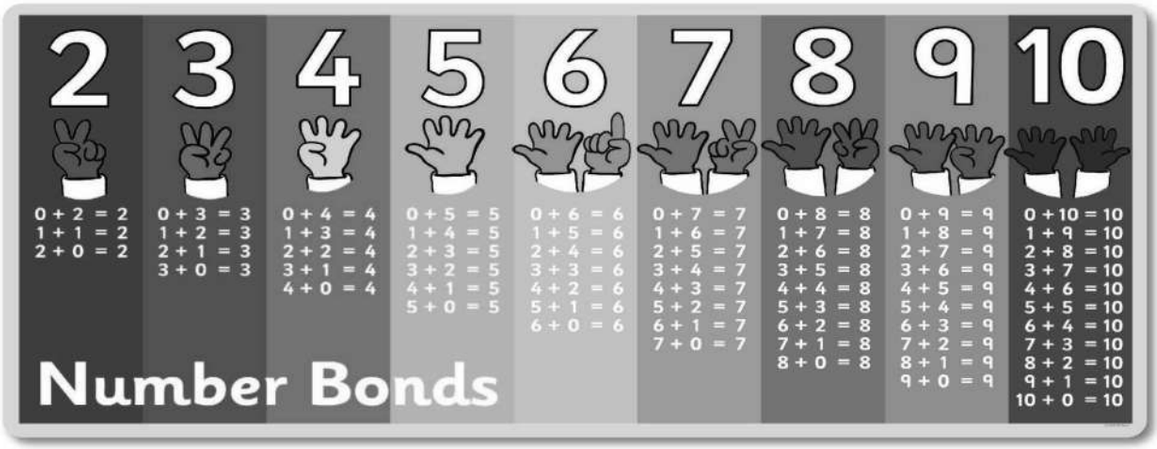
**6.b To be able to visit the ten, children need to know even more number facts about 10:**

- The **next multiple of 10**, and
- The **previous multiple of 10**.
- **How far to the next 10**, and
- **How far to the previous 10.**
- All the **bonds to 10.**

They put the first number in their heads. They find the next ten. They work out how far to the next ten. They break up the second number, to visit the ten.

For example: For 8 + 6 = ..., they must break up 6 to visit the 10. So they need to know the bonds for the second number. In this case they must know that 6 = 2 + 4.

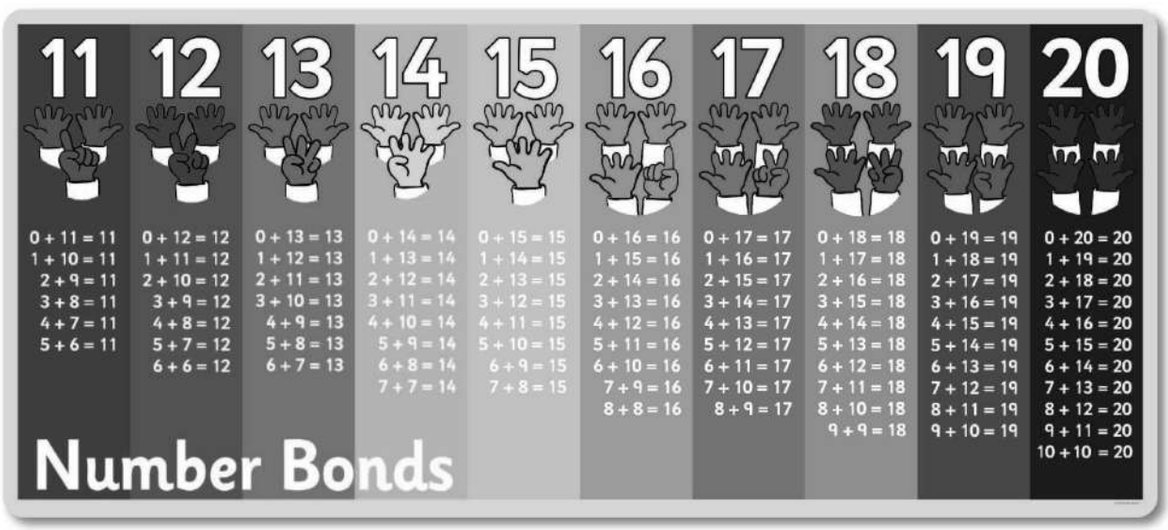
So to visit the 10, they must know all the bonds to 10.




**7 To be able to split number into 10s and 1s, children need to know all the bonds to 20.**

Because they know their bonds to ten fluently, they should also be able to work out all the bonds to twenty.

I know that 9 + 1 = 10. Therefore I know that 19 + 1 = 20



## Place value and starting to work in columns

Several of the formal written algorithms require working in columns. Each column represents the place value unit: ones, tens and hundreds.

At first it is helpful to children to label the columns

Example

| H | T | O |
|---|---|---|
| 1 | 4 | 7 |
| | 8 | 4 |

Children need practice to write and say the numbers in the correct place value column.

Placing 84 below 147 may be difficult. They need to learn to say the numbers with their place value.

Example

147 = 1 hundred and 4 tens and 7 ones

84 = 8 tens and 4 ones

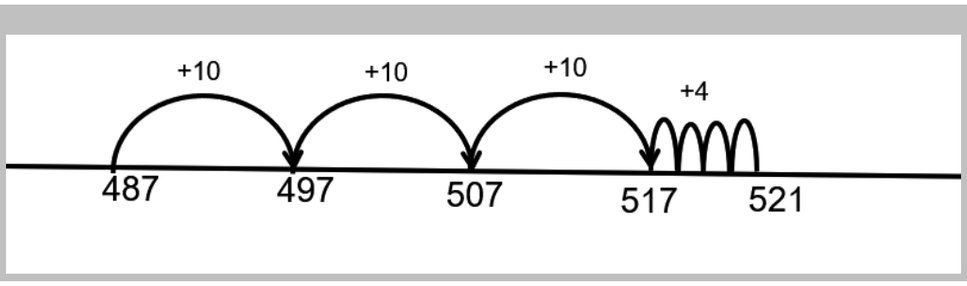
This is easier / transparent in African languages, but not obvious in English.

Writing the numbers starting with the ones and moving from right to left is the opposite to how children read and write numbers. So it is better to read from left to right, and name the first digit (using 100s, 10s, and 1s) and the children put that digit into the correct column on the left. They should check that they have filled up the columns to the right and that the number goes all the way to the ones column. If the ones column is empty, they have made a mistake.

Practicing the go-to strategy or jump in columns is an important step, before working on other written methods.

**THE GO-TO STRATEGY ON A NUMBER LINE: 521 – 487**

Example



## The go-to strategy in columns: 487 + 34

First break down the second number 34 = 30 + 4. The calculation is adding (+) to we add each part.

Example

If children are used to writing the calculation in columns.

| They may write this first: | Then they re-write like this: |
|---|---|
| 487<br>+ 34 | 487<br>+30 Add 30<br>517<br>+4 Add 4<br>521 |

Notice that when breaking down the second number for the go-to strategy:

- The first number is kept whole. You work with 487 (4 hundred and 8 tens and 7 ones, not "four eight seven").
- The second number is broken down into parts. You write down one part at a time.
- No re-grouping is needed.

The whole concept of regrouping is only needed if you start to work in each column (each place value) separately. Re-grouping is only needed if you break down both numbers (See Umthamo 4.3).

## Some ideas of your classroom

All of the expanded methods for breaking down both numbers require excellent knowledge and understanding of expanded notation and of re-grouping. Here are some activities which you can use with your class to build these skills.

Encourage children to use both pathways into number. They should be able to use number pictures, number lines and number sentences (horizontally or in columns). They may choose "their best strategy", that they always use, but they should understand when a teacher or child explains another strategy too.

Make sure children can explain what they have done. Point out when a strategy is inefficient and encourage them to work efficiently.

Activity for learners

### Number talk

Once a week you can offer a **number talk** with children. Give them a calculation, and time to try it for themselves. Then choose children to explain what they did. The most efficient strategy will depend on the numbers involved. We want children to be flexible.

38 + 27 =...

45 – 28 = ...

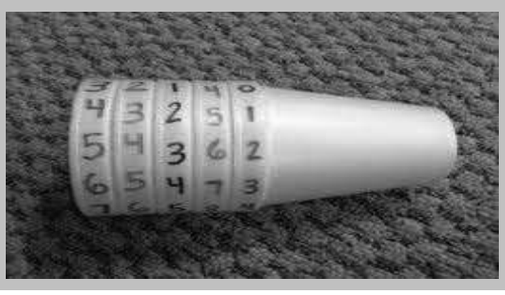
They could use number pictures, number lines and number sentences or columns).
They can use:

- Go-to strategy. Break up the second number
- Visit the tens
- Round and adjust
- Split / break up both numbers: Expanded notation / Write all totals / Shift up / SWA

Activity for learners

Make an expanded notation twister

**You need:** 1 polystyrene cup. A pen. A group of 3.

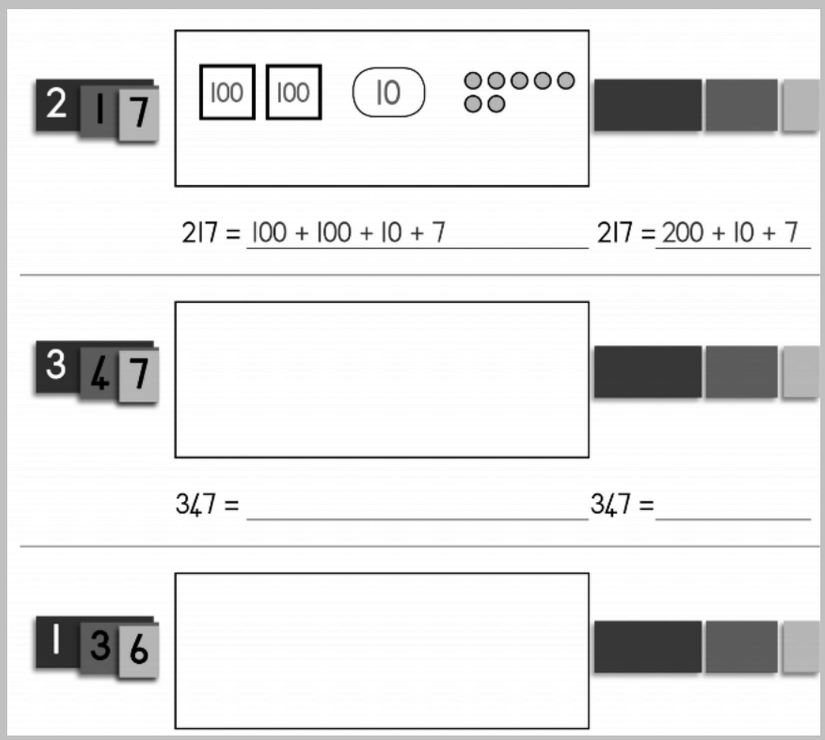



Watch this video for a great teacher resource for expanded notation

SOAN ME

- In groups of 3.
- Decide who will make the ones, who will make the tens, and who will make the hundreds.
- Follow the instructions to make your own expanded notation twister.
- Player 1 twists. The others in the group must say the number name.
- Swop over.
- Player 1 says a number. Player 2 must make that number using the twister. Swop over.

Activity for learners

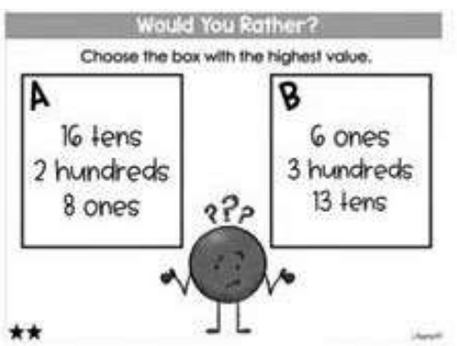


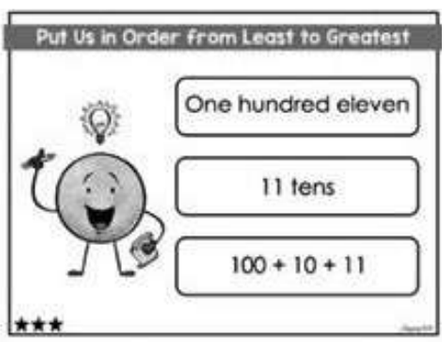

Use or make flash cards to pack out for place value

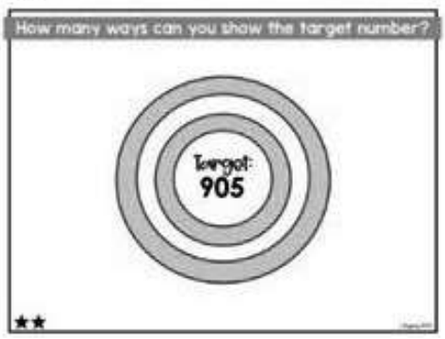
For each example, give one alternative way to break down the numbers. Regroup the numbers.

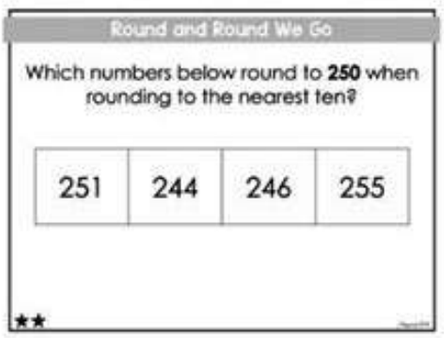
Eg. 217 = 1 hundred + 1 ten + 7 ones AND 217 = 1 hundred + 11 tens + 7 ones

Number talks



## Tasks 4.1.2

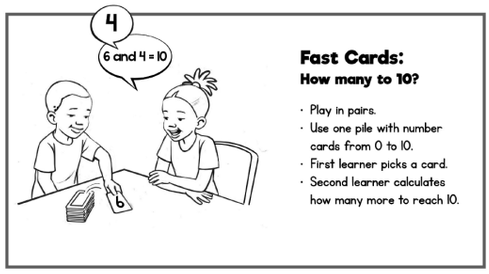
### 100 addition facts

Children need to know 100 addition facts. These mental fluencies are built up slowly over Grades R, 1 and 2. "My number" is a whole number, between 1 and 10.

## 1. Planning mental starters for Grade 1

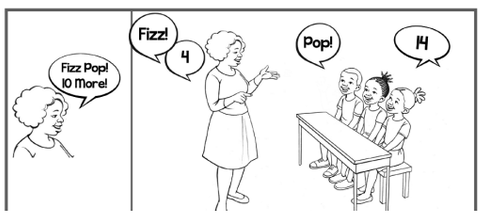
The Foundation Phase head asks you to plan the mental starters for each Grade.

**a.**




**b.**

**Fizz Pop**
**10 More!**
- Start with numbers 1 to 20.

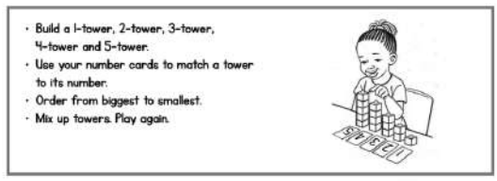



**c.**

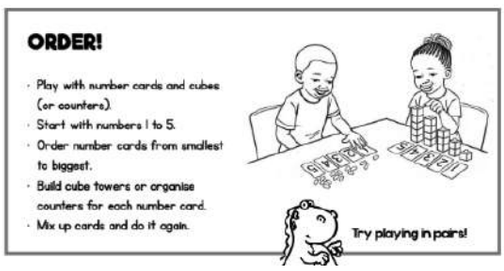
**Fizz Pop**
**1 MORE and 1 LESS**
- Start with numbers from 1 to 20.
- Extend to 50. Extend to 100. Can you go over 100?


**d.**

**Counting: Building Towers!**

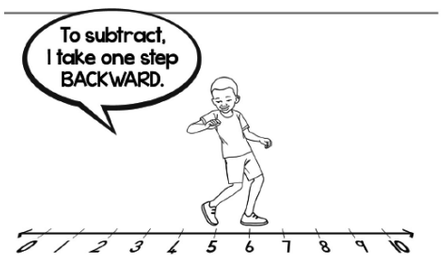



**e.**



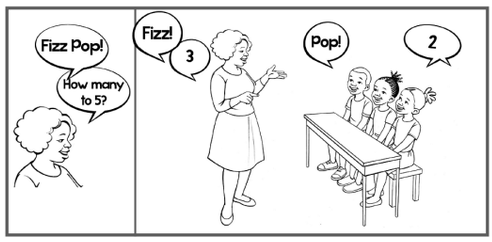

**f.**


**g.**




**h.**

**i.**




**j.**


**k.**

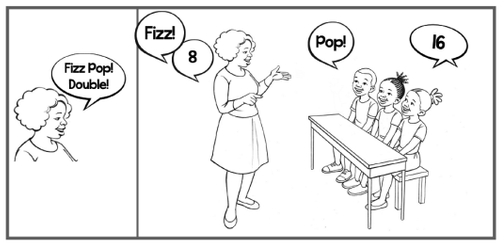
## Counting

### Forward to 10

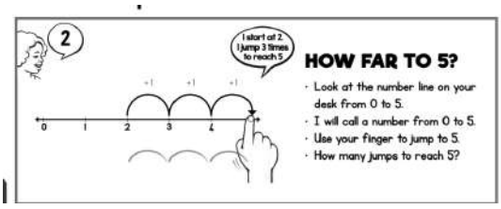
Count forwards from any number between 0 and 10. Stop and stand up on the 10. How many to reach 10?

- Call a number from 0 to 10 (start with 6, 5, 7). 6!
- Ask children to put the number in their head. Tap your head to lock it in.
- Count forward in 1s until we reach 10: 7, 8, 9, 10! That is 4 counts forward!
- We keep track using our fingers.
- Yes 6 add 4 is 10!


**l.**




**m.**




**n.**

## Counting

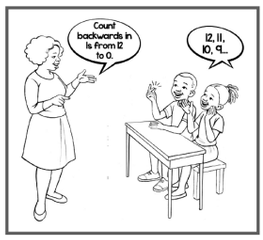
### Backwards in 1s

Count backwards in 1s. Call a number between 0 and 20. Learners count backwards from this number to 0. Clap on the 10.

**Example:**

Count backwards in ones from 12 to 0.

- Start with numbers 0 to 20. Then go to 30.




**o.**

| | | |
|---|---|---|
| 1. | 88 + ☐ = 92 | Fill in 15, 5 and 20 into the number sentences below (11 - 14)<br>15 \| 5<br>20 |
| 2. | 42 – 4 = ☐ | 11. ☐ – 5 = ☐ |
| 3. | 86 + 5 = ☐ | 12. ☐ + 5 = ☐ |
| 4. | 17 + ☐ = 23 | 13. ☐ – ☐ = 5 |
| 5. | 199 + ☐ = 201 | 14. 5 + ☐ = ☐ |

**p.**

## Counting

### Forward to 10

Count forwards from any number between 0 and 10. Stop and stand up on the 10. How many to reach 10?

- Call a number from 0 to 10 (start with 6, 5, 7). 6!
- Ask children to put the number in their head. Tap your head to lock it in.
- Count forward in 1s until we reach 10: 7, 8, 9, 10! That is 4 counts forward!
- We keep track using our fingers.
- Yes 6 add 4 is 10!


She asks you to

- Order these activities from easiest to hardest.
- Show which tasks are about cardinality (discrete objects) and which tasks are about ordinality (in sequence for a continuous measurement)
- Write which fluency each task is building.

| Sequence | Card letter | Cardinality versus ordinality | Which fluency is this building? |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |
| 13 | | | 1 |
| 14 | | | |
| 15 | | | |
| 16 | | | |

**2. Complete these calculations (from MSAP Linking addition to subtraction pre-test).**

1. $88 + \square = 92$

2. $42 - 4 = \square$

3. $86 + 5 = \square$

4. $17 + \square = 23$

5. $199 + \square = 201$

Fill in 15, 5 and 20 into the number sentences below (11 - 14)

| 15 | 5 |
|---|---|
| 20 | |

11. $\square - 5 = \square$

12. $\square + 5 = \square$

13. $\square - \square = 5$

14. $5 + \square = \square$

3. **2 more than**

Remember to derive facts. You can re-order addition.

a. I know that 5 + 2 = 7, therefore I know that _____.

b. Complete all the "2 plus my number" facts that start with 2 + ... = ... .
Write them in the addition table. Write all the 2 + "my number" = …number sentences next to the table.

c. Complete the family of 4 numbers facts for 3 + 2 = 5

| 7 | |
|---|---|
| 3 | |

| | |
|---|---|
| 7 - _____ = 4 | 7 - _____ = 3 |
| 4 + _____ = 7 | 3 + _____ = 7 |

| 10 | |
|---|---|
| 6 | |

| | |
|---|---|
| 10 - _____ = 4 | 10 - _____ = 6 |
| 4 + _____ = 10 | 6 + _____ = 10 |

Do you remember what happens when we break into equal parts?

| 10 | |
|---|---|
| 5 | |

| | |
|---|---|
| 10 - _____ = 5 | 10 - _____ = 5 |
| 5 + _____ = 10 | 5 + _____ = 10 |

d. Write the other 4 number facts that also are in the 3 + 2 = 5 family.

### 5. Bonds of 5

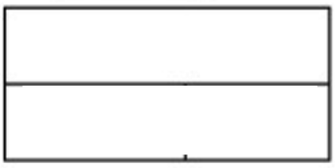
Another key family of facts that children should know fluently are the bonds of 5. These are all the pairs of numbers which add together to make 5.

a. I know that 4 + 1 = 5, therefore I know that _____.

b. For 4 + 1 = 5, complete the wpp diagram and family of equivalent number sentences

Complete the wpp diagram

1 + 4 = 5



Write down the family of 8 number sentences

| | |
|---|---|
| 1 + 4 = 5 | 5 – 1 = 4 |

a. Use known facts to derive new facts. Complete these subtraction calculations

| | | |
|---|---|---|
| 5 – 4 = _____ | 5 – 2 = _____ | 5 – 3 = _____ |
| 50 – 40 = _____ | 50 – 20 = _____ | ___ – ___ = ___ |

Say aloud: I know 5 – 4 = 1 Therefore 50 – 4 = 10

### 6. Bonds of 10

Another key family of facts that children should know fluently are the bonds of 10. These are all the pairs of numbers which add together to make 10.

Remember to derive facts.

a. I know that 8 + 2 = 10, therefore I know that _____.

b. Draw the whole-part part diagram for 1 + 9 = 10.
Write all eight equivalent number facts.

c. Use known facts to derive new facts. Complete these calculations

Say aloud. I know that 6 + 4 = 10
Therefore I know that 60 + 40 = 100

| | | |
|---|---|---|
| 10 – 4 = ____<br>**10**0 – **4**0 = ____ | 10 – 7 = ____<br>**10**0 – **7**0 = ____ | 10 – 5 = ____<br>___ – ___ = ____ |
| 9 + 1 = ____<br>**9**0 + **1**0 = ____ | 8 + 2 = ____<br>___ – ___ = ____ | 3 + 7 = ____<br>**3**00 + 700 = ___ |

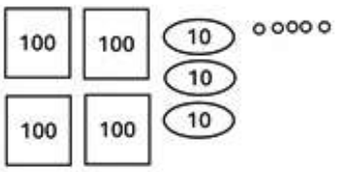
7. **Use > and < and = signs to make these number sentences correct.**

4 tens _____ 40 ones
2 tens _____ 9 ones
49 ones _____ 5 tens
3 tens and 2 ones _____ 2 tens and 12 ones
5 tens and 7 ones _____ 4 tens and 15 ones
tens and 6 ones _____ 3 tens and 16 ones

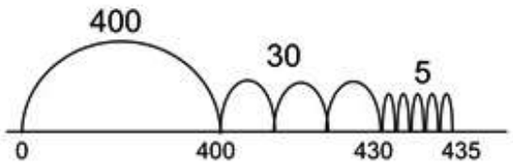
**Place value table**

| Hundreds | Tens | Ones |
|---|---|---|
| 4 | 3 | 5 |

**Number picture**




**Number line**



## Tasks 4.1.3

### Investigate patterns in the 100 square

This is a 100 square. It lists the numbers from 1 to 100.

Notice: It is the not the same as the addition table.

We put the smaller numbers at the bottom.

- Move up the square. The numbers get bigger. Add 10 each time.
- Move down the square. The numbers get smaller. Subtract 10 each time.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 21 | 22 | 23 | **24** | 25 | 26 | 27 | 28 | 29 | 30 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

| | | |
|---|---|---|
| | Choose a number on the 100 square. | **IF** my number is **24,** |
| a. | Find the sum of the numbers on either side of your number. | 23 is to the left of my number<br>25 is to the right of my number<br>**THEN** the sum of the numbers on either side of my number is _____ |
| b. | Choose another number<br>Find the sum the numbers on either side of your number. | **IF** my number is _____,<br>_____is to the left of my number<br>_____is to the right of my number<br>**THEN** the sum of the numbers on either side of my number is _____ |

c. Choose another number

Find the sum of the numbers on either side of your number.

**IF** my number is _____,

_____is to the left of my number

_____is to the right of my number

**THEN** the sum of the numbers on either side of my number is _____

_____

d. Your number is 'n'.

Find the sum of the numbers on either side of n.

IF my number is n,

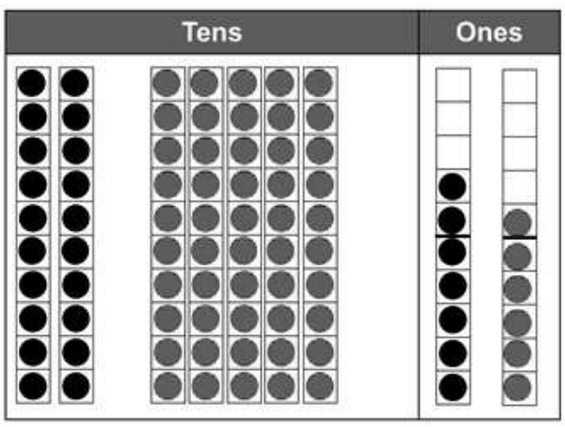
_____is to the left of my number. _____is to the right of my number.

**THEN** the sum the numbers on either side of my number is _____

**Break up the second number** is a good strategy as the same approach ca be used for both addition and subtraction.

## Addition using Base Ten Kits

**27 + 56 =**




| | T | O | |
|---|---|---|---|
| | 2 | 7 | |
| + | 5 | 6 | |
| | 1 | 3 | O: 7 + 6 = 13 |
| | 7 | 0 | T: 20 + 50 = 70 |
| | 8 | 3 | |

## Tasks 4.1.4

### Learning to be flexible with place value

1. Explore by writing down as many different ways that you can expand 1942 flexibly.

This one is obvious: 1942 = 1 thousand 9 hundreds 4 tens and 2 ones.
What are some other ways?

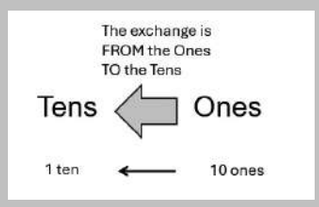
Children need to be able to break up numbers flexibly.
They need to understand how to exchange for when they work in columns.

**To exchange for addition:**

- The exchange is taken from the column to the right.
- **10 ones** (from the column to the right) is exchanged for **1 ten**
- The exchange is given to the column to the left.

For example: 6 tens and 18 ones = _____ tens and _____ ones

| T | O | | | |
|---|---|---|---|---|
| 6 | 18 | is the same as | T | O |
| | | | 7 | 8 |

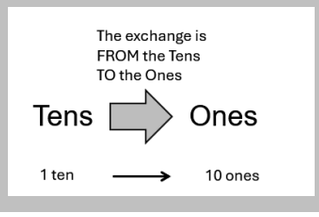



**To exchange for subtraction:**

- The exchange is taken from the column to the left.
- **1 ten** (from the column to the left) is exchanged for **10 ones**
- The exchange is given to the column to the right.

For example:7 tens and 8 ones = 6 tens and _____ ones

| T | O | | | |
|---|---|---|---|---|
| 7 | 8 | is the same as | T | O |
| | | | 6 | 18 |



3. Practice exchanging for addition. Complete the Tens and one columns.

- 6 tens and 17 ones = _____ tens and _____ ones

| T | O |
|---|---|
| 6 | 17 |

| T | O |
|---|---|
| | |

- 5 tens and 14 ones = _____ tens and _____ ones

| T | O |
|---|---|
| 5 | |

| T | O |
|---|---|
| | |

- 8 tens and 13 ones = _____ tens and _____ ones

| T | O |
|---|---|
| | |

| T | O |
|---|---|
| | |

4. Practice exchanging for subtraction. Draw in the Tens and Ones columns

- 6 tens and 7 ones = 5 tens and _____ ones

- 5 tens and 4 ones = 4 tens and _____ ones

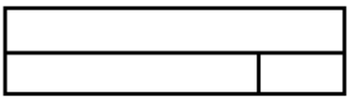
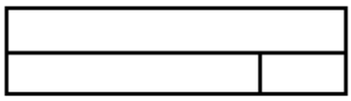
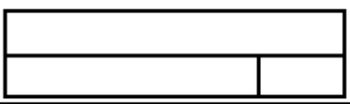
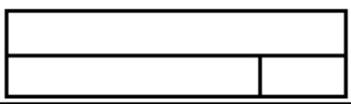
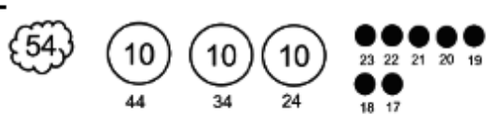
- 3 tens and 2 ones = 2 tens and _____ ones

- 8 hundreds 6 tens and 7 ones = 7 hundreds _____ tens and _____ ones

## Tasks 4.1.5

### Two min tango – flexible place value

To be able to work in columns, being flexible with place value is very important.

| | | | |
|---|---|---|---|
| 1 | 7 + 5 = 1 ten and ____ ones | 11 | 92 = 8 tens and ___ ones |
| 2 | 9 + 8 = ____ ten and ____ ones | 12 | 47 = 3 tens and ___ ones |
| 3 | 7 tens + 2 tens = 8 tens and ____ ones | 13 | 56 = ___ tens and 16 ones |
| 4 | 7 tens + 5 tens =10 tens and ___ ones | 14 | 83 = ____ tens and 13 ones |
| 5 | Complete the whole part part diagram for 65 – 18 = ____  | 13 | Complete the whole part part diagram for ____ – 18 = 65  |
| 6 | Show 65 – 18 = ____on a number line _____ | 16 | Show ____ – 18 = 65 on a number line _____ |
| 7 | Complete the whole part part diagram for 65 + 18 =  | 15 | Complete the whole part part diagram for ____+ 18 = 65  |
| 8 | Show 65 –+18 = ____on a number line _____ | 16 | Show ____ + 18 = 65 on a number line _____ |
| 9 |   Write down the calculation shown in this number picture. _____ | 19 |   Write down the calculation shown in this number picture. _____ |
| 10 | T O 8 17 ⇨ T O | 20 | T O 8 7 ⇨ T O 7 |

## Summary 4.1

1. List the order in which children usually learn their number facts.
2. Give an example of using place value to break up a number flexibly.

# UMTHAMO 4.2 NEAR DOUBLES, ROUND AND ADJUST

In this umthamo you learn the last few mental calculation strategies, which children could use.

There are excellent resources to help you build children's mental maths and number sense. These are presented in detail in the *Teach Maths in a Classroom* module, which support your teaching practice, or work integrated learning.

## Magic Classroom Collective (MCC): Lesson Starters

Daily routines to learn the 100 addition facts in a Foundation Phase classroom.

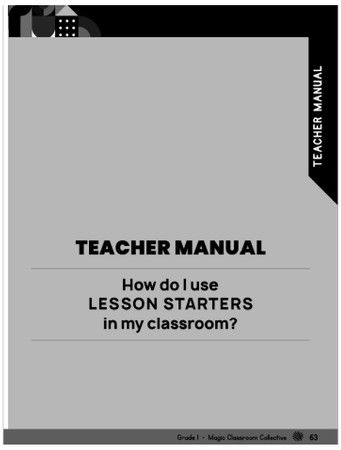
Kim Porteus, Nobuntu Mazeka, Qhama Ngcobo (2024) *Magic Classroom Collective - Maths Lesson Starters.* Nelson Mandela Institute

Lesson starters are available in: isiXhosa; isiZulu; Sesotho; Sepedi; Xitsonga; English

They are suitable for Grades 1, 2 and 3.

https://tinyurl.com/yc7yzcdv

MCC Lesson Starters


## Mental Starters Assessment project (MSAP)

3-week sprints to develop calculation strategies in an Intermediate Phase classroom.

Mellony Graven, Hamsa Venkat, Mike Askew, Lynn Bowie, Samantha Morrison, Pamela Vale (2020) *Mental Starters Assessment Project*, Department of Basic Education

MSAP materials are available in all South African languages.

https://www.education.gov.za/MSAP2022.aspx . They are suitable for grade 3 and 4


Key ideas

At the end of this umthamo, you should:

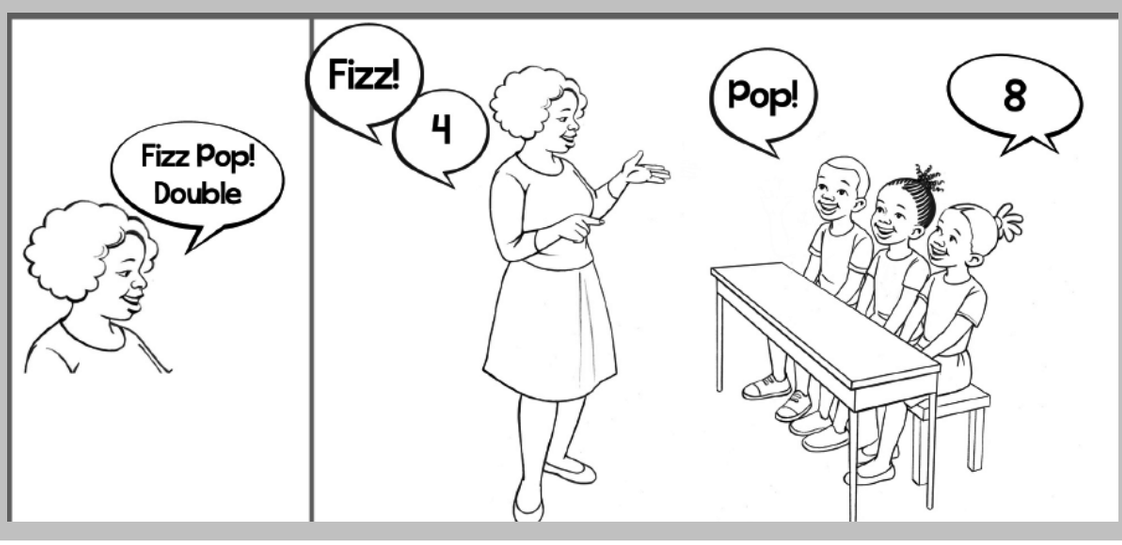
- Know the **doubles** and
  - » therefore know the **halves;** and
  - » therefore know the "**near double**" facts.
- Be able to **round off** (to the nearest 10)
- Be able to explain how to **round and adjust.**
  - » Know how to **add or subtract 10 and 100s**, and
  - » therefore know how to add and subtract 9 and 99.

## Learning trajectory for doubling and halving

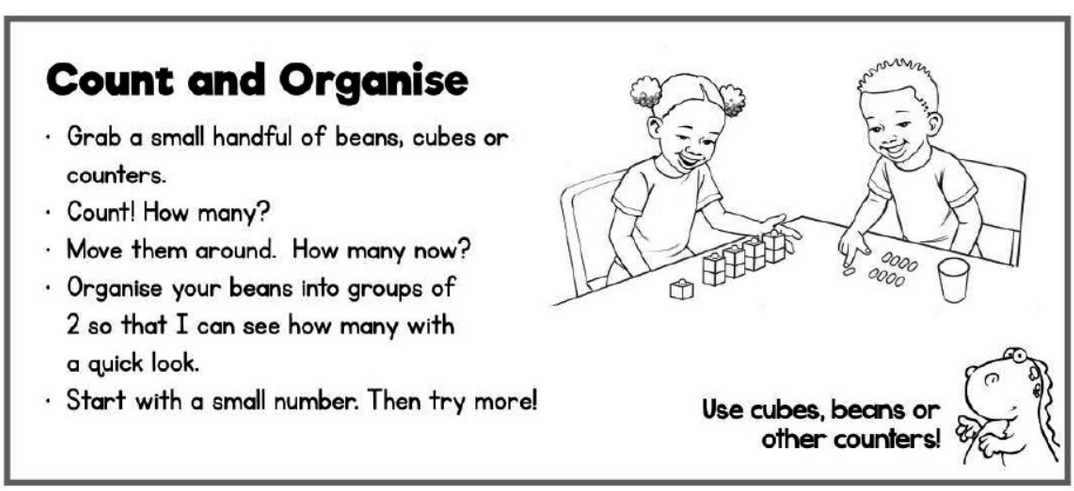
There is a MCC Lesson Starter on doubling and halving (Grade 1 and Grade 2).

There is an MSAP unit on doubles and near doubles (Grade 3 and Grade 4)

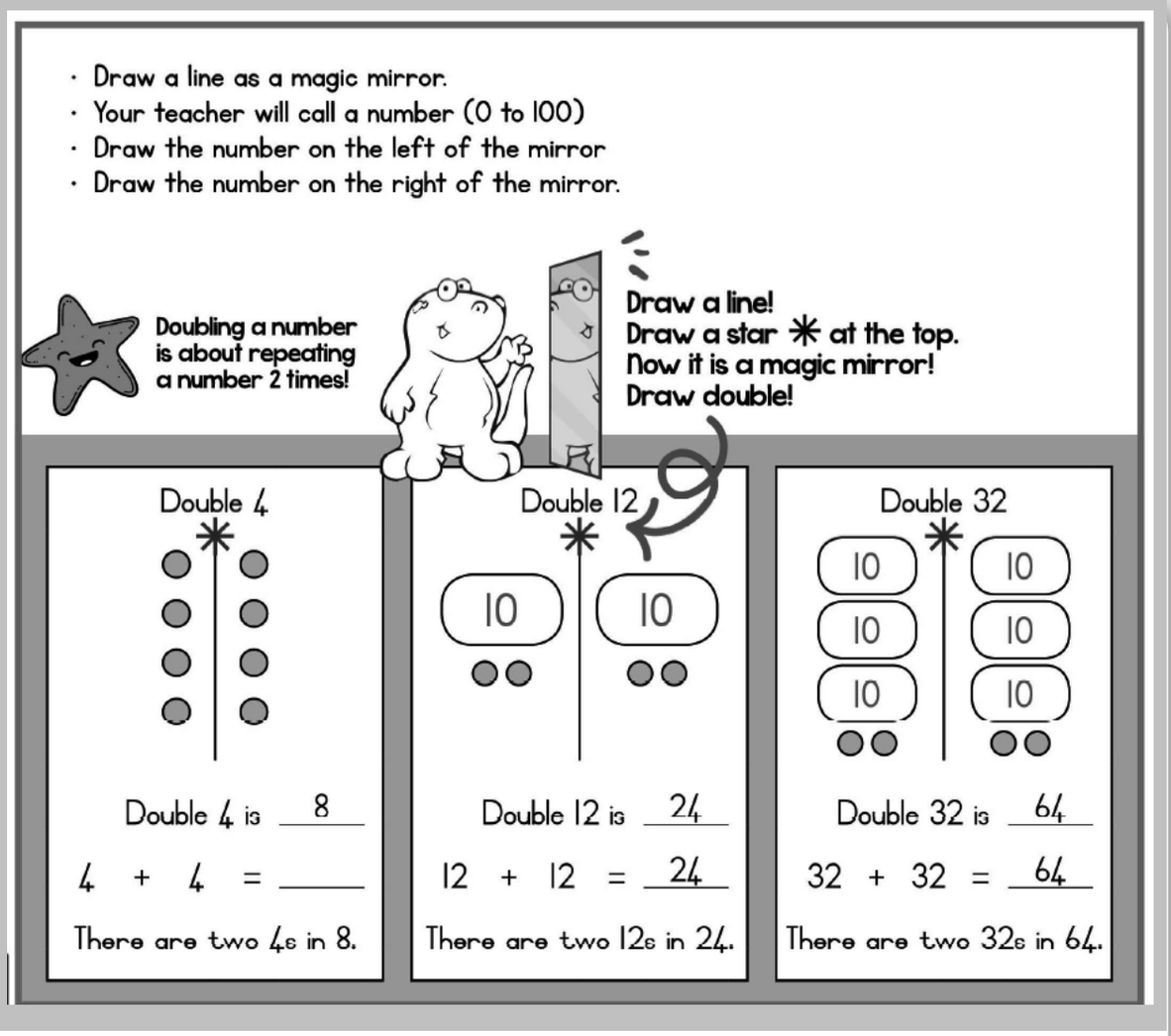
Example




Example



Activity for learners




There are 8 children, how many eyes?

There are 2 fours in 8. AND There are ___ twos in 8.

There are 12 children, how many eyes?

There are 2 _____ in 12. AND There are ___ twos in 12.

There are 32 children, now many eyes?

There are 2 _____ in 32. AND There are ___ twos in 32.

Knowing doubles helps us do other calculations quickly.

When planning doubling tasks you should think about the level of difficulty for the children in your class.

- Level 1 (Grade R and beginning Grade 1) must be known well, before you move to
- Level 2 (late Grade 1)
- Level 3 (early Grade 2)
- Level 4 is the most difficult (late Grade 2 early Grade 3).

| Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|
| Numbers: 0 to 10 | Numbers: 10 to 20 | Numbers: Tens | Two Digit Numbers |
| Double 3 ____ | Double 13 ____ | Double 20 ____ | Double 24 ____ |
| Double 5 ____ | Double 16 ____ | Double 40 ____ | Double 31 ____ |
| Double 2 ____ | Double 12 ____ | Double 50 ____ | Double 26 ____ |

**Notice:** You will probably have children at all levels in one class.

If a child is on level 2, you help them move to level 3.

When the doubles are known facts, the children can derive new facts.

Level 3 is derived from Level 1 and Level 2...

I know that double 3 is 6. Therefore I know that double 30 is 60

- I know that double 3 is 6.
- I know that double 10 is 20.
- Therefore I know that double 13 is 26

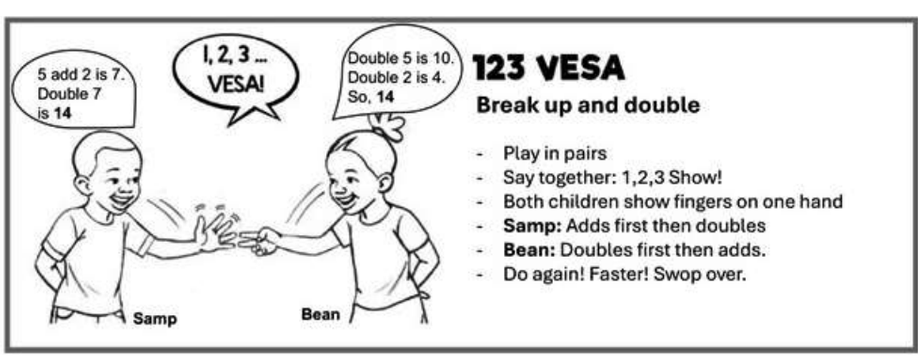
So, **to double a number I can break it down and double each part**.

Activity for learners

### Double my number

You need: 2 players (one leader and one follower)

- Both players put their hands behind their backs
- The leader says "1, 2, 3 DOUBLE"
- When the leader says "DOUBLE", the leader shows a number of fingers (either using one hand or two hands)
- If the number is bigger than 5, the leader has to show a full 5 on one hand (i.e. 7 should be shown as 5 and 2 and NOT as 3 and 4)
- The follower has to show the same number of fingers, in the same arrangement
- The pair has to work out the total number of fingers
- If a number bigger than 5 is shown, the pair can group the two 5s to make a 10 to calculate the answer quicker



Investigation

**Investigate: Does this always, sometimes or never happen?:**

Half of my number is also the number of twos in my number.

If my number is 6. Then half of 6 is 3. AND There are 3 twos in 6.

If my number is _____. Then half of my number is _____. AND there are _____ twos in my number .

If my number is _____. Then half of my number is _____. AND there are _____ twos in my number .

Once we know our doubles, it is easy to calculation near doubles.

Because we know that: 3 + 3 = 26

It is easy to work out: 3 + 4 = 3 + 3 + 1 = 7.

The second number being added is one more, so the answer will be one more.

## Round off

It is often easier to work with the nearest multiple of 10. These are the friendly numbers.

Then you get a good estimate for your calculation.

To round off to the nearest 10 means – to use the nearest multiple of 10

To round off to the nearest 100 means – to use the nearest multiple of 100

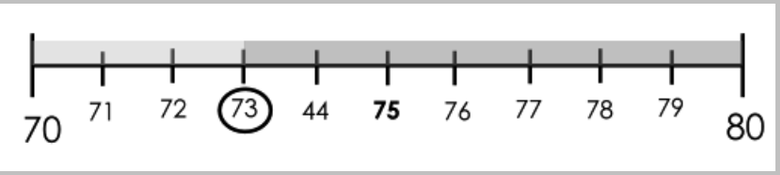
If your number is exactly in the middle of two multiples, then you round up to the bigger multiple.

Example

**Example: Round off**

73 rounds down to 70.
73 is closer to 70 than it is to 80.

75 is in the middle. It rounds up to 80.



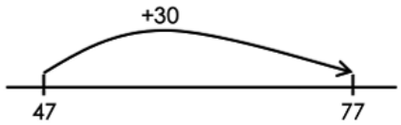
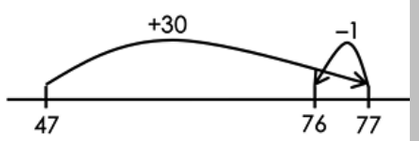

76 also rounds up to 80. 76 is closer to 80 than it is to 70.

## Round and adjust

The round and adjust strategy uses friendly numbers. Instead of adding or subtracting 9 you can round off 9 to the friendly number 10. Instead of adding or subtracting 99, you can round off to the friendly number 100. Round and adjust is a strategy included in MSAP

Once children have the idea of rounding off for 9, they can use round and adjust for other numbers that are close to the multiples of 10.

Example

| | |
|---|---|
| Teacher: What friendly number is 29 close to?<br>Learners: 30<br>Teacher: If we round 29 to 30 and then jump 30 forwards from 47 where would we be?<br>Learners: 77(show this on the board)<br>Teacher: Is 77 the final answer or must we **adjust**? | 47 + 29<br>  |
| Ask the learners to explain the logic of adjusting the answer: We added an extra one, so we must subtract one from the answer to get 76.<br>Show this on the number line on the board.<br>Tell the learners this round and adjust method works well when one number is very close to a friendly number. | Add to the number line:<br>  |

*Source: Mental Starters Assessment Project (MSAP)*

If you know the **medium number song**, it is easy to add ten to any number.

## Add/subtract 1 ten starting at any number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Medium number song

Small number song

In both pathways we emphasise

1s, 10s and 100s.

Children must know the

- Small number song (in 1s);
- Medium number song (in 10s); and
- Large number song (in 100s);

forwards and backwards,

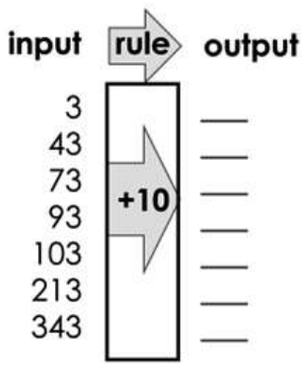
starting from any number.

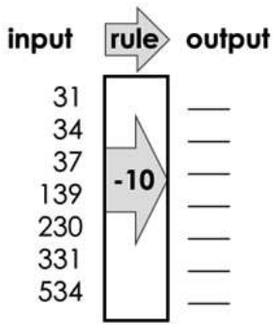
**PrimTEd Std NA 3.3**
Demonstrate knowledge of equivalent relations in general and equality, in particular in relation to addition and subtraction. The function '+ 99' is equivalant to the function '+100 then - 1'

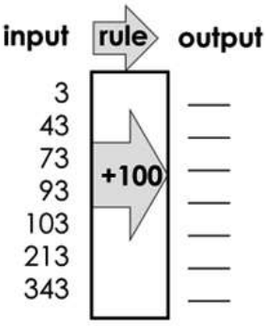
**PrimTEd Std NA 8.1**
Apply knowledge of relations and functions in order to understand the nature of mathematical operations. The rule '-99' has one step. The rule '- 100 then + 1' has two steps.

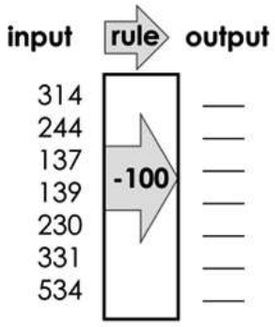
Activity for learners

It can be helpful to think about add 10, subtract 10, add 100 and subtract 100 as a function. This function an be used in fizz pop games. It can be used with Funky the function machine.




The function machine picture is helpful for algebra and functions in high school. It is also helpful to think about inverse functions. And help to reason about 2 step functions.

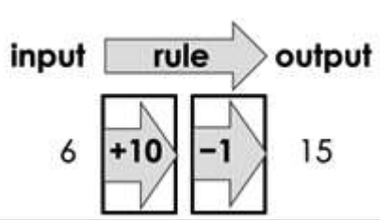
## Add or subtract 9 (one less than 10)

Adding 9 is the same as adding 10 and then taking away 1 from the answer.
Because it is easy to add 10, and because it is easy to take away 1, it is also easy to add 9.

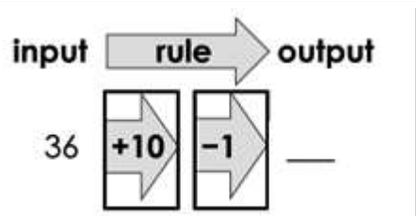
Example

Example: 6 + 9 = ...

6 + 9
= 6 + 10 – 1
= 16 – 1
= 15



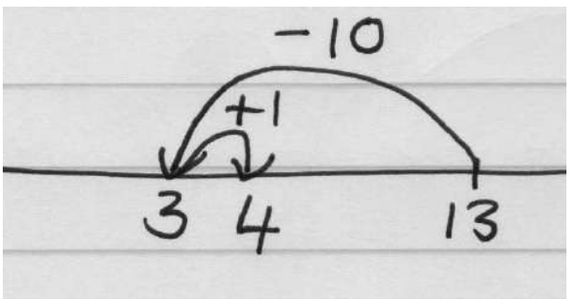

36 + 9
= 36 + 10 – 1
= 46 – 1
=



Subtracting 9 is the same as subtracting 10 and then adding 1 to the answer. Because it is easy to subtract 10, and because it is easy to add 1, it is also easy to subtract 9. In 'How numbers work' we look at why subtracting 10 is easy.

Example

Example: 13 − 9 = ...




13 − 9

= 13 − 10 + 1

= 3 + 1

= 4

13 − 9 = 4

Subtracting 10, instead of subtracting 9, is subtracting too much, so to adjust you have to add 1.

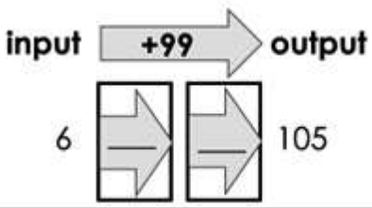
## Add and subtract 99 (1 less than 100)

6 + 99

= 6 + 100 – 1

= 106 – 1

= 105

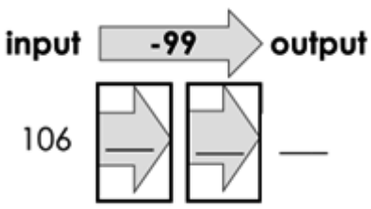



436 - 99

= 436 - _____ + 1

= _____ + 1

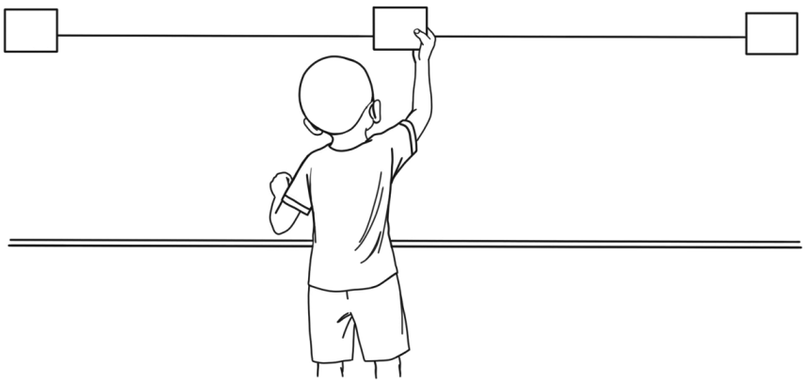
=_____



## IDEAS FOR YOUR CLASSROOM

There are two main routes to developing children's number sense:

- **A counting pathway** where the starting point is discrete quantities (discrete objects or discrete events); and
- **A measurement pathway** where the starting point is continuous quantities that can be measured using a particular attribute such as length or volume.



My number is 6. Which number is **1 less** than 6? _____. Does it go on the left or right of 6?

My number is 6. Which number is **1 more** than 6? _____. Does it go on the left or right of 6?

Activity for learners

Visit nrich.maths.org to find some mathematical thinking and engaging tasks for learners.

- KS1 refers to Key Stage 1 in the UK (Foundation Phase in South Africa)
- KS2 refers to Key Stage 2 in the UK (Intermediate Phase in South Africa)

SCAN ME

Activity for learners

## Five steps to Fifty

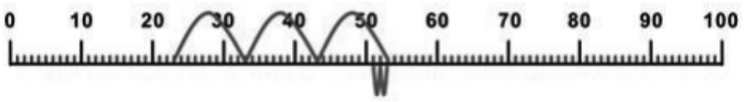
This challenge is about counting on and back in steps of 1, 10 and 100.

Roll a dice twice to establish your starting number - the first roll will give you the tens digit and the second roll will give you the units digit.
You can then make five jumps to get as close to 50 as possible.
You can jump forwards or backwards in jumps of 1 or 10 or 100.

Compare your strategy with a friend.
Did you jump forwards or backwards?
Can you land on 50 exactly?
How far from 50 were you?
Could you do it another way?
Could you get even closer?
Which numbers can get you to 50?
Which can't?

Roll the dice again and have another go!

For example:
I roll a dice and get a 2 then a 3, so my starting number is 23.
I make the following jumps to get as close to 50 as possible:
Starting number is 23
Jump one is +10 to get me to 33
Jump two is +10 to get me to 43
Jump three is +10 to get me to 53
Jump four is -1 to get me to 52
Jump five is -1 to get me to 51




*Source: nrichmaths.org.*

SCAN ME

**Round and adjust** is one of the MSAP strategies. You can try and 3 week sprint with a grade 4 or 5 class.

All of the MSAP strategies are in the MSAP teacher guide

**M4PT Adding and Subtracting**

**STUDENT FOLDER**

By working through the **Maths 4 Primary Teachers** *Adding and subtracting* module, you had an opportunity to practice mental fluences needed for these calculation strategies:

- Break up the second number: Jump or go-to strategy;
- Visit the tens; and
- Re-ordering.


**Tango 8**

Jump 1


**Tango 9**

Jump 2


**Tango 10**

Visiting the tens 1

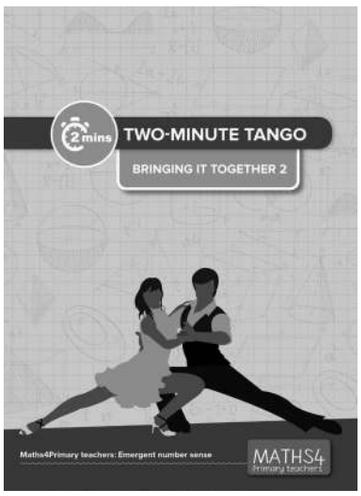

**Tango 11**

Visiting the tens 2


**Tango 12**

Re-ordering

**Tango 13** is a good revision to practice the mental fluencies, needed for the basic calculation strategies. Every quiz, and the post-test, includes a mental fluency section.




**Jumptrak**

In this course you use jumptrak for:

**Two minute TANGO practice;**

Use jumptrak online.

Or go to your app store and download: Jumptrak.

## Tasks 4.1.2

### Doubles, near doubles and 'round and adjust'

1. **Doubles**

Children find it easy to remember the doubles. By the end of Grade 1 they should know how to double to 10 and halve to 20 fluently.

1 more than a double', is called a 'near double' fact.

I know that 3 + 3 = 6. Therefore I know that 3 + 4 = 3 + 3 + 1 = 6 + 1 + 7

a. I know that 4 + 4 = _____. Therefore I know that 4 + 5 = _____.

b. I know that double 2 = _____. Therefore I know that 2 + 3 = _____.

c. I know that 7 + 7 = _____. Therefore I know that 7 + 8 = _____.

d. Complete all the 'near doubles' facts in the 100 addition facts table. Write down the near doubles facts.

2. **Double these numbers.** Show your working

| | | |
|---|---|---|
| a. Double 6 | b. Double 26<br><br>Double 20 = 40<br><br>Double 6 = 12<br><br>40 + 12 = 52 | c. Double 76 |
| d. Double 8 | e. Double 18 | f. Double 58 |

| g. Double 3 | h. Double 43 | i. Double 83 |
|---|---|---|

3. **Practice Near Doubles**

| |
|---|
| 7 + 8 = **7 + 7** + 1 = **14** + 1 = 15 |
| 12 + 13 = **12 + 12** + 1 = **24** + 1 = ____ |
| 49 + 50 = **50 + 50** - 1 = **100** – 1 = _____ |
| 23 + 24 = **23 + 23** + 1 = _____ + 1 = _____ |
| 17 + 18 = ____ + ____ + 1 = ____ |
| 28 + 29 = |
| 62 + 63 = |
| 213 + 214 = |
| 57 + 58 = |
| Your own near double example: |
| Your own near double example: |

4. **Practice adding 9** (add 10 then subtract 1)

| | |
|---|---|
| 3 + 9 = | 1056 + 9 = |
| 17 + 9 = | 423 + 9 = |
| 62 + 9 = | 619 + 9 = |
| Your own example | Your own example |

5. Use an empty number line to show the **round and adjust strategy** for

35 + 9 =

---

6. **Practice subtracting 9**
   Show your working

| | |
|---|---|
| 15 - 9 = | 1 056 - 9 = |
| 517 - 9 = | 323 - 9 = |
| 72 - 9 = | 741 - 9 = |
| 555 - 9 = | 234 - 9 = |

7. **Use an empty number line**

35 - 9 =

8. **Use an empty number line**

57 – 9 =

9. **Practice adding/ subtracting 99**

| | |
|---|---|
| 53 + 99 = | 1056 - 99 = |
| 17 + 99 = | 423 - 99 = |
| 62 + 99 = | 619 - 99 = |
| 865 + 99 = | 44 + 99 = |

10. **Use an empty number line: 15 + 99 =**

11. **Use an empty number line 359 – 99 =**

12. Draw a 1-step function machine for '- 99'. Give 5 inputs and 5 outputs.
Then draw a 2-step function machine for '-100, then + 1' . Use the same 5 inputs.

## Tasks 4.2.1

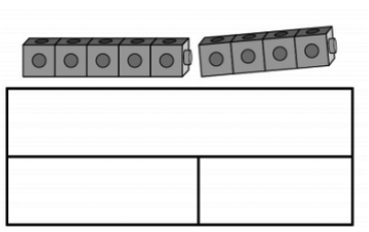
### Big ideas for adding and subtracting

1. Use the example 43 – 27 = ..., to explain the two different pathways to learn about numbers.

   a. Explain what is meant by the **counting pathway.**
   Make up a word problem for children to solve which involves the counting pathway for 43 – 27 = ...
   Then show how to calculate 43 – 27 using number pictures.

   b. Explain what is meant by the **measurement pathway.**
   Make up a word problem for children to solve which involves the measurement pathway for 43 – 27 = …
   Then show how to calculate 43 – 27 using a number line.

2. Complete the whole-part-part diagrams. Write down the family of 8 number sentences for each diagram. Link addition and subtraction. There are 4 addition and 4 subtraction number sentences.



Addition

Subtraction

There are four different types of additive relations word problems. Some are dynamic. There is a start, a change and a result. Some are static. There is a part-part-whole

| **Models for additive relations** | **Additive relations word problem types** | |
|---|---|---|
| ACTION / CHANGE<br>like a video<br>**start-change-result** | Change (increase)<br>Combine/join | Change (decrease)<br>Separate/take away |
| NO ACTION/ STATIC<br>Like a photo<br>**part-part-whole** | Collection | Compare |

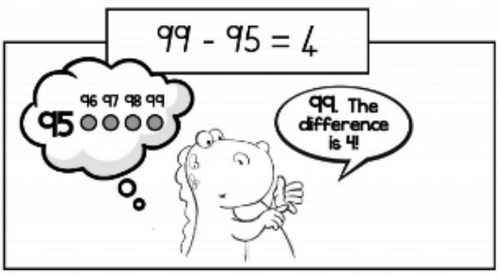
In the table below. Read the word problem. Complete the word problem type (chane increase, change decrease, compare or collection). Then look at the problem context, which pathway into number is used:

- a measurement pathway (M) or
- a counting pathway (C)?

| **Word problem** | **Type** | **Pathway** |
|---|---|---|
| a. It is 454 kilometres from Mbizana to East London. Ta'Jola has driven 312 kilometres. How many more kilometres until he reaches east London? | | |
| b. Uncle Neo has R450. He buys petrol for R150. How much money does he have left? | | |
| c. There are 22 cars in the parking lot. 15 cars are white. How many cars are not white? | | |
| d. There are 16 pots, but only 11 lids. How many pots do not have lids? | | |
| e. Anita baked 34 scones. Then she baked 30 more. How many scones altogether? | | |
| f. My car has 32 litres of petrol. I pour 24 litres more. How many litres of petrol altogether? | | |
| g. Maya has R85. She buys food for R21. How much money does she have now? | | |
| h. Njogu is 145 cm tall. Mary is 152 cm tall. How much taller is Mary than Njogu? | | |

4. Complete the calculations

**Do you remember?**
**Subtraction means take away AND compare!**
**When numbers are close together, it is easier to calculate the DIFFERENCE between the numbers.**



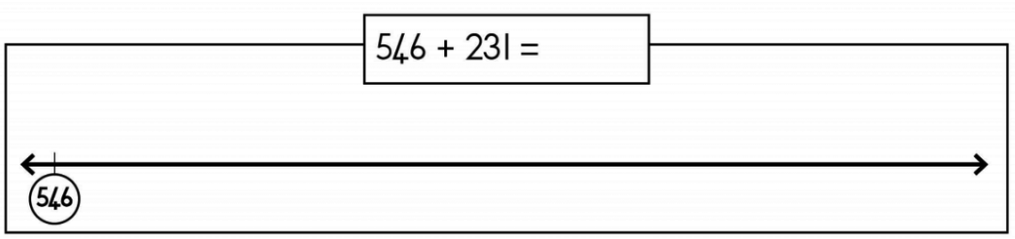

**The small number is the starting point.**
**The big number is the target.**
**We put the smaller number in our head.**
**We count forward to reach the second number.**
**This is the difference!**

| | | |
|---|---|---|
| 203 - 199 = _____ | 387 - 381 = _____ | 403 - 397 = _____ |
| 578 - 572 = _____ | 604 - 597 = _____ | 802 - 796 = _____ |

5. **Add by showing on a number line**

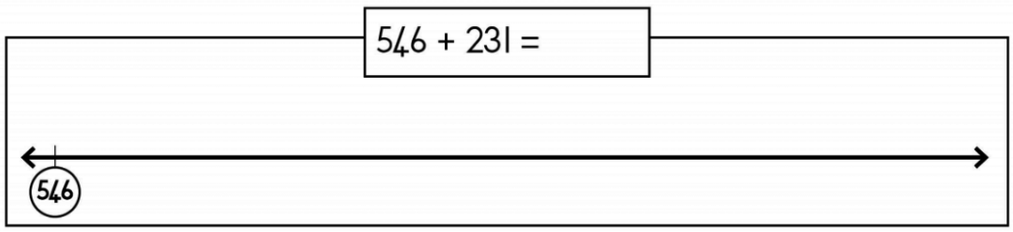
You and your learners need to flexible in how you add and subtract. You should know all the different strategies. All your learners should know the go-to strategy.

a. Break up the second number into 100s, 10s and 1s.
Use the jump or **go-to strategy**.




b. Break up the second number into 100s, 10s and 1s. Use the jump strategy in columns.

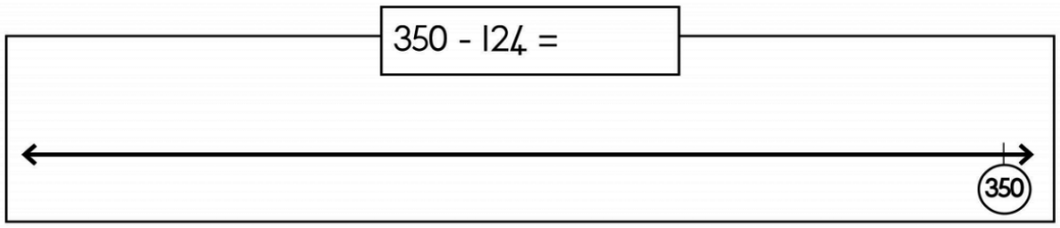
c. Break up the second number to **visit the next ten.**



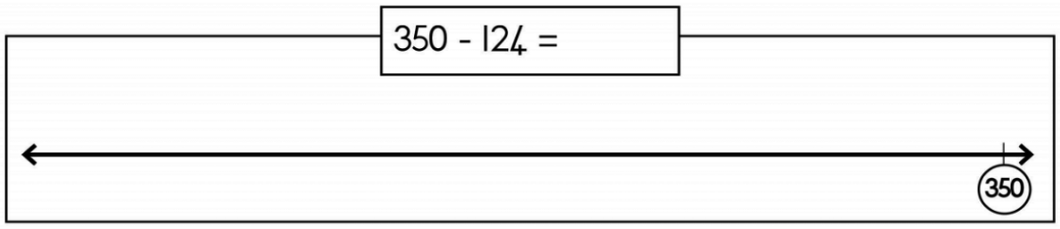

d. Break up the second number using place value. Write in columns

6. **Subtract by showing on a number line.**

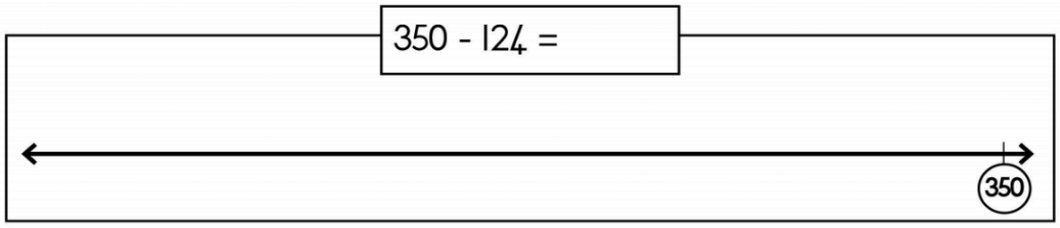
a. Use a **take-awa**y model. Break up the second number into 100s, 10s and 1s. Use the jump or **go-to strategy**.




b. Use a **take away** model. Break up the second number to **visit the previous hundred, or previous ten.**




c. Use a **find the difference** model. Link addition and subtraction. Compare 124 and 350 and find the difference.

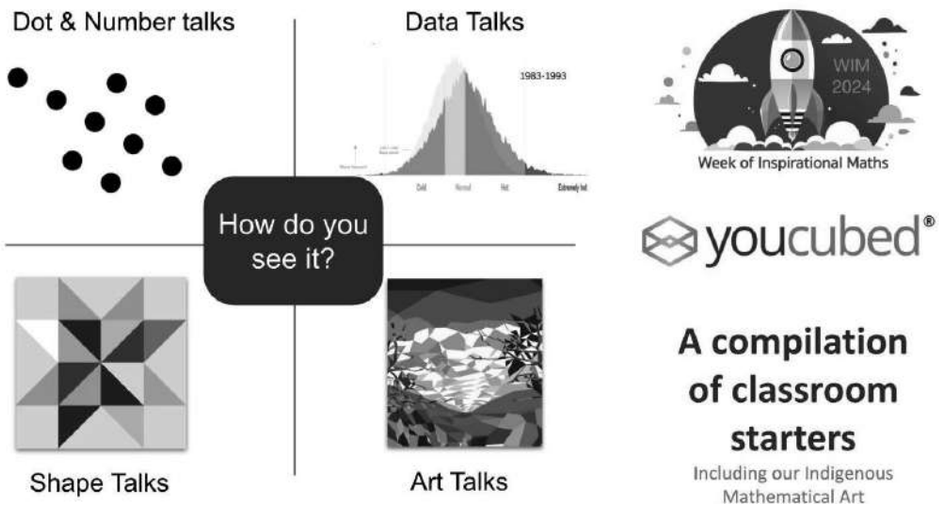


## Tasks 4.1.2

### Two minute tango – Near doubles, Round and adjust

| | | | |
|---|---|---|---|
| 1 | 9 + 8 = ___ | 11 | 13 + 14 = ___ |
| 2 | 7 + 7 = ___ | 12 | 28 + 29 = ___ |
| 3 | 137 – ___ = 107 | 13 | 28 + 99 = ___ |
| 4 | 29 ___ 5 = ___ | 14 | 328 + 199 = ___ |
| 5 | 49 + ___ = 53 | 15 | 28 + ___ = 48 |
| 6 | 48 = ___ – 2 | 16 | 78 + 12 = ___ |
| 7 | 76 + ___ = 80 | 17 | 90 – 78 = 10 + ___ |
| 8 | 97 = 100 – ___ | 18 | 198 + 78 = 200 + ___ |
| 9 | 27 + ___ = 30 | 19 | 899 – 203 = 900 – ___ |
| 10 | 97 – 70 = ___ | 20 | 899 + 203 = 900 + ___ |

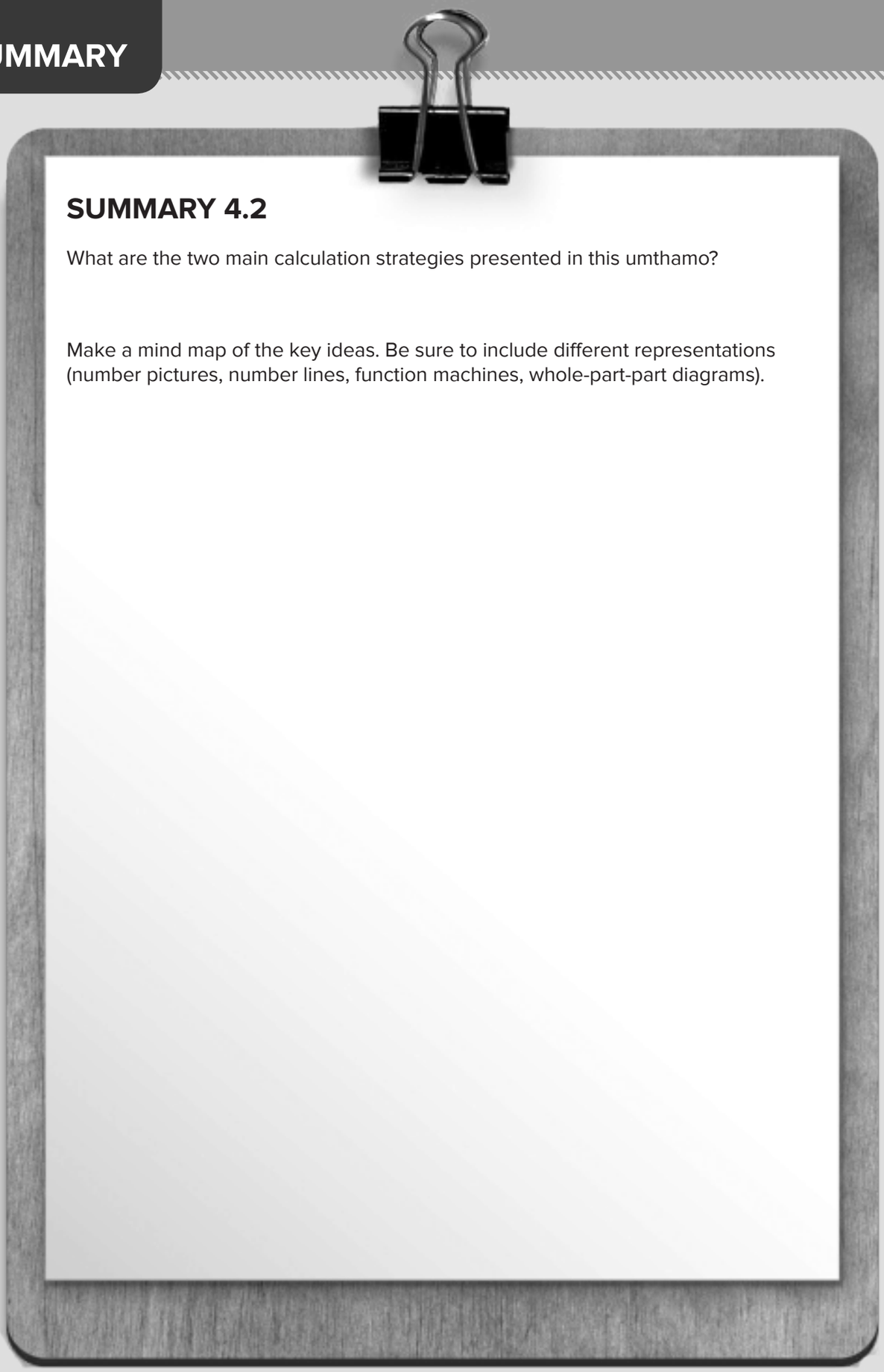
When you **round and adjust** and **visit the tens,** you are using knowledge of the friendly numbers. How far to the next ten? How far to the previous ten? You need to know the **bonds of ten** very well.




SKAN ME

You-cubed has excellent resources for number talks. Children should be flexible about what strategy they use. They should explain their strategies to you and others.

https://www.youcubed.org/tasks/what-do-you-see/



## SUMMARY 4.2

What are the two main calculation strategies presented in this umthamo?

Make a mind map of the key ideas. Be sure to include different representations (number pictures, number lines, function machines, whole-part-part diagrams).

## BREAK UP BOTH NUMBERS (SPLIT)

Up to this point, all of the flexible strategies to add and subtract involve breaking up the second number. The first (or bigger) number is kept whole. Children put the first number in their head. They break up the second number.

- Almost all children can master 'break up the second number' strategies.
- 'Break up the second number' strategies are excellent for **mental fluency and flexible calculation**.
- In the 21st century, children need to have number sense, and work flexibly with numbers mentally.
- 'Break up the second number' strategies are what learners need for success in algebra in secondary school.
- 'Break up the second number' strategies help children learn place value. The break up the second number using place value. They break up the second number into 100s, 10s and 1s.

**In the past,** we used to need learners to also be able to add and subtract large numbers accurately. **However in the 21st century, we now use calculators**, mobile phone or spreadsheets to add long lists of large numbers. Before calculators, people would calculate efficiently by breaking up both numbers. This is called split. In this umthamo you focus on the split strategy – where you break up both numbers. This can be done using write all totals, or a condensed column method.

**In the 21st century** it is more important for learners to have **number sense – flexible mental calculation strategies,** than it is for them to accurately and quickly add or subtract lists of large numbers. It is useful to include the split strategy when learning maths, but it is not important for all children to master it.

At high school in South Africa, learners will use a calculator in maths class. They need flexible mental methods, to check that their calculator work is correct. In their workplace and in daily life adults use calculators, mobile phones and spreadsheets. The skill of splitting and working in columns is not a 21st century skill.

Key ideas

At the end of this umthamo, you should:

- Break down large numbers into expanded notation;
- Break down large numbers flexibly; and
- Be able to use and explain the break down both numbers (split) strategy for adding and subtracting large numbers including:
  - » Write all totals.
  - » The condensed column method

The split strategy should only be used when children have developed deep conceptual knowledge and can fluently add and subtract by breaking down the second number. Note there are several **dangers that make the split strategy difficult** for some children in your class:

- For many calculations children have to "carry" or "borrow" or "regroup" or "exchange" as they work with each digit separately.
- The children need a good understanding of place value to keep track of their work.
- Children start using a digit-wise conception of number. Instead of thinking about 189 as 1 hundred and 8 tens and 9 ones they start thinking about one eight nine. They lose the sense that it 189 is more than 150 and close to 200.
- If they make an error – even when they get a ridiculous answer – they don't notice as they are no longer thinking about the actual numbers and what they mean.

## Break up both numbers (Split for Adding)

To prepare children to able to add by breaking up both numbers, it is helpful to make sure that children can:

1. **Add two single digits fluently.** They should know all their bonds very well. These are the number facts in Umthamo 4.1. The most important number facts are:
   a. **One more than.** Count forwards in 1s starting at any number**.**
   b. **The bonds of 5.** How far to the five?
   c. **The bonds of 10.** How far to get to the ten?
   d. **Doubles and near doubles**
   e. **Add 9.** Add 10 and subtract 1. This is round and adjust.
   f. **All the bonds to 10.** Connect addition to subtraction. Whole-part part diagrams and families of number sentences.
   g. **All the bonds to 20.** There are 100 addition facts. They connect together.
   h. Know that **order does not matter when adding.** Addition is commutative. I know 3 + 4 = 7. Therefore I know 4 + 3 = 7.
2. Work with 10 as a friendly number
   a. **The bonds of 10.** How far to get to the ten? How far to get to the next multiple of ten?
   b. **Add a single digit to 10.** I know that 10 + 7 = 17. Therefore, I know that 7 + 10 = 17.
   c. **Add 10s together.** This uses knowledge of bonds. I know that 4 + 3 = 7. Therefore, I know that 40 + 30 = 70. 4 tens + 3 tens = 7 tens.
   d. **Add 10 to any number. 10 more than.** Eg. 34 + 10 = 44. Count forwards in 10s. Start at any number.
   e. **Add a single digit to any multiple of 10.** Eg. 9 + 30 = 39.

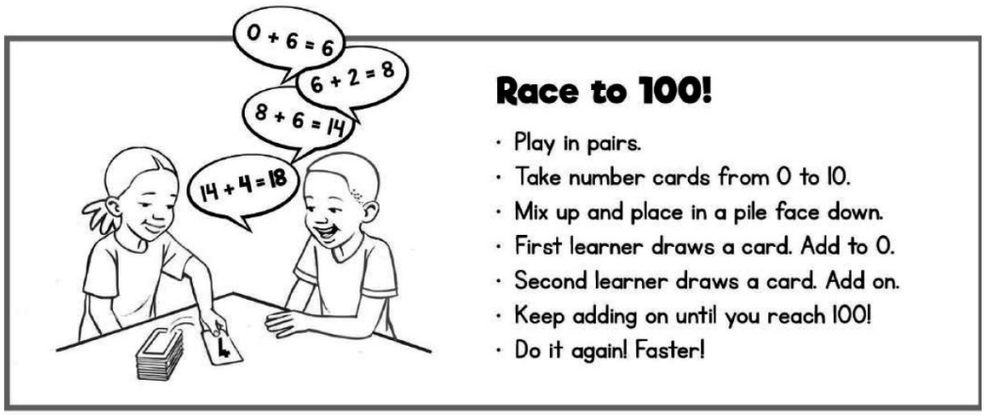
3. **Add a single digit to any number.** Eg 9 + 34 = What is the next 10?
   How far to the next ten? They should be able to **visit the next ten.**
   Eg. 34 + 8 = ...For this they need to know the next ten. And how far to the next ten.
   This strategy is in Umthamo 3.3.

Here is a table of all the mental fluencies needed for adding. It also shows which mental maths fluencies (number facts) children must know to be able to use each calculation strategy.

**Table: Which mental fluencies are needed for which addition calculation strategies?**

| ADDING FLUENCIES | Count all | Count forwards in 1s. Add 1s | Add multiples of 10s | Count forwards in 10s. Add 10s. | Rapid recall. Known facts | Jump. Add 10s. Then sdd 1s | Visit the tens | Round and adjust | Split – Write all totals | Split to add– Condensed columns method |
|---|---|---|---|---|---|---|---|---|---|---|
| Count on in 1s. Start at 1 | ■ | ■ | | | | | | | | |
| Count in 1s. Start at any number. | | ■ | | | | ■ | | | | |
| Doubles | | | | | ■ | | | | | |
| Bonds of 5 | | | | | ■ | | | | | |
| Near doubles | | | | | ■ | | | | | |
| Count on in 10s. Start at 10 | | | ■ | | | | | | | |
| Count on in 10s. Start at any number. | | | | ■ | | ■ | | | | |
| Place value. Break down into 10s and 1s | | | | | | ■ | ■ | | | |
| Find the next ten | | | | | | | ■ | ■ | | |
| How far to the next ten | | | | | | | ■ | ■ | | |
| Bonds of 10 | | | | | | | ■ | ■ | | |
| Add 9. Add 10 subtract 1. | | | | | | | | ■ | | |
| All bonds to 10 | | | | | | | ■ | | | |
| All bonds to 20 | | | | | | | | | ■ | ■ |
| Exchange ten 1s for one 10. Re-group to add | | | | | | | | | | ■ |

Remember that primary maths should build number sense. Chidlren should be flexible and playful with how they calculate with numbers. They should not just know one way – and follow a recipe. They need to work with understanding. They should know both rules, and reasons.




In this table, we summarise some of the advantages and disadvantages of the main calculation strategies.

**Table: Advantages and disadvantages of jump, visit the tens and the condensed column method**

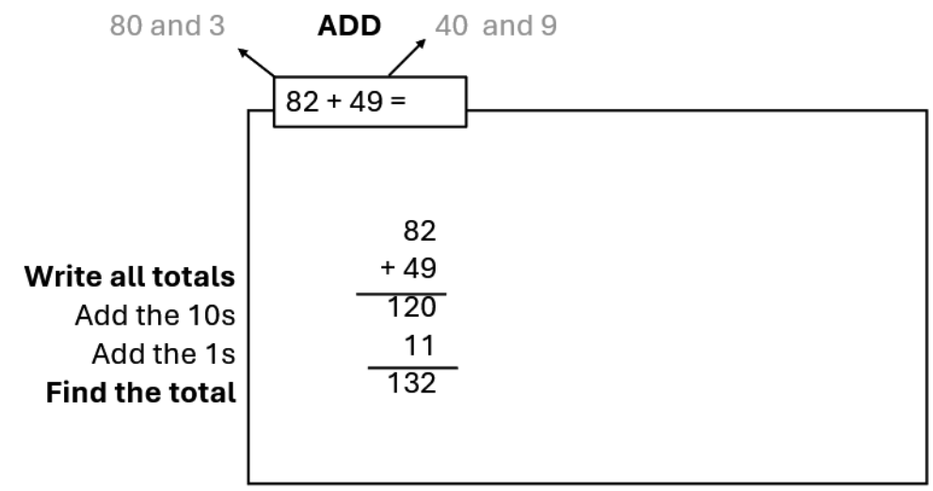
| Break up the second number using jump | Break up the second number using visit the tens | Break up both numbers using the condensed column method |
|---|---|---|
| **Advantages** | | |
| • Only need to know a few mental fluencies.<br>• Learn place value. They can break up the 2nd number into 10s and 1s.<br>• Work in any order. They can start with the 1s. Or they can start with the 10s. They can be flexible.<br>• Works the same way every time. Works for all numbers.<br>• Helps algebra. Children need to know that they can break up a number and add each part. and that order does not matter.<br>• a + b + c = (a + b) + c = a + c + b | • Break up a number in any way. They can be flexible.<br>• Builds number sense, and mental fluency.<br>• Helps rounding off and estimation.<br>• This is a 21st century skill. | • Learn place value. They can break up both number into 10s and 1s.<br>• Learn the rules of what to do and do the same thing every time...but be careful about exchanges.<br>• Is easy with the right numbers, when no exchanges are needed.<br>• eg. 23 + 35 = 58<br>• When this is well known, it can be done fast, with little thinking. |

| Break up the second number using jump | Break up the second number using visit the tens | Break up both numbers using the condensed column method |
|---|---|---|
| **Disadvantages** | | |
| • May not progress from counting on in 1s. So 7 + 4 is always: "7 in my head, 8, 9, 10, 11". They do not learn 7 + 4 = 11<br>• May not progress from counting on in 10s. So 7 + 40 is always: "7 in my head, 17, 27, 37, 47". They do not learn 7 + 40 = 47. | • Need to know bonds of 10 very well<br>• Need to know the next 10, and how far to the next 10<br>• Need to know all bonds to 10, as they break up the second number in a special way to visit the ten. | • Need to know all 100 number facts. (Bonds to 20)<br>• Only break up a number using place value. They are not flexible.<br>• Only works in one order. They must first add the 1s then add the 10s. They start right to left. And you read a number from left to right.<br>• Is difficult, when exchanges from 1s to 10s are needed.<br>• eg. 28 + 35 = 63<br>• This is not a 21st century skill.<br>• This does not help algebra. In high school children use calculators. |

## Split – Using write all totals (Adding)

For addition, there is an easy strategy to split both numbers called **write all totals**.

- This strategy involves **breaking up both numbers**.
- And it **does not require any exchanges** from 1s to 10s.




This strategy can be done in any order.

- Children can start with the 10s. This is helpful as working from left to right is the same way that they learn to read, and say the numbers (in English and African languages).
- Children can start with the 1s. This is working from right to left. This may be helpful in Afrikaans, where the 1s are said before the 10s.

Example

**Write all totals for addition:**

Break up both numbers.

547 = 500 + 40 + 7
284 = 200 + 80 + 4
700 + 120 + 11 = 831

547
+284
700
120
11
831

In this approach it does not matter if you start with the hundreds, then tens then ones, or start the other way around. This is a break down both numbers strategy. It does not work very well for subtraction as regrouping is difficult.

Example

**Write all totals for addition**

| | |
|---|---|
| 547 | |
| +284 | |
| 700 | Add the100s |
| 120 | Add the 10s |
| 11 | Add the 1s |
| 831 | |

Using this method, children can start on the left (which is the same way the read and say the number names).

| | |
|---|---|
| 547 | |
| +284 | |
| 11 | Add the 1s |
| 120 | Add the 10s |
| 700 | Add the 100s |
| 831 | |

Using this method, children can also start on the right.

This is a good stepping stone towards the standard written algorithm or the condensed column method.

## Split - Using base ten blocks (Adding)

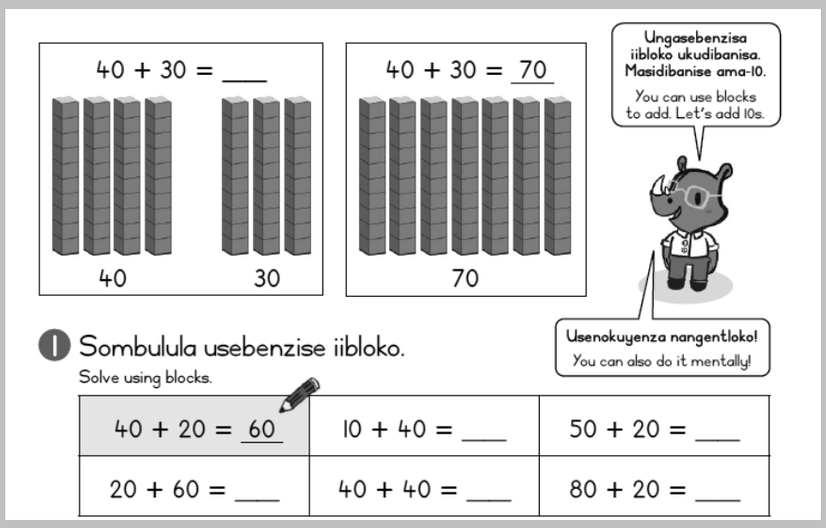
Some learning materials use split as the go-to strategy for all children.
To get children using this strategy fluently, they need to know all their bonds to 20 well.
This strategy is built up slowly using base ten blocks.

- You can use blocks or cubes and make sticks or trains or 10 blocks.
- You can use paper: squares for 1s, strips of 10, and a big square for 100s.

You will probably find that about half of children in any Grade 2 or 3 class will find addition in columns with exchanges difficult. They can add much more reliably using the break up the second strategies. They should be encouraged to keep using break up the second number strategies.

Example

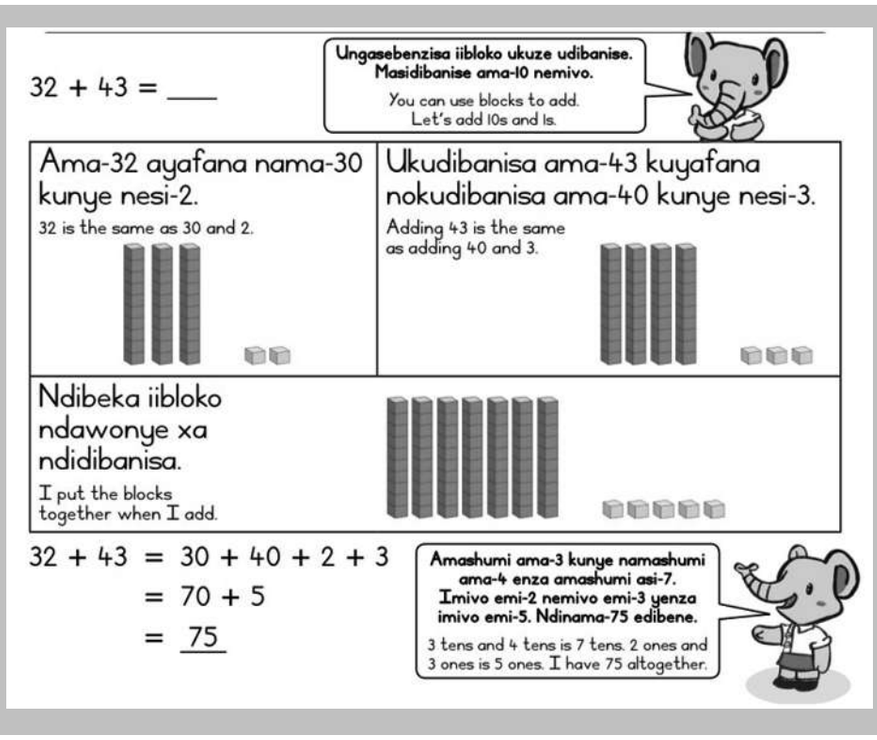
**First, children practice adding multiples of 10 together.**




Children should notice that they don't need to keep using the blocks. They can learn these as derived facts: I know 4 + 2 = 6. Therefore I know 40 + 20 = 60.

**Second, children practice with base ten blocks, to calculate addition problems, without exchanges.** They do not yet write in columns.

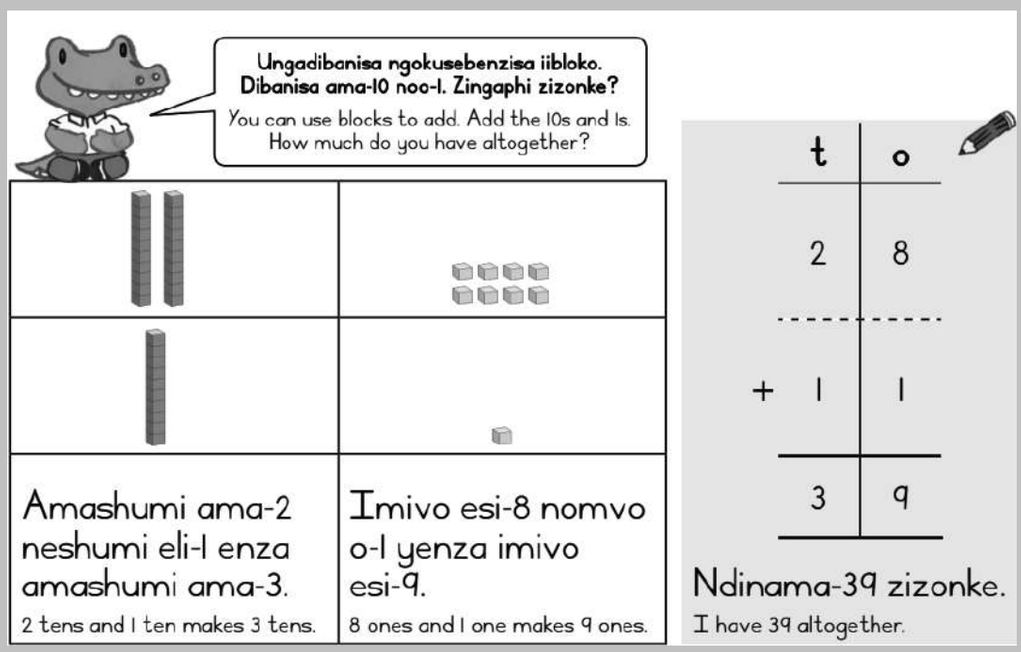
Example




Look carefully at the number used in this example. The digits to be added do not need to visit the ten. The sums are only to 9 or less. This is usually very easy for children, who know their bonds to 10.

**Third, children practice adding in columns, still with no exchanges.**
**They learn to write in columns.**

Example




**Notice from this example:**

- The teacher talk carefully uses **2 tens** and **3 tens**. The teacher talk is not about the digits 2 and 3.
- The teacher talk carefully uses **8 ones** and **1 one**. The teacher talk is not about the digits 8 and 1.
- Look carefully at the number used in this example. The digits to be added do not need to visit the ten. The sums are only to 9 or less. This is very easy for children.

**Finally, children practice adding in columns, with exchanges.**
Expanded notation is not the only way that a number can be broken down.

Break down flexibly is also called re-group. We put the number into new groups.

- 462 is not just 4 hundreds 6 tens and 2 ones.
- 462 is also 3 hundreds 16 tens and 2 ones.
- 462 is also 2 hundreds 15 tens and 12 ones, etc.

As the formal written methods often use columns, it is important for children to practice regrouping in columns.

Example

| H<br>Hundreds | T<br>Tens | O<br>Ones | |
|---|---|---|---|
| 4 | 6 | 2 | = 400 + 60 + 2 |
| 3 | 16 | 2 | = 300 + 160 + 2 |
| 2 | 15 | 12 | = 200 + 150 + 12 |
| | | | Etc... |

Why would you need to be able to re-group or break down flexibly?

- We break down flexibly when we round and adjust.
- We break down flexibly when we use doubles and near doubles.
- We break down flexibly when we break down both numbers (split) to add.

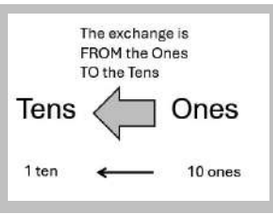
**To exchange for addition:**

- The exchange is from the column to the right.
- 10 ones (from the column to the right) is exchanged for 1 ten
- The exchange is given to the column to the left.

Example

6 tens and 18 ones = _____ tens and _____ ones

| T | O | | | |
|---|---|---|---|---|
| 6 | 18 | is the same as | 7 | 8 |




## Break up both numbers (Split) for Subtracting

### Split – Using write all totals (Subtracting)

For subtraction, **write all totals is not as easy as for addition**.

- This strategy involves **breaking up both numbers**.
- And it **does not require any exchanges**.
- But, it requires understanding **negative numbers!**

This example would require an exchange. But if you understand negative numbers, you can use write all totals.

Example

90 and 2 **SUBTRACT** 40 and 7

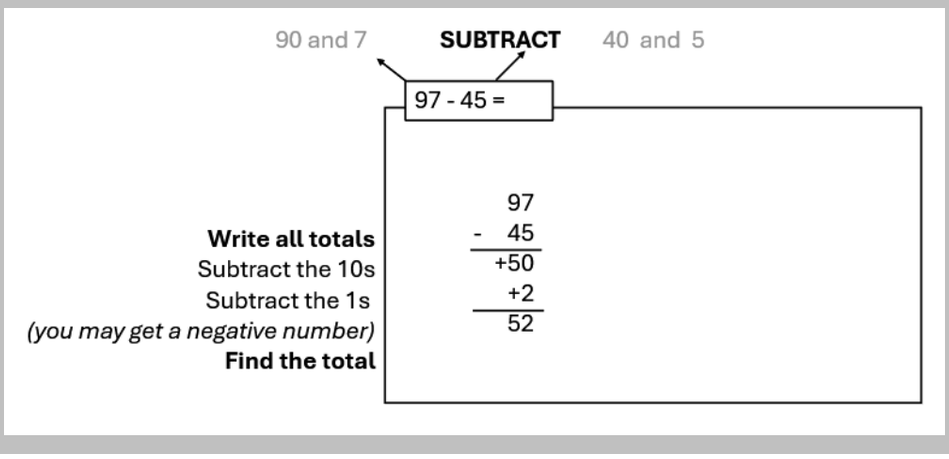
92 - 47 =

**Write all totals**
Subtract the 10s
Subtract the 1s
*(you may get a negative number)*
**Find the total**

92
- 47
+50
-5
45

This example would not require an exchange. You can use write all totals.

Example




With **write all totals** It is important to keep track of whether the totals are negative or positive numbers. **We don't recommend using this strategy with primary school children.** However, some children may invent this for themselves. They may learn it from parents or older siblings.

Write all total for subtraction requires knowledge of integers (negative numbers).
So we do not use that in primary school!
As a teacher you should understand write all totals.

## Split – Using base ten blocks (Subtracting)

To get children using this strategy fluently, they need to know all their bonds to 20 well.

This strategy is built up slowly using base ten blocks.

- You can use blocks or cubes and make sticks or trains or 10 blocks.
- You can use paper: squares for 1s, strips of 10, and a big square for 100s.

You will probably find that about half of children in any Grade 2 or 3 class will find subtraction with exchanges difficult.

- They can subtract much more reliably using the break up the second strategies.
- They can also link subtraction to addition and find the difference.
- They should be encouraged to keep using the strategies that they understand well.

To build up confidence with using the split strategy in columns, text books and teachers usually follow the same sequence as with learning this for addition.

**First, children practice subtracting multiples of 10 together.**

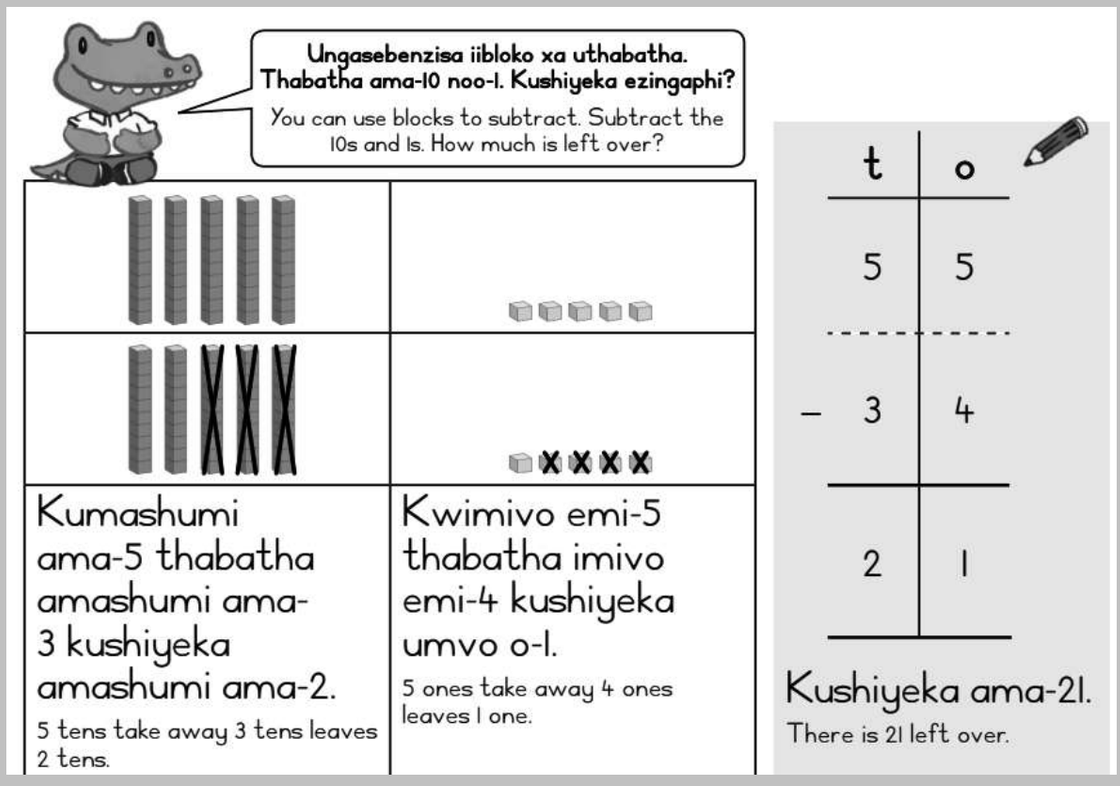
Children should notice that they don't need to keep using the blocks. They can learn these as derived facts: I know 4 + 2 = 6. Therefore I know 40 + 20 = 60.

**Second, children practice with base ten blocks, to calculate subtraction problems, without exchanges.** They do not yet write in columns.

Look carefully at the number used in this example. The digits to be added do not need to visit the ten. The sums are only to 9 or less. This is usually very easy for children, who know their bonds to 10.

**Third, children practice subtraction in columns, still with no exchanges.**
**They learn to write in columns.**

Example




Notice from this example:

- The teacher talk carefully uses **5 tens** and **3 tens**.
  The teacher talk is not about the digits 5 and 3.
- The teacher talk carefully uses **5 ones** and **4 ones**.
  The teacher talk is not about the digits 5 and 4.
- Look carefully at the number used in this example.
  The digits being subtracted always have the bigger number subtracting a smaller number.
  This is very easy for children.

**Finally, children practice subtracting in columns, with exchanges.**

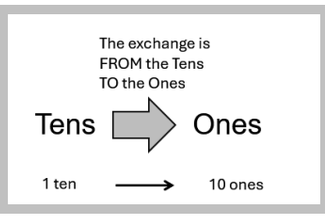
To exchange for subtraction:

- The exchange is taken from the column to the left.
- 1 ten (from the column to the left) is exchanged for a 10 ones
- The exchange is given to the column to the right.

Example

7 tens and 8 ones = 6 tens and _____ ones

| T | O | | T | O |
|---|---|---|---|---|
| 7 | 8 | is the same as | 6 | 18 |



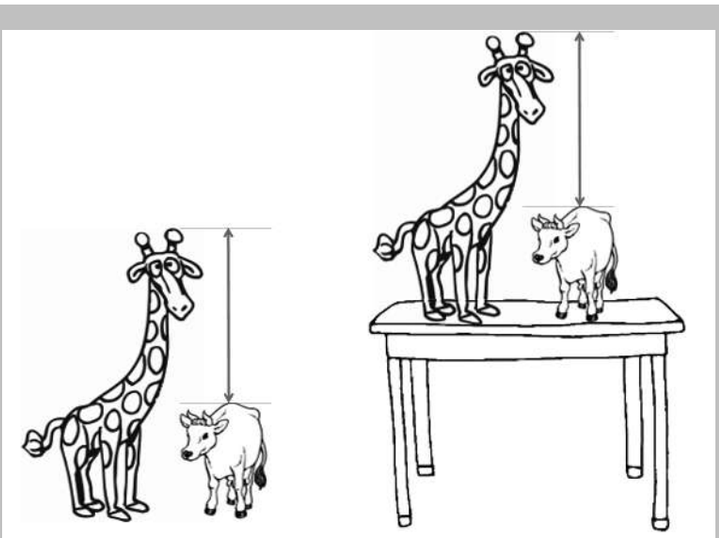
## Split - Shifting down (Subtracting)

Children often make mistakes as they don't understand exchanges for subtraction.
They struggle to explain how and why to regroup. One way to avoid regrouping for subtraction is to shift the calculation up or down.

This giraffe is taller than the cow. We can work out the difference between their heights.

If we shift the animals up – and put the giraffe and the cow on a table – their difference in height stays the same.

Diagram



So when we are subtracting (finding the difference) we can shift up or shift down to make the calculation easier.

- As long as I **add** the same amount from both numbers, the difference stays the same. The giraffe and the cow **shift up** onto the same table.
- As long as I **subtract** the same amount from both numbers, the difference stays the same. The giraffe and the cow **shift down** from the table, to the floor.

Example

**Teacher talk to explain**

Subtract is find the difference
Shift the whole calculation down by the same amount.
The difference stays the same.

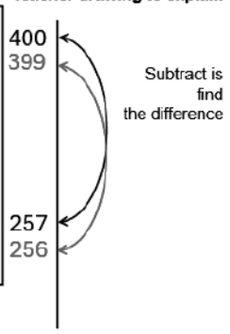
Subtract 1 from 400
So I must also subtract 2 from 257

Now I can subtract easily.

400 - 257 =

400 subtract 1 399
- 257 subtract 1 - 256
143

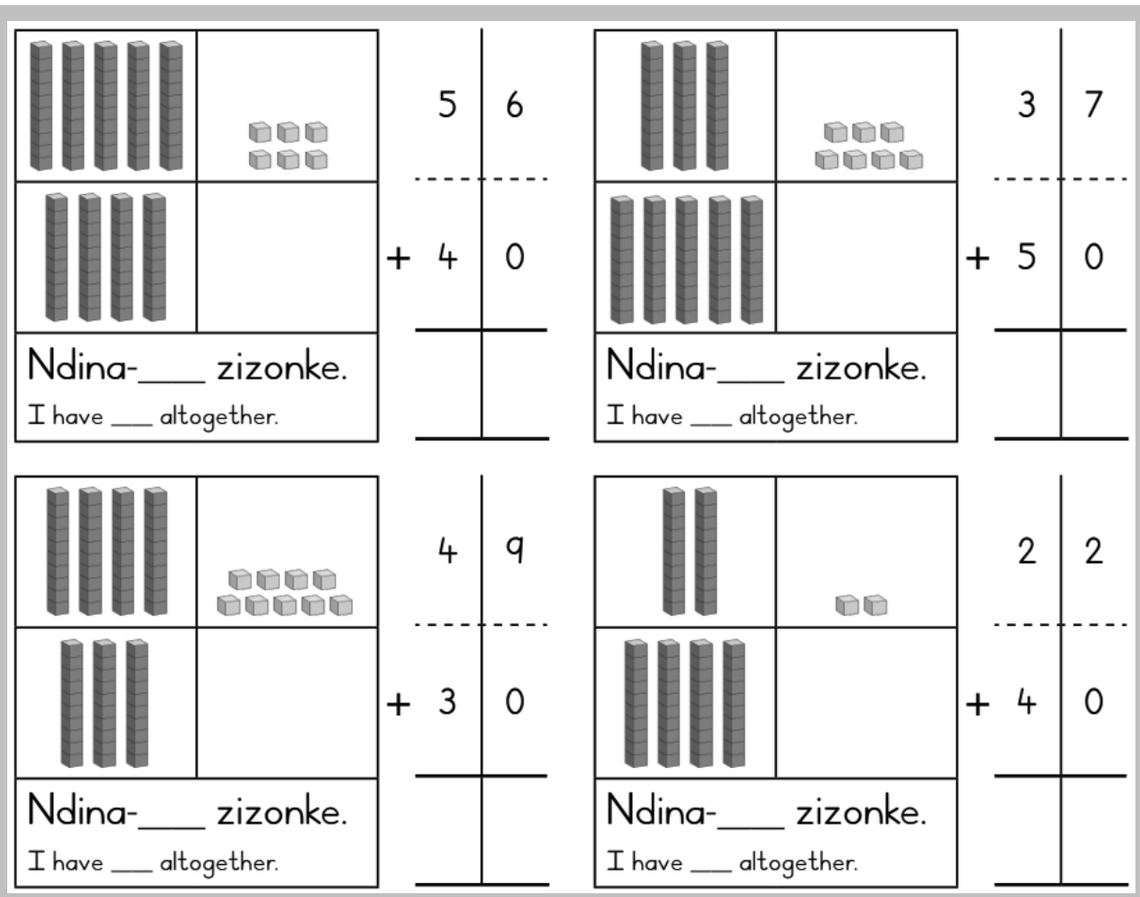
**Teacher drawing to explain**




To use the **shift up** strategy well, you need to be able to quickly work out what is a sensible number to subtract.

## SOME IDEAS OF YOUR CLASSROOM

Activity for learners




So children can practice exchanges for addition, with examples like this:

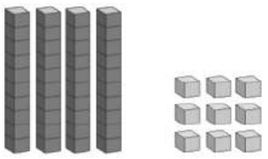
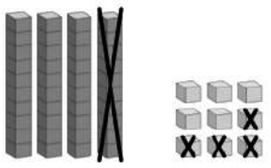
- 6 tens and 17 ones = _____ tens and _____ ones
- 16 tens and 7 ones = _____ hundreds _____ tens and _____ ones
- 15 tens and 14 ones = _____ hundreds _____ tens and _____ ones

Children can practice exchanges for subtraction, with examples like this:

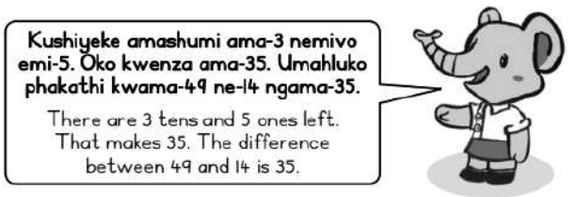
- 6 tens and 7 ones = 5 tens and _____ ones
- 8 hundreds 6 tens and 7 ones = 7 hundreds _____ tens and _____ ones
- 6 hundreds 5 tens and 4 ones = 5 hundreds _____ tens and _____ ones

Activity for learners

$49 - 14 = \_\_\_\_$


| Ama-49 ayafana nama-40 kunye ne-9. | Ukuthabatha i-14 kuyafana nokuthabatha i-10 kunye nesi-4. |
|---|---|
| 49 is the same as 40 and 9. | Subtracting 14 is the same as subtracting 10 and 4. |
|  |  |

$$\begin{aligned} 49 - 14 &= 49 - 10 - 4 \\ &= 39 - 4 \\ &= \underline{35} \end{aligned}$$




SCAN ME

DBE presentation (December 2025). This is work undertaken by Asiya Hendricks and Aki Itagaki for the Teaching maths with understanding project


**Scaffolding for addition and subtraction of 2-digit and 3-digit numbers**


Review the powerpoint developed by the Department of Basic Education on the steps needed before adding and subtracting in columns.

View slides 27-29 on slide show mode. Watch the animations to see how to model subtraction with re grouping using base ten blocks. Notice that first children write all totals for addition (slide 28). They exchange the tens for ones, and write all totals for subtraction (slide 29).

## Tasks 4.3.1

### Adding bigger numbers

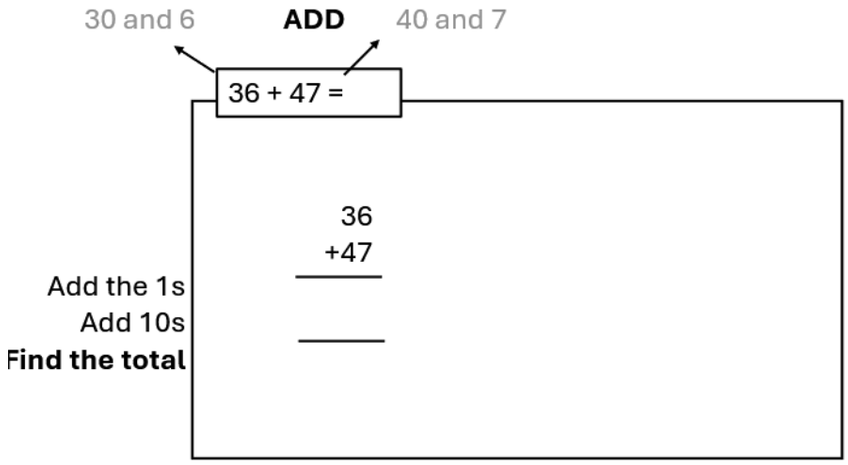
1. Show how you were taught addition, when you were at primary school.

287 + 196 = ___

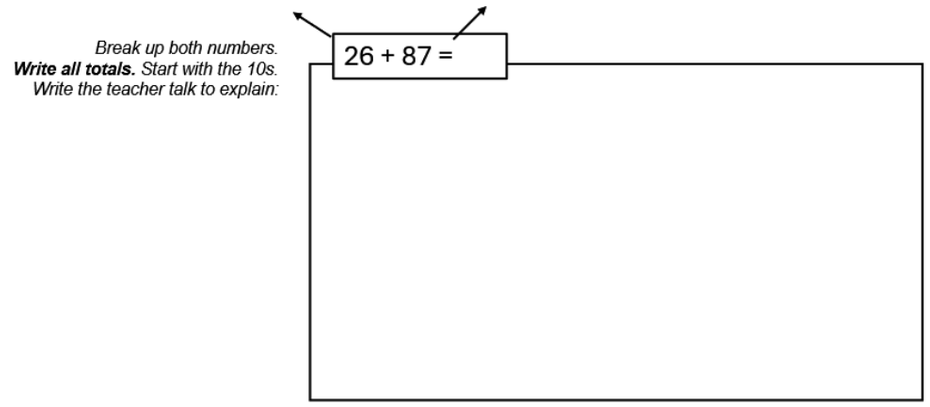
2. Explain how to do addition using your method.

| 3. For this method: | YES/NO |
|---|---|
| | |
| How to **count forwards in ones**. Start at any number? | |
| How to **count forwards in ones**. Start at any number? | |
| The **next ten**. How far to the next ten? | |
| Did it help you with **rounding off** and **estimating**? | |
| Did you learn to **link subtraction to addition**? (Find the difference) | |
| Could you work in any order and be **flexible**? | |
| Could you do this **mentally**, or did you have to write it down? | |
| Did it help you with **number lines** or **coordinates for a graph** at high school? | |
| Did it help you with **algebra** at high school? | |

4. As a teacher, you need to be able to work flexibly. You need to be able to explain well. Break up both numbers. **Write all totals.** .Add the 1s first.




5. Break up both numbers. **Write all totals.** Add the 10s first.
Write down the teacher talk to explain



6. Break up both numbers. **Write all totals.** .Add in any order.
   Write down the teacher talk to explain.

*Write down the teacher talk to explain*

48 + 57 =

36
+47

7. Break up both numbers. **Write all totals.** .Add in any order.
   Draw **number pictures** to explain.

*Teacher drawing to explain*

68 + 74 =

8. Put the first number in your head. Break up the 2nd number. **Jump.**
   Draw a number picture or a **number line** to explain.

*Teacher drawing to explain*

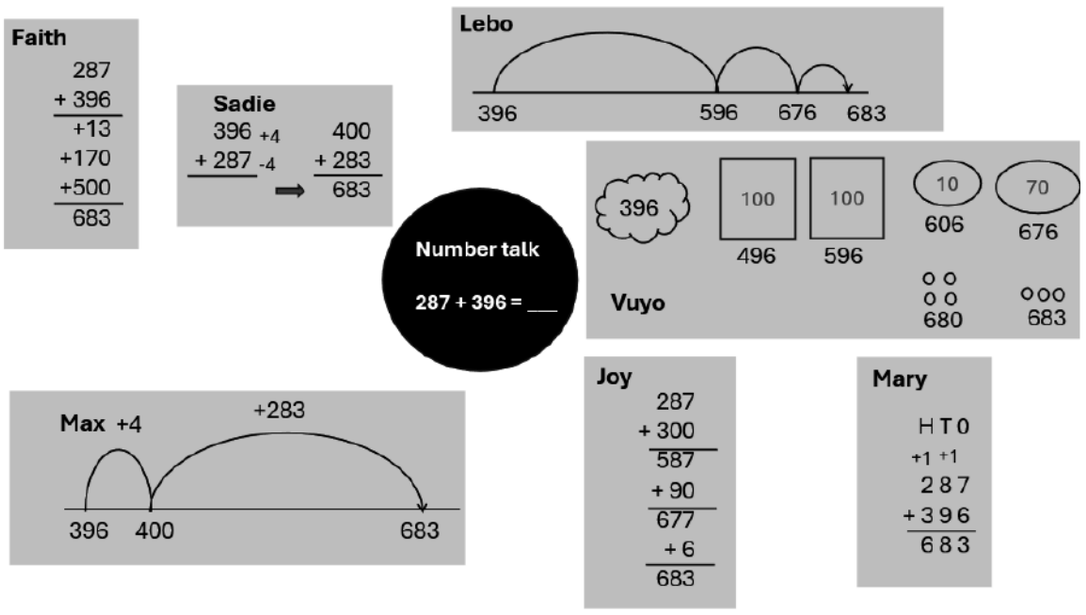
68 + 74 =

9. Put the first number in your head. Break up the 2nd number. **Jump.**
   Work in columns.

*Write down the teacher talk to explain*

92 + 57 =

10. A teacher does a number talk for 287 + 396.
    He chooses some examples to put onto the board and discuss.




a. Which children **started with the bigger number**?

b. Which children used a **visit the ten** strategy?

c. Which 4 children **broke up both numbers**?

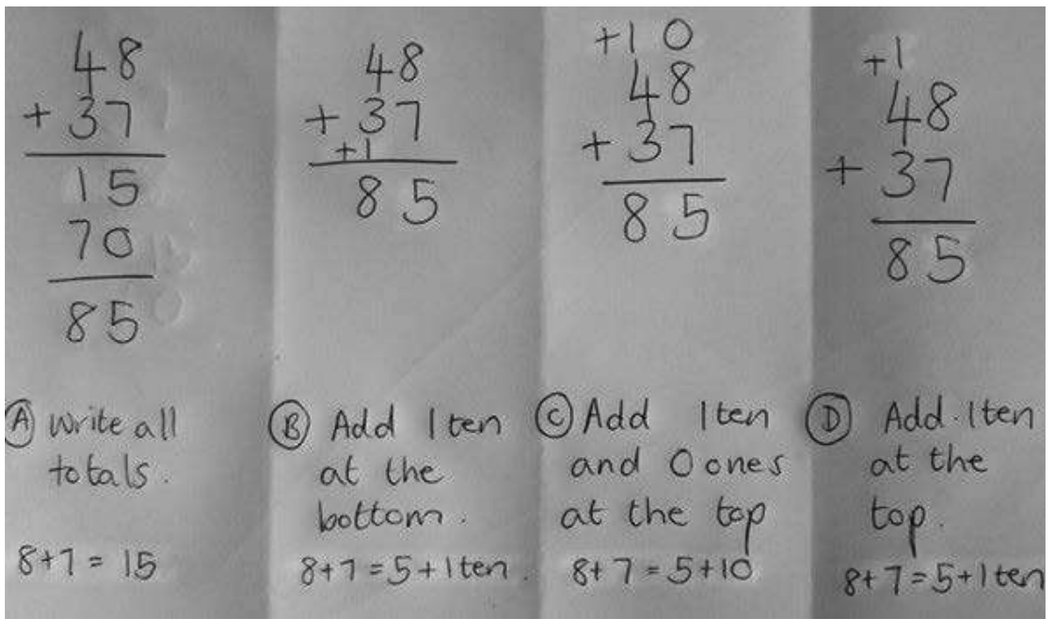
d. Which children **broke up the 2nd number**?

e. Which child **wrote all totals**?

f. Draw a number line to explain what Sadie did

11. About half the children in Miss Zama's Grade 3 class make mistakes when they have to add and exchange in columns. Miss Zama is planning how to help these learners. She looks on the Internet for different ways to add in columns. She finds 4 different ways.

She has taught her class **Option D**. Children have to know that 8 + 7 = 15 = 5 + 1 ten.
That was how she was taught, when she was at school.
But so many children don't understand it, and make mistakes.




a. Which option do you think she should try with her struggling learners?

b. Give your reason for choosing that option.

c. For the calculation 67 + 39 = ... Show all the options that Ms Zama used. Show what children have to know about 7 + 9 = ....

| a. Write all totals | b. Add 1 ten at the bottom | c. Add 1 ten and 0 ones at the top. | d. Add 1 ten at the top. |
|---|---|---|---|
| | | | |

## Tasks 4.3.1

### Subtracting bigger numbers

1. Show how you were taught subtraction, when you were at school.

– 196 = ____

2. Now write down each step, to explain how to do subtraction using your method.

| 3. For this strategy and representation: | YES/NO |
|---|---|
| Did you need to know | |
| • How to **count backwards in 1s**. Start at any number? | |
| • How to count **backwards in 10s**. Start at any number? | |
| • The **previous ten**. How far to the pervious ten? | |
| Did it help you with **rounding off** and **estimating**? | |
| Could you work in any order and be **flexible**? | |
| Could you do this **mentally**, or did you have to write it down? | |
| How to **link subtraction to addition**? So how to **find the difference**? | |
| Did it help you with **number lines** or **coordinates for a graph** at high school? | |
| Did it help you with **algebra** at high school? | |

4. A child breaks up the second number and subtracts each part.
   Complete their working out in columns.
   Now draw a number line to explain their working.

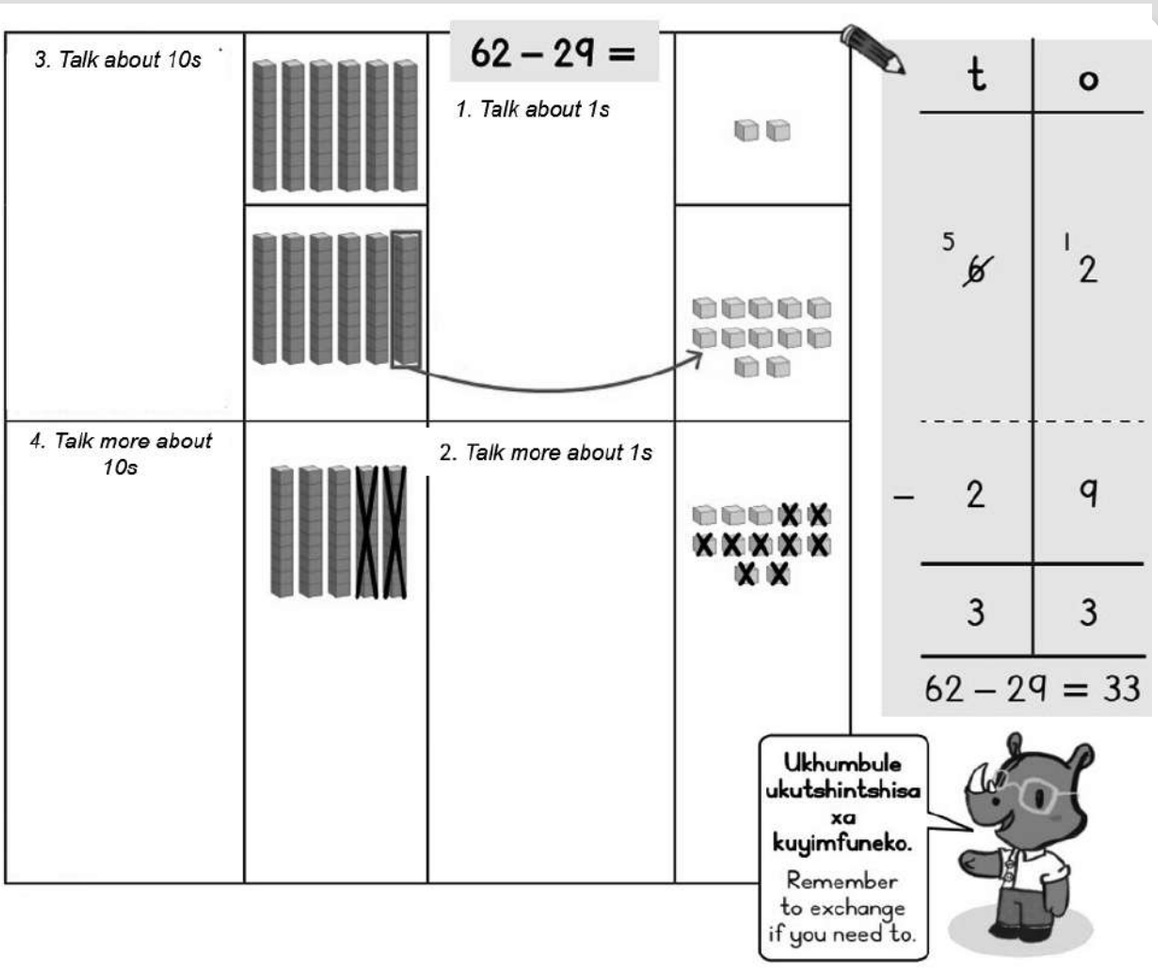
92 **SUBTRACT** 40 then subtract 7

92 - 47 =

92
-40

First subtract the 10s
Then subtract the 1s

5. A teacher is modelling how to **break up both numbers and exchange** 10s for 1s.
   She works through steps 1 to step 4. Write down what she should say to explain each step.




6. A teacher is writing on the board to show her class how to **break up both numbers and exchange** 1 ten for 10 one. Complete the worked example, and write down what she should say to explain

Write down the teacher talk to explain:

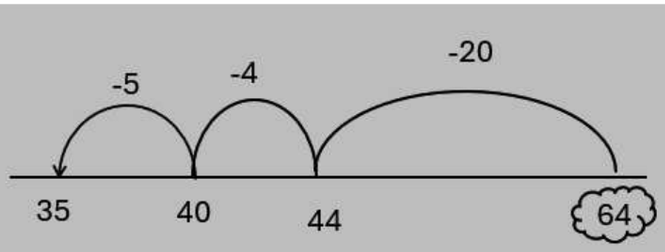
64 – 29 =

| | t | o |
|---|---|---|
| | | |
| – | | |
| | | |

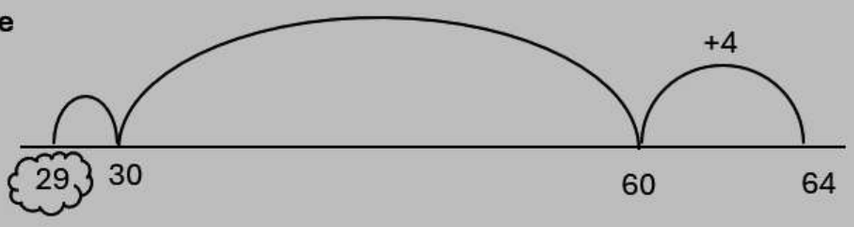
7. Some learners do the same calculation on a number line.
Mpho and Refilwe argue about who did it correctly.

64 – 29 =

**Mpho**




**Refilwe**




a. Circle the answer on Mpho's work.
b. Finish Refilwe's work. Circle where you find her answer.

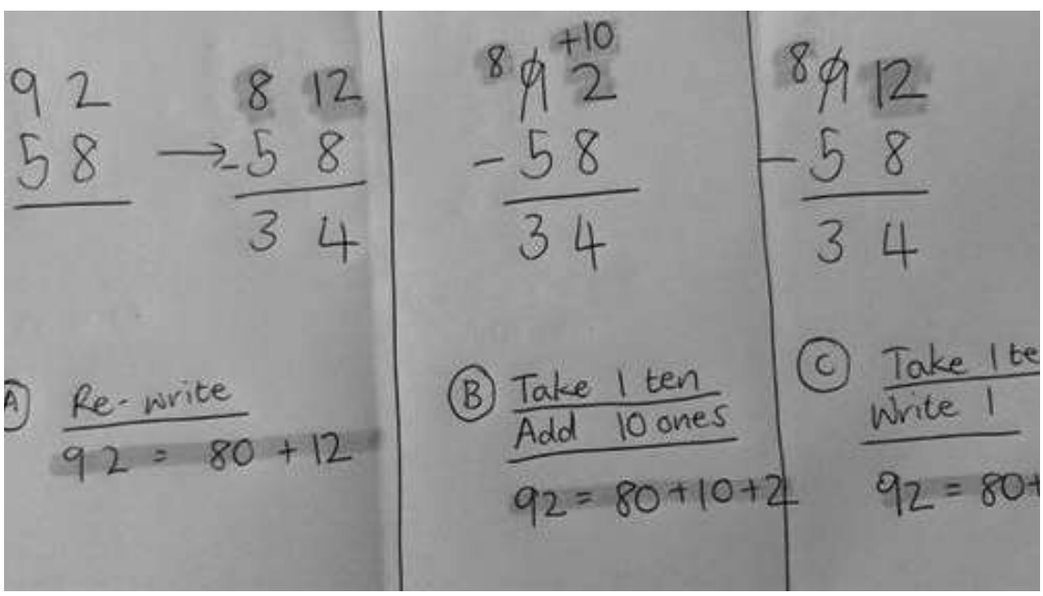
c. Which learner broke up the 2nd number to **take away**? _____

d. Which learner worked out 29 + ___ = 64, to **find the difference**? _____

e. Show how to use a numebr line to round and adjust for 64 - 29 = _____

8. You set this calculation for a number talk. Show at least 5 different ways that you think chidlren might answer it.


9. Miss Zama marked her Grade 3 class test. She was shocked to find that only 5 children could subtract and exchange in columns correctly. She went to look for different ways to split and write down subtraction in columns.




She has taught her class **Option D**. Children have to know that they cant subtract 8 from 2. So they must chane 92 to be 80 and 12. But so many children don't understand it, and make mistakes.

a. Which option do you think she should try with her struggling learners?

b. Give your reason for choosing that option.

c. Almost all of your class can subtract correct if they **break up the second number**. Show Miss Zama how to do this for 92 – 58 = ... Show her three different ways to write this down.

| 92 – 58 = ___<br>**Break up the second number.**<br>Use a **number picture** | 92 – 58 = ___<br>**Break up the second number.**<br>Use **columns** |
|---|---|
| | |

**Break up the second number.**
Use a **number line**

d. For the calculation **67 - 39** = ___ Show all the options A to C that Ms Zama used. Show what children have to know about 7 – 9 = ___

| A. Re-write | B. Take 1 ten.<br>Add 10 ones | C. Take 1 ten.<br>Write 1. |
|---|---|---|
| | | |
| D. Show 2 other ways that you think children find easier than columns | | |

10. You FP Department Head is concerned that the chidlren in Foundation phase don't know their number facts for addition and subtraction. She asks you to plan the mental starters for Grade R to 3 for the first term next year.

a. The Foundation Phase DH gives you the "Table: Which mental fluencies are needed for which addition calculation strategies?" which is on p59 She asks you to complete the table for subtraction calculation strategies.

Table: Which mental fluencies are needed for which subtraction calculation strategies?

| **SUBTRACTING FLUENCIES** | Count all | Count forwards in 1s. Add 1s | Add multiples of 10s | Count forwards in 10s. Add 10s. | Rapid recall. Known facts | Jump. Add 10s. Then sdd 1s | Visit the tens | Round and adjust | Link subtraction to addition (the difference) | Split – Write all totals | Split to subtract – Exchange 10s for 1a | Split to subtract – Shift down |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Count backwards in 1s. Start at 10 | | | | | | | | | | | | |
| Count backwards in 1s. Start at any number. | | | | | | | | | | | | |
| Doubles | | | | | | | | | | | | |
| Bonds of 5 | | | | | | | | | | | | |
| Near doubles | | | | | | | | | | | | |
| Count backwards in 10s. Start at 10 | | | | | | | | | | | | |
| Count backwards in 10s. Start at any number. | | | | | | | | | | | | |
| Place value. Break down into 10s and 1s | | | | | | | | | | | | |
| Find the previous ten | | | | | | | | | | | | |
| How far to the previous ten | | | | | | | | | | | | |
| Bonds of 10 | | | | | | | | | | | | |
| Whole-part-part diagram | | | | | | | | | | | | |
| Families of number sentences | | | | | | | | | | | | |
| All bonds to 10 | | | | | | | | | | | | |
| All bonds to 20 | | | | | | | | | | | | |
| Exchange one 10 for ten 1s | | | | | | | | | | | | |
| Re-group to subtract | | | | | | | | | | | | |

b. She says she wants to plan for mental fluencies for these calculation levels, and asks you to complete the table she started.

| Calculation strategy level | Calculation strategies | Grade and terms | Mental fluencies needed |
|---|---|---|---|
| Level 1 | Count all | GR, T1&T2<br><br>G1, T1&T2 | |
| Level 2 | | GR, T3&4<br><br>G1, T3&4 | |
| Level 3 | | G2, T1&2 | |
| Level 4 | | G2, T3&4 | |
| Level 5 | | G3, T1&2 | |
| Level 6 | | G3, T3 | |
| Level 7 | | G3, T4 | |

c. Next, she asks you to plan the 8 weeks of mental starters for the first term of Grade 3 Create the 8 week plan for Level 5. Rember to include lots of repitition.

| LEVEL 5<br>Week | Fizz pop function | Strategic counting | Game in pairs |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

d. But you are worried because you know that Grade 3s don't yet know Level 4 how to visit the tens!

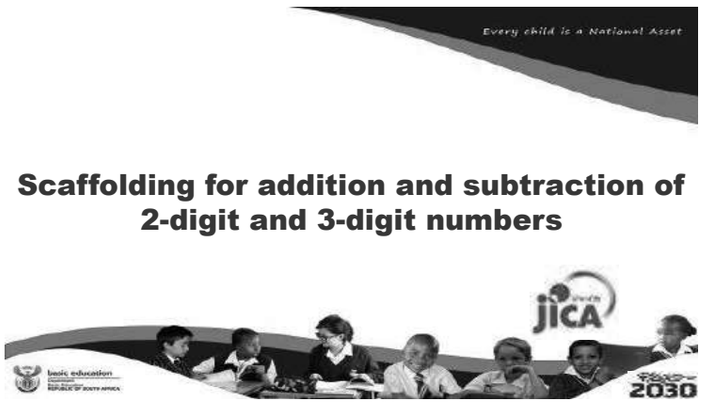
Create another 8 week plan for Level 4.

| LEVEL 4<br>Week | Fizz pop function | Strategic counting | Game in pairs |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

## Tasks 4.3.3

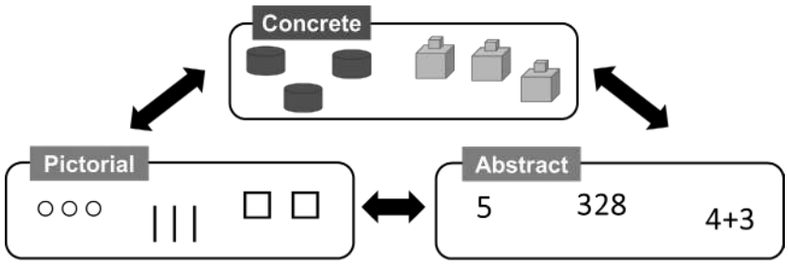
SCAN ME

DBE presentation (December 2025). This is work undertaken by Asiya Hendricks and Aki Itagaki for the Teaching maths with understanding project




Review the powerpoint developed by the Department of Basic Education on the steps needed before adding and subtracting in columns.

1. The DBE encourages moving between concrete pictorial and abstract (CPA) representations.




Draw a **number picture** to show 328. Use Zibalo's trays of scones.

Draw a **number picture** to show 328. Use the DBE's example (slide 31).

Show 328 on a **number line**. Show big leaps for 100s, jumps for 10s, and little hops for 1s.

2. In this **M4PT adding and subtracting** manual we have focused on 7 levels for adding and subtracting calculation strategies. These connect with the DBE presentation.

Write down the main ideas for each schema in this table.

| **Fuson's schema** | **DBE slides** | **Main ideas** | **Levels in the calculation strategy trajectory** (draw at least 1 representation) |
|---|---|---|---|
| Counting | Slides 4-9 | Count objects one by one<br>One to one correspondence..... | **Level 1 Count all** |
| Part whole | Slides 10-17<br><br>Slides 25 and 26 | | **Level 4 Visit the ten** |
| Additive reasoning | Slides 18 - 24 | Meaning of addition<br><br>Meaning of subtraction | **Level 5 Moves to flexibility**.<br>See also the additive word problem types. |
| | Slides 27 - 34 | | **Level 7 Split** |

Notice that this particular DBE presentation does not include **Level 2 count on**, or **Level 3 count on in 10s and 1s (the jump strategy)**, but these are in the CAPS policy.

3. On slide 30 there is an exercise to use the base 10-kits for 2 calculations.

Use **Base Ten Kits** to perform the following calculations.

a) 87 + 32

b) 129 – 75

| Hundreds | Tens | Ones |
|---|---|---|
| | | |

Use base ten kits (unifix connected in groups of 10s) or the base ten printables at the back of the book.

a. Make a short “hands and voice” video to show how you explain: 87 + 32.
b. Make another shift “hands and voice” video to show how you explain: 129 - 75.

If you prefer you can use powerpoint with animations and voice over recording (so using the same methods shown in Slide 28, but with a voice recording).

Write the link to the folder with your work here:

4. Complete the exercise on slide 34, using the simplified pictorial approach

68 + 35

| Hundreds | Tens | Ones |
|---|---|---|
| | | |

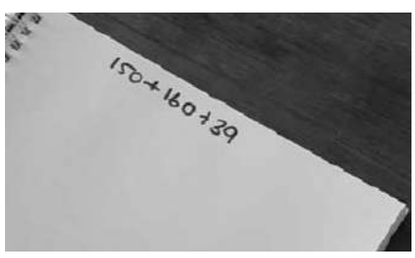
104 – 25

| Hundreds | Tens | Ones |
|---|---|---|
| | | |

## Two minute tango

| | | | |
|---|---|---|---|
| **1** | 8 + 4 = ____ ones and 1 ten | **11** | 43 = 3 tens and ___ ones |
| **2** | 6 + 8 = ____ ten and ____ ones | **12** | 57 = 4 tens and ___ ones |
| **3** | 7 tens + 3 tens = 9 tens and ____ ones | **13** | 96 = ___ tens and 16 ones |
| **4** | 7 tens + 8 tens = 14 tens and ____ ones | **14** | 23 = ____ tens and 13 ones |
| **5** | Complete the whole part part diagram for<br>53 – 28 = ____ | **13** | Complete the whole part part diagram for<br>____ – 28 = 53 |
| **6** | Show 63 – 28 = ____on a number line | **16** | Show 28 + ____ = 63 on a number line |
| **7** | Complete the whole part part diagram for<br>59 + 19 = | **15** | Complete the whole part part diagram for<br>____+ 19 = 59 |
| **8** | Show 59 + 99 = ____on a number line | **16** | Show ____ + 18 = 100 on a number line |
| **9** | T O<br>4 19 ⇒ T O | **19** | T O<br>6 8 ⇒ T O<br>5 |
| **10** | Re-write this sum, to show the exchange, for split<br>T O<br>6 7<br>+ 8 9 ⇒ T O | **20** | Re-write this difference, to show the exchange, for split<br>T O<br>6 7<br>- 4 9 ⇒ T O |

SCAN ME




This Australian maths bub video discusses calculating flexibly: 150 + 160 + 39

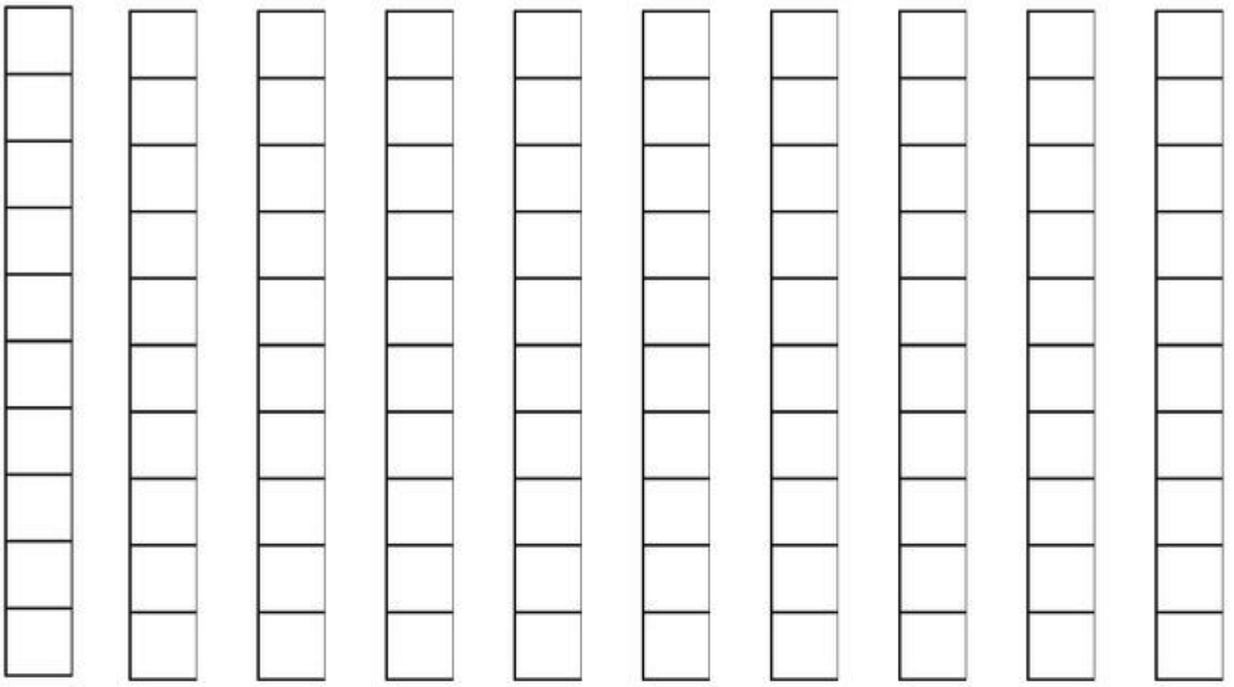
## SUMMARY 4.3

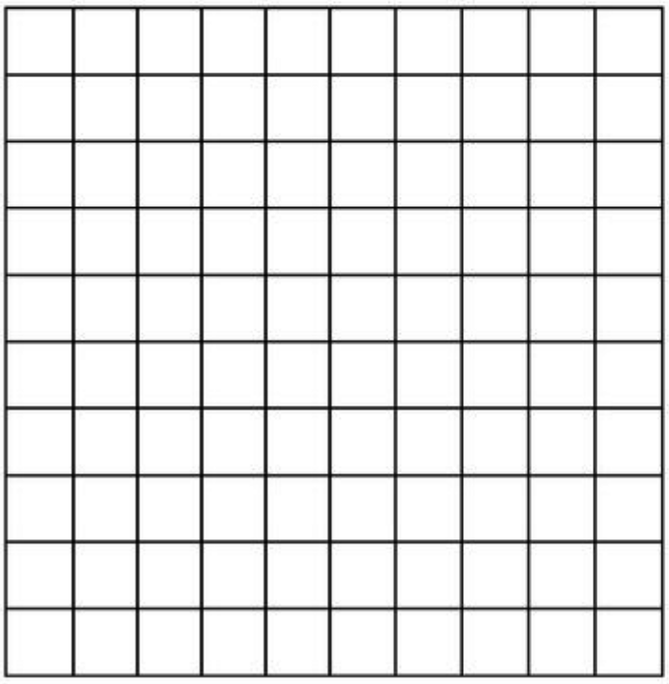
Show at least 2 ways to calculate 569 + 387 = ..., using the split (break down both numbers) strategies. For one of the ways write what you would show on the board. Write what you would say to the class.

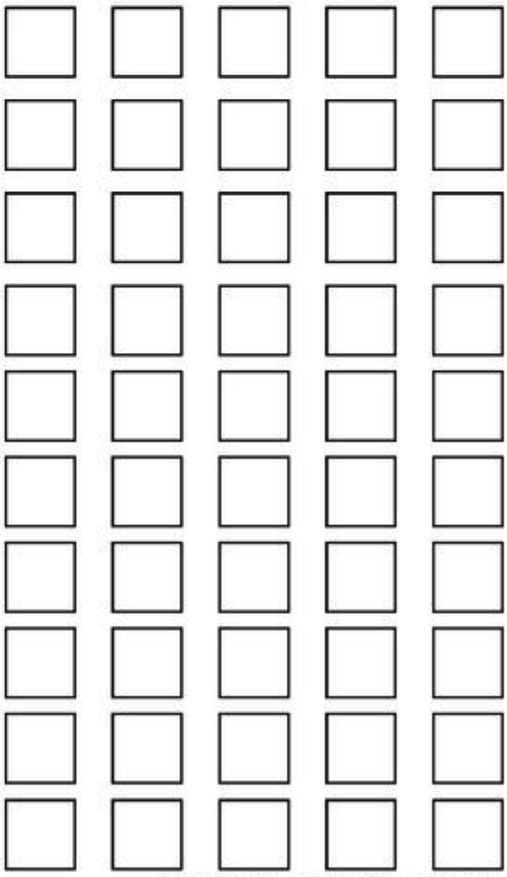
Show how at least 2 ways to calculate: 561 – 387 = ..., using the split (break down both numbers) strategies. For one of the ways write what you would show on the board. Write what you would say to the class.

Skill – Base 10

Name: _____







| 91 | 92 | 93 | 91 | 95 | 96 | 97 | 98 | 99 | 100 |
|---|---|---|---|---|---|---|---|---|---|
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

**Maths4Primary teachers** is a learning series for Bachelor of Education students in Foundation Phase and Intermediate Phase specializations. Each module includes online materials, games, two minute tangos and short quizzes for students and lecturers in initial teacher education.

**Maths4Primary teachers** is aligned to the **Primary Teacher Education (PrimTEd)** standards which have been agreed as the minimum core content for teachers of mathematics in primary schools in South Africa.

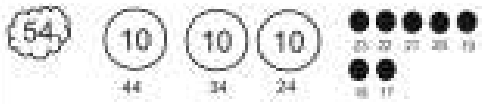
| Children use increasingly efficient **calculation strategies.** | |
|---|---|
| **Level** | **Calculation Strategy** |
| Level 1 | **Count all:** Count all in ones. |
| Level 2 | **Count on:** Put the first number in my head, then count on with the small number song (in ones) |
| Level 3 | **Jump:** Break up the second number and jump forwards or backwards in 10s and 1s. |
| Level 4 | **Visit the 10s:** Break up the second number to visit the next or the previous multiple of 10. |
| Level 5 | **Moves to flexibility:**<br>Links subtraction to addition<br>21 – 19 = ___ is the same as 19 + ___ = 21<br>Reorder:<br>7 + 16 + 23 = 23 + 7 + 16 |
| Level 6 | **Efficient strategies:**<br>Depends on the numbers involved, including strategies such as<br>• near doubles,<br>• round and adjus |
| Level 7 | **Split strategies:**<br>**To add**<br>• Break up the 2nd number<br>• Write all totals<br>• Exchange<br>**To subtract**<br>• Break up the 2nd number<br>• Exchange |

Children use increasingly efficient **representations**

**54 – 37 = ...**

For **break up the second number,**

I can use number pictures




54 – 37 = 17

For **break up the second number,**

I can use number lines.


54 – 37 = 17

For **break up the second number,** I can use number sentences

Tens: 54 – 30 = 24

Ones: 24 – 7 = 17

54 – 37 = 17

**Add: Split**
Write all totals

76
+47
13
110
123

**Subtract**
Break up the 2nd number

63 − 47 ⇒ 63 − 40 = 23; 23 − 7 = 16

**Add: Split**
Exchange

+1
76
+47
123

**Subtract: Split**
Exchange

+10
63
−47
16

**Adding and subtracting** is an introduction to the teaching of addition and subtraction. It answers questions such as:

- How do young children learn to add and subtract?
- How do we move children away from counting in ones, to using more structured and efficient ways of calculating?