MATHS4
Primary teachers

Emergent number sense

2026

MATHS 4 PRIMARY TEACHERS
PRACTICE WORKBOOK

This books belongs to:

Name: ______________________________

Surname: ______________________________

Contact: ______________________________

University: ______________________________

Find additional resources for this module here:
M4PT Emergent Number Sense
STUDENT FOLDER
https://tinyurl.com/bdhdzche

This Emergent Number Sense module aligns to the following PrimTEd standards

General pedagogical standards for mathematics teaching	GPM1	**Plan** effective learning experiences
	GPM 2	Take **learners' knowledge** into account
	GPM 3	**Engage learners productively** in mathematics
	GPM 4	Teach a **balanced** mathematics curriculum
Mathematical acting and thinking	MAT 1	**Playful engagement** to search for, and develop, mathematical insight
	MAT 2	**Represent** and use mathematics
	MAT 3	**Reason** mathematically
	MAT 4	**Reflect** for action
Number and algebra	NA 1	Knowledge and use of **emergent number awareness** of learners
	NA 2	Knowledge of **number systems**
	NA 3	Key ways of **representing** numbers and Show how **counting** is used as a fundamental mediating resource
	NA 7	**Early algebra**
	NA 8	**Overarching mathematical ideas** for school arithmetic
Geometry and measurement	GM 2	Knowledge of **measurement**

In addition the module offers a basic introduction to fluencies and representations for NA3 Additive relations, and NA4 Multiplicative relations.

This *Maths 4 Primary Teachers* course is for Bachelor of Education students
is undertaken as design-based research initiative.

2026 Fifth intervention cycle

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UNIT 1: Introduction

Emergent Number Sense focuses on **how young children** (from babies to 5 and 6 year olds) **learn mathematics**. Children make meaning from the world around them. They come to school (even pre-school/creche and grade R) with many mathematical experiences from their life experience. Over time they are inducted – learning from adults and their peers – into a tradition of more formal mathematics.

Key Ideas

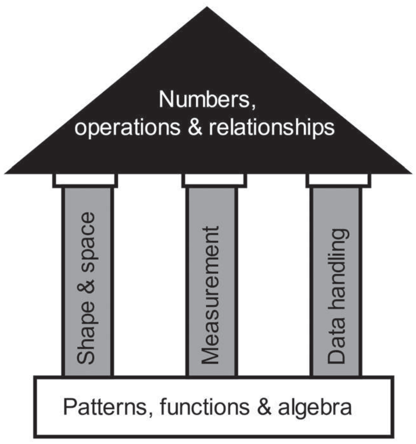
This umthamo has the following key ideas:

1. Explain **what mathematics is,** and agree on some **ground rules (classroom norms)** for this maths course.
2. Know **how to succeed in this maths course**: what to do each week and how you are assessed.
3. Know that for mathematics to make sense, we need **a clear mental map or picture** for numbers and relationships. There are **two main pathways for learning about numbers:**
    a. A **discrete pathway;** and
    b. A **continuous pathway.**

What is mathematics?

We know that many students taking this course have not liked mathematics at school. Many students find maths difficult and stressful. We believe everyone can do mathematics. Even if a person has been taught badly, and hated maths before, with effort and opportunity that person can do maths.

But, what is mathematics? Are we all talking about the same thing when we say "maths"?
Mathematics involves thinking about and working with numbers, shape and space.
The South African Curriculum and Assessment Policy (CAPS) identifies five content areas for primary school mathematics.



Numbers operations and relationships is the focus in primary school. it is given the most time

- Numbers can be whole, natural, rational or integers;
- Operations are add, subtract, multiply and divide; and
- Relationships include equal, unequal contexts, more than and less than.

There are 3 contexts in which to use numbers, operations and relationships:

- Shape and space
- Measurement and
- Data handling.

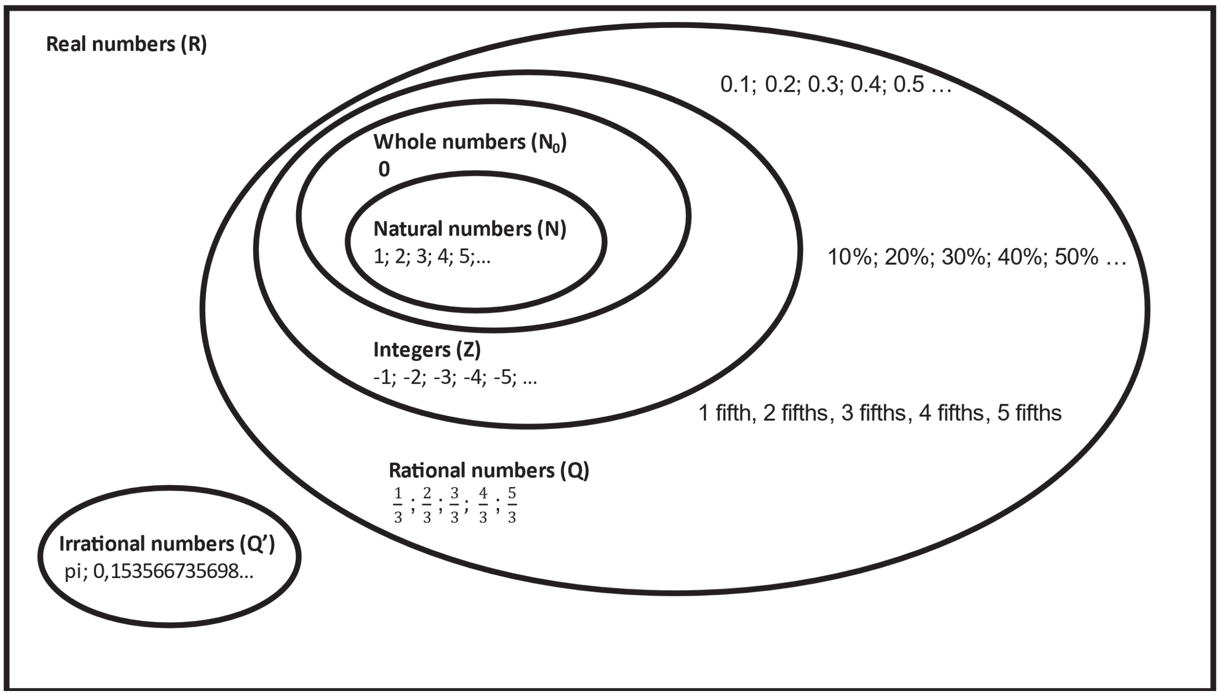
Patterns functions and algebra is the foundation. It is the way of learning about numbers, operations and relationships in the 3 different contexts.

The South African curriculum and assessment policy (CAPS) defines mathematics as follows:

> Mathematics is a language that makes use of symbols and notations for describing **numerical, geometric and graphical relationships.** It is a **human activity** that involves observing, representing and investigating **patterns and quantitative relationships** in physical and social phenomena and between mathematical objects themselves. It helps to develop **mental processes** that enhance logical and critical thinking, accuracy and problem-solving that will contribute to decision-making. (DBE, 2011).

As teachers you need to know the **different types of numbers** which children encounter in primary and secondary school. This diagram shows how the different **number systems** fit together.

<u>Diagram: Number systems</u>



This course is designed to help you to learn the mathematics you need to teach primary school children, well. Learning mathematics involves learning ways of thinking. Carpenter, Franke et al. (2003) write that learning mathematics

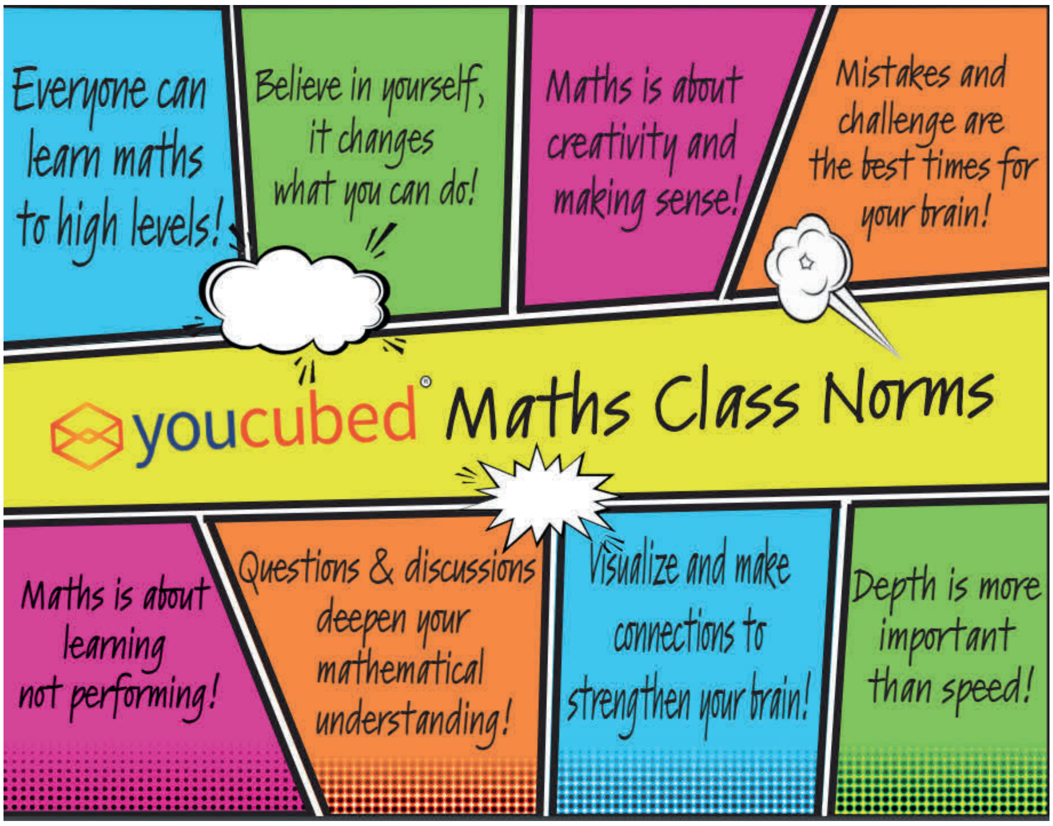
> involves learning **powerful mathematical ideas** rather than a collection of disconnected procedures for carrying out calculations. But it also entails **learning to generate those ideas, how to express them using words and symbols, and how to justify to oneself and others** that those ideas are true. (Carpenter, Franke et al. 2003, p. 1)

Learning mathematics is learning **ways of thinking** about **quantities (discrete objects/events** and **continuous quantities** which we can **measure).**

Learning mathematics is **learning ways of thinking** about **shapes,** their arrangement, how they are positioned and can change (transform) in **space**. In **Task 1.2** you read an article about mathematical powers. Once you have thought about your relationship with maths (how it makes you feel, right now) and you have agreed on what maths actually is, it may be worth working on some classroom norms. How do you want to be treated, and how will you agree to behave in this maths class? What kind of a maths teacher do you want to be?

These are some examples of classroom norms. Your class might agree to some of these, add some more. You may want to elect a class representative. Start a whatsapp group to help each other.

Diagram: Some classroom norms



How to succeed in this course

Weekly rhythm for learning

To be successful in this university course, you will need to work every week. We want you to pass and to pass well. We give you lots of opportunities to

- learn how to teach maths and
- improve your own maths.

If your own maths is shaky and if you hate maths you wont be able to teach it well. Now is your chance to work hard to develop a better relationship with maths, be better at doing maths (maths should make sense); and learn to teach primary maths well.

SCAN ME

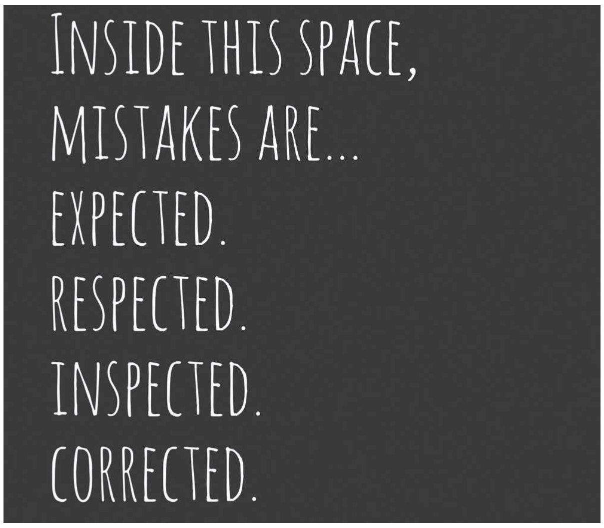
Jumptrak

Get your jumptrak username and password from your lecturer.
Make sure you use it on a computer or mobile.
https://library.jumptrak.co.za/

You will get feedback on your work through tutorials, marking and reflecting on tasks, and from your quizzes. Use this feedback to improve.

Mistakes are good

In this course there are many opportunities to practice, check what you know, get feedback and improve.



All mathematicians make mistakes.

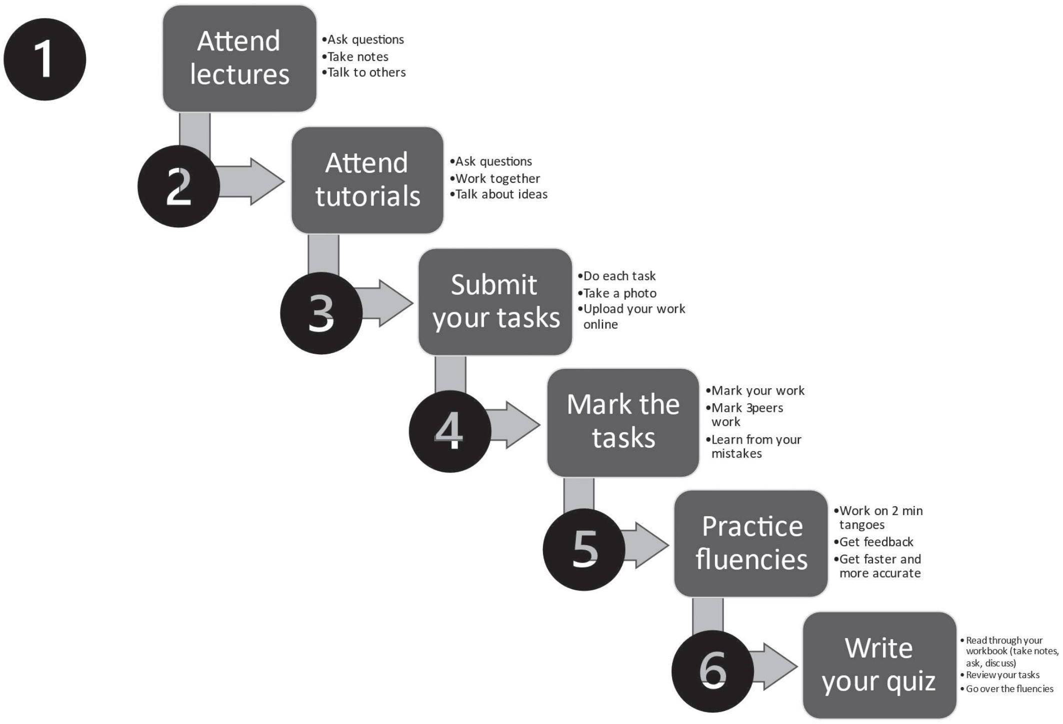
No one gets everything correct at first.

Mathematicians learn from their mistakes.

If you get everything correct you are not learning (you already knew the work).

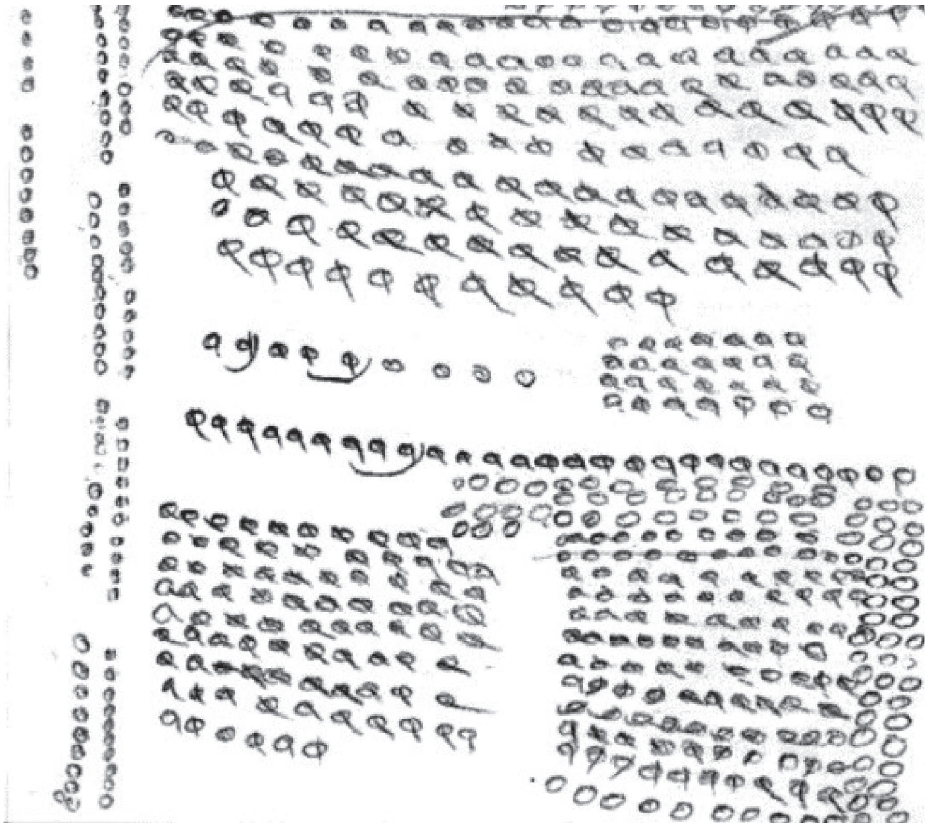
To learn something new you must try, practice, get things wrong, and learn from your mistakes.

You get feedback when you mark your own work, and when you mark other students work. You also get feedback when you practice 2 min tangoes online (the computer tells you if you are correct or incorrect). You can practice until you are getting most questions correct. You also get feedback when you write a weekly quiz. All of this helps you to learn and become more fluent with mental calculations.

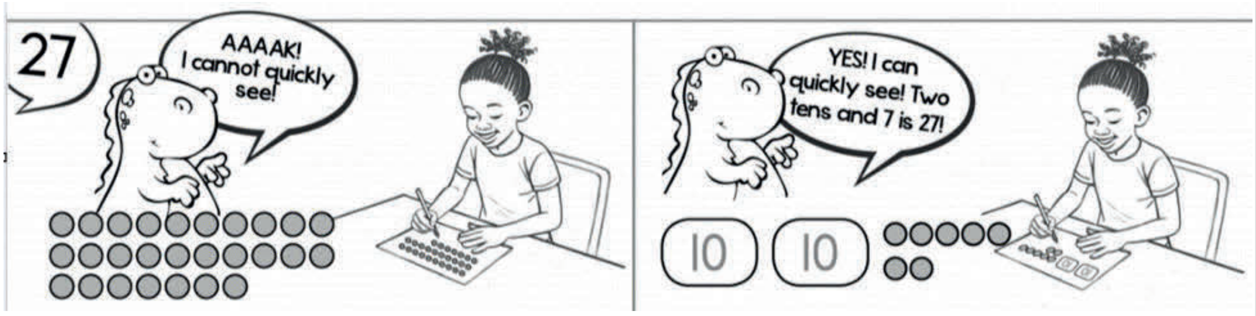


If you follow these 6 steps, for every umthamo each week, you will succeed

In this module we focus on how to teach children to make sense of numbers.
In South Africa, many children imagine numbers as a looooong string of 1s.
They calculate in 1s, like this:



But this is very inefficient.
We **DON'T want** learners to draw number pictures as a long string of 1s!
We **DO want** learners to use number symbols and structure into 100s, 10s and 1s.



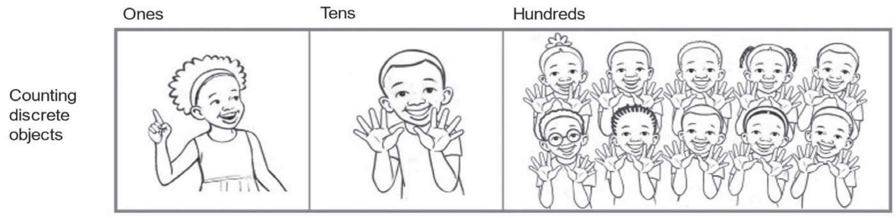
As teachers, we help learners structure their number pictures into 100s, 10s and 1s.
As a teacher you should <u>**ALWAYS**</u> **structure** number pictures into 100s, 10s and 1s.
We **NEVER** want to see teachers working like this:

Building mental maps or pictures

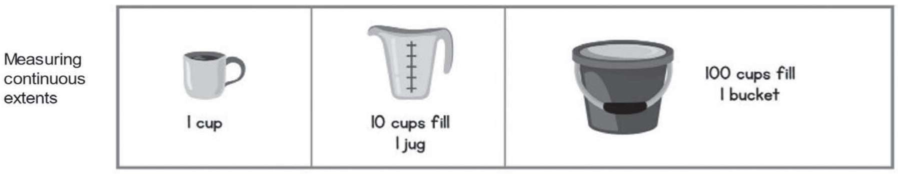
Many of us have learnt maths by memorising. Many of us don't have a mental map for maths which goes beyond counting in ones. **Our first job in this course to see maths as meaningful.** For maths to make sense we each need a coherent mental map or picture. **Our second job in this course is to help children build meaningful mental maps for learning mathematics.**

There are two main routes to developing children's number sense:

- **A counting pathway** where the starting point is discrete quantities (discrete objects or discrete events); and

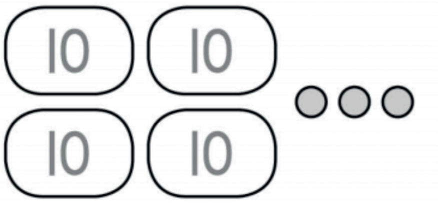


- **A measurement pathway** where the starting point is continuous quantities that can be measured using a particular attribute such as length or volume.



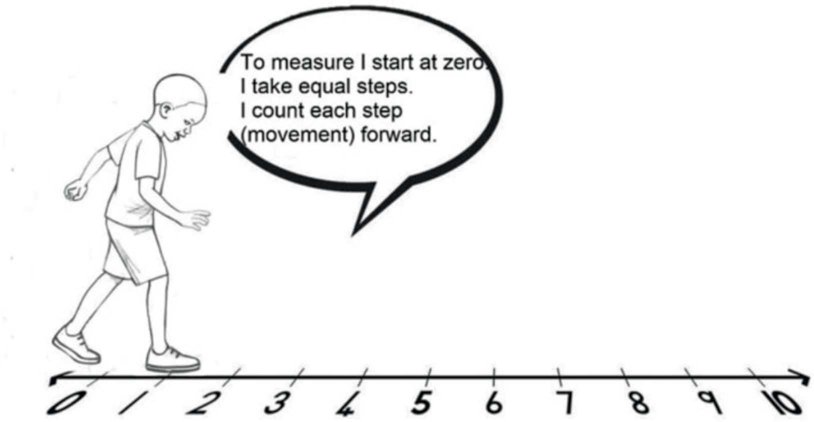
In this module we consider both pathways, and an underlying approach of structuring numbers to identify, describe and generalise the patterns of 2s, 5s and 10s.

The counting pathway uses number pictures.



Children should structure number pictures in 10s. They write the number symbol 10.

The measurement pathway uses number lines.



Children should structure number lines in 10s. Multiples of 10 are "friendly numbers".

The two pathways are related. Children need to learn both meanings of the number: **cardinal (size of a set)** and **ordinal (the sequence of numbers)**. They need to use and move between different representations of number.

Let's talk maths

What does the English word 'cardinal' mean? How is 'cardinal' different from 'ordinal' and 'nominal'?

- **Cardinal** is used to indicate **how many**. One, two, three.
- **Ordinal** numbers are words used to **arrange** numbers in a sequence – to rank or order. First, second, third.
- **Nominal** numbers are used to **identify** something, like an ID number, or telephone number.

SOME IDEAS FOR YOUR CLASSROOM

The CAPS and the learner workbooks have activities which use **discrete objects**, and activities which use continuous measurements. Look through a workbook, or a grade in the CAPS and find an example that uses the counting pathway and an example that uses the measurement pathway.

Discrete objects and discrete events
Examples of discrete objects are: people, fingers, apples, counters.
Examples of discrete events are: steps, claps, breaths, chimes of a bell, a clock ticking.
In this activity for learners, grade 1 children count the discrete objects in the picture.

Activity for learners

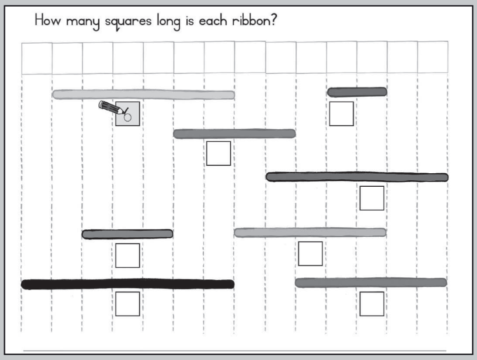
Source: DBE Rainbow book

Notice that the children "count the number of each different type of animal and draw the same number of dots". Each dot represents a discrete object.

What else could children count in this picture? What questions could you ask your class about this picture?

Continuous quantities which can be measured
Examples of continuous quantities or extents are: the area of field, the length of a stick, the volume of water, the height of a wall, the mass of a bag of sand

Activity for learners



In this learner activity the children compare the lengths of each ribbon by measuring them using squares as a unit of measurement.

What questions could you ask your class about this picture?

TASK 1: Conceptual understanding

Mathematical disposition

Many of us have maths trauma or wounded maths stories.
With support from lecturers and peers we can change our maths stories.

Task 1.1. Drawing "mathematics" and creating a pattern

1. Imagine that could hold "mathematics" in your hand. Or imagine that you could meet "mathematics" walking down the street. Reflecting your experience so far, draw a picture of this object or character called "mathematics". Show what mathematics looks like to you.

Now, discuss: How does mathematics make you feel?
Now, take a photo of Task 1.1 Question 1. Upload the photo.

Task 1.2: Reading an article

When you study to be a teacher, you also learn to be a reader and a researcher. Good teachers read. They take and use ideas from others. They share their ideas with their colleagues. They reflect on their teaching. So, this course includes some readings. Read each article carefully. Read it more than once. Notice the authors and the date. Make notes. Highlight key ideas. Look up words that you don't understand. Discuss it with friends.

Mathematical disposition
Good mathematics teachers help their children see mathematics as sensible, useful and worthwhile. Good teachers believe that they can always learn more.

Read Mason and Johnston-Wilder (2004) on *Mathematical Powers.* (on p.13&14)

1. Who wrote this article? ______________________and_________________________.

2. When did they publish this article? ____.

SCAN ME

Mason & Johnston-Wilder (2003)

We use mathematical power across all maths topics, and at all levels (from ECD to university).

3. Give an example from your own experience of when you have made a **conjecture and convinced** someone (or yourself).

4. Describe a situation where you have used your powers of **organising and classifying.** It can be from any context (in life, in school, or at home, …)

5. Have you ever played **charades, Pictonary or 30 seconds?**

Choose one simple concept or word. It can be any concept (apple, car, triangle, smile, walk, anger). First **imagine** that concept (think about it yourself – what does the concept refer to, or look like?). Now - without writing down your word - **express** your concept by drawing or writing.

6. Generalising: "All white men can't jump". Specialising: "Elon Musk can't jump"
Write down your own example of 2 sentences which show the power of **generalising and specialising.**

PrimTEd MAT3

To reason mathematically we: specialise - generalise - conjecture - justify/refute - prove

Reading 1: Mason & Wilder-Johston (2004) Johnston-Wilder

John Mason has identified a set of 8 "Mathematical Powers" that all children possess and which we need to foster and develop in order to create "able mathematicians" who are able to reason about maths and problem solve. The powers, which come in pairs, are as follows:

Conjecture

Children should be encouraged to **make conjectures,** that is say what they think about what they notice or why something happens. E.g., "I think that when you multiply an odd number by an even number you are always going to end up with an even number."

and

Convince

Children should then be encouraged to **convince**, that is to persuade people (a partner, group, class, you, an adult at home etc) that their conjectures are true. In the process of convincing, children may use some, or all, of their other maths powers.

Organise

Children should be encouraged to **organise,** putting things (numbers, facts, patterns, shapes) into groups, in an order or a pattern, e.g., sorting numbers or shapes

and

Classify

Children should then be encouraged to **classify** the objects they have organised e.g., identifying the groups odd and even numbers, irregular and regular shapes, etc.

Imagine

Children should be encouraged to **imagine,** objects, patterns, numbers and resources to help them solve problems and reason about mathematics.

and

Express

Children should be encouraged to **express their thinking,** that is show and explain their thinking and reasoning e.g., about a problem, relationship or generalisations.

Specialise

Children should be encouraged to **specialise,** that is to look at specific examples or a small set examples of something. For example, looking at the odd number 7 and the even number 8 to test their conjecture that an odd X even number = odd number. Children can also specialise in order to start to see patterns

and

Generalise

Children should be encouraged to **generalise,** that is to make connections and use these to form rules and patterns. For example, from their specific example they could generalise that any odd number multiplied by an even number gives an even number. Children should also be encouraged to use algebra to

Mason, J and Johnston-Wilder, S (2004) Learners Powers in: *Fundamental Constructs in Education,* London:Routledge Falmer pp115-142

Key Pedagogies

Prompting childrens's thinking through questions:
What do you notice?
What is the same and what is different?

Providing opportunities for children to:
Engage in talk (listen, analyse, discuss)
Manipulate, experience, visualise

Developing children's thinking through:
Investigation
Scaffolding

Enabling learning through:
Drawing attention to....
Developing reasoning and making connections

Developing Reasoning

Reasoning and conceptual understanding
Encouraging children to reason in maths helps to support children to develop a conceptual and relational understanding of maths: an understanding of why maths "works", rather than just following a set of instructions. This leads to a far greater understanding and confidence in maths.

Key Strategies

Always, sometimes, never
Give the children a statement and ask whether it is always, sometimes or never true.

Is it always sometimes or never true that the product of two even numbers is even?

Another, another and another
Give the children a statement and ask them to give you examples that meet the statement, and then ask for another example, and another.........

Can you give me a way of partitioning the number 3104? Another, another, another.....

Convince me
Make a statement to the children and ask them to decide whether it is accurate or not, then explain their reasoning to convince you.

Convince me.....that multiplication is commutative.

Hard and Easy
Ask the children to give you an example of a 'hard' and 'easy' answer to a question, explaining why one is 'hard' and the other 'easy'

Give me a hard and easy example of a 4 digit subtraction number sentence.

If this is the answer, what's the question?
Give children an answer and ask them to come up with as many questions as possible that could have that answer.

The answer is 36, what could the possible questions be?

Task 1.3: Show and explain a calculation

1. You are a maths teacher in Grade 3. Show children how to calculate 92 - 58 = ..., without using a calculator

2. Then write a paragraph explaining to your class they way that you solved the problem.

3. 

How many squares and rectangles?

Working out: Find each unit. Then count how many repeats.

Small squares: _____

Big squares: _____

Rectangles: _____

Total: _____

PrimTEd GPM4

Teach a balanced curriculum
Communicate concepts and procedures through various modes of representation.

Task 1.4: Two minute tango

1	200 - 1 = ______	11	______ = 256 - 100
2	______= 20 - 1	12	100 more than 23 is ______
3	1 more than 300 is ______	13	If I decrease 134 by 100, I get ______
4	If I decrease 56 by 1, I get ______	14	207 ; 197 ; 187 ; ______
5	127 ; 126 ; 125 ; ______	15	4 plus 100 = ______
6	10 more than 50 is ______	16	If I increase 146 by 1, I get ______
7	______ = 105 - 10	17	9 + 6 + 1 = 10 + ____
8	If I decrease 50 by 1, I get ______	18	99 + 26 + 1 = 100 + ____
9	If I increase 25 by 10, I get ______	19	89 subtract 1 = ______
10	4 ; 14 ; 24 ; ______	20	55 - 10 = ______

Mathematical disposition

Many of us have maths trauma or wounded maths stories.
With support from lecturers and peers we can change our maths stories.

Representations

Number pictures and number lines for the
- small (1s)
- medium (10s)
- large (100s)

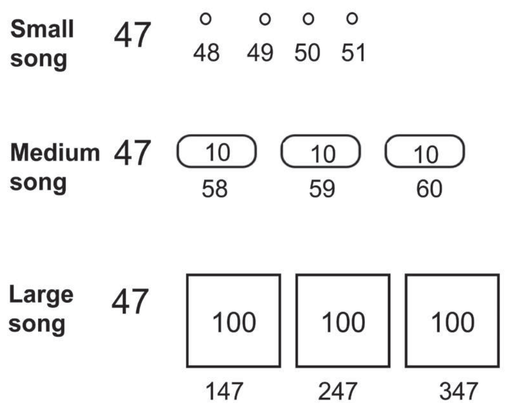
number songs

Children should learn the **small, medium and large counting songs.**

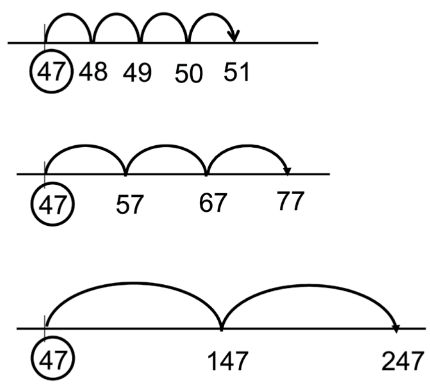
- Counting forwards in 1s is the same as adding 1 each time
- Counting forwards in 10s is the same as adding 10 each time
- Counting forwards in 100s is the same as adding 100 each time

They should have a mental image of the number songs as a number picture and as a number line.

The counting pathway (number pictures)



The measurement pathway (number lines)



SUMMARY

Summarise in your own words 4 main ideas in this introductory unit.

What are the WEEKLY activities you must complete to succeed in this course?

UNIT 2: The counting pathway

The counting pathway

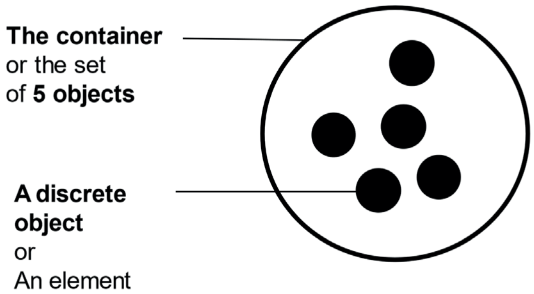
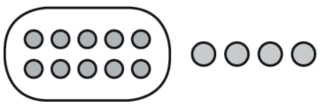
A counting pathway is a route for learning about numbers, where the starting point is **discrete** quantities (discrete objects or discrete events).

Key Ideas

The key ideas in Unit 2 are:

3. Following the **counting pathway** to learn about numbers, children count discrete objects or discrete events.
4. **Children learn to 'count with meaning'** by moving along a counting trajectory.
5. **Children first learn to count in 1s**. But to understand number names beyond ten, they need to count and structure in 1s, 10s and 100s. They also count and structure in 2s and 5s.

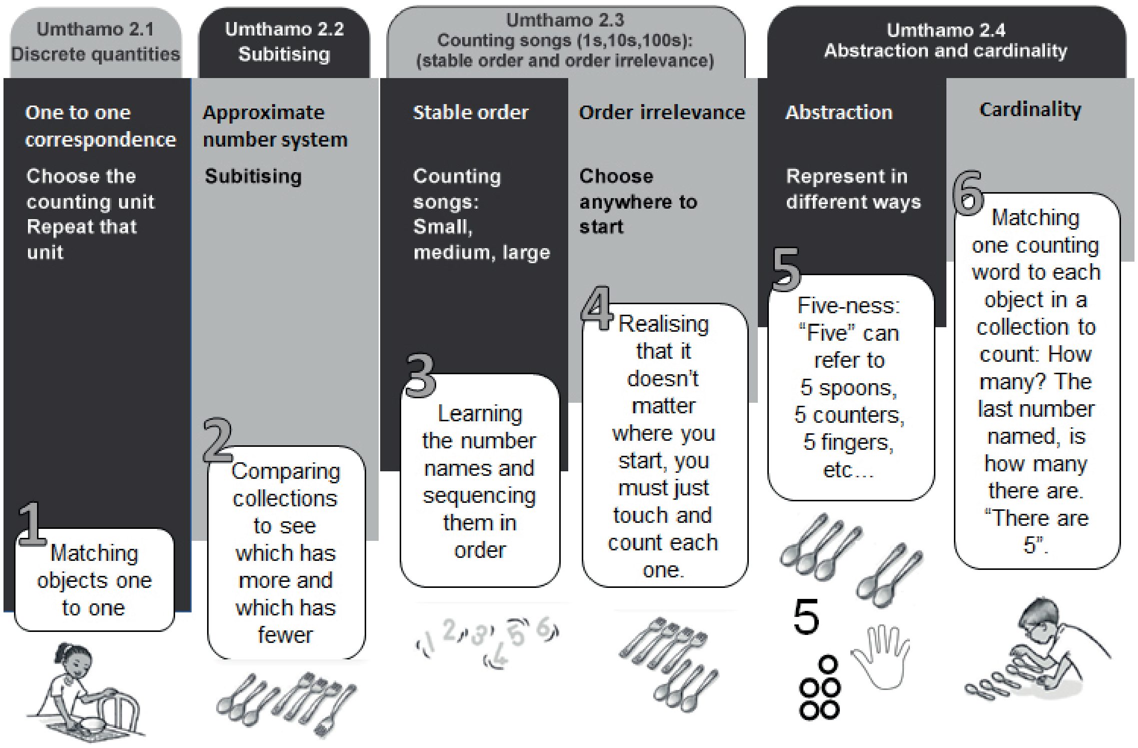
These are the main features of the counting pathway.

Quantities	Discrete quantities that can be counted
Pathway into number sense	Counting pathway
Unit	Counting unit (often in 1s, 2s, 5s or 10s)
Metaphor	A container 
Representations	Number pictures 
Number meaning	**Cardinality** Number is the size of a set/collection of discrete quantities.

Children learn to ‘count with meaning’ by moving along a counting trajectory

Children learn to count, moving along a **counting learning trajectory:**

Stage 1: Choose what to count, and **match** objects one to one
Stage 2: **Compare** collections (by subitising) to identify which has more or fewer
Stage 3: Learn the **number song** (say the small number sequence to count in ones)
Stage 4: Realise that **it does not matter where you start**, you must touch and count each one. So they notice where they start and end to count a set of objects (so they don’t count the same object twice) (order irrelevance)
Stage 5: **Abstract** the number-ness of a number. Know that five can refer to five objects or five pictures or five events or 5 steps etc.
Stage 6: Know how many there are. Match one counting word to each object in the collection and realise that the last number named is how many there are in the set (**cardinality**).



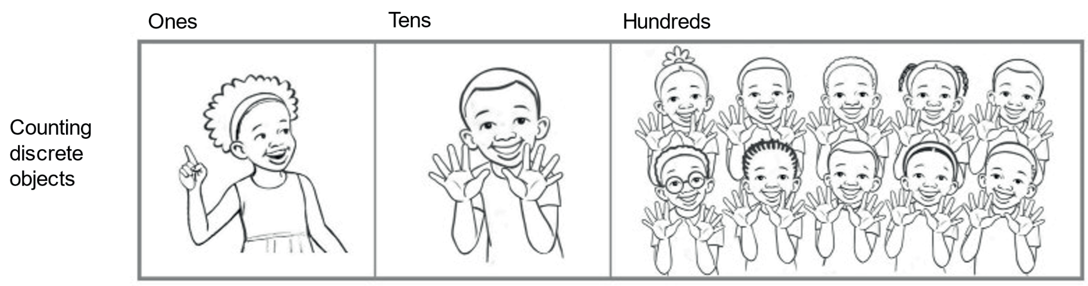
Source: Adapted from Kuhne, et al. (2017) R-Maths Concept Guide, p. 62

Children first learn to count in 1s. But very soon they need to structure and count in 2s, 5s, 10s and 100s.

Children first learn to count with meaning for a counting unit of 'one'. They count forwards and backwards in ones and realise that they last number they say is the quantity. But to understand our base ten number system (and so count beyond 9), they need to be comfortable using different counting units (not just ones). They learn three counting songs:

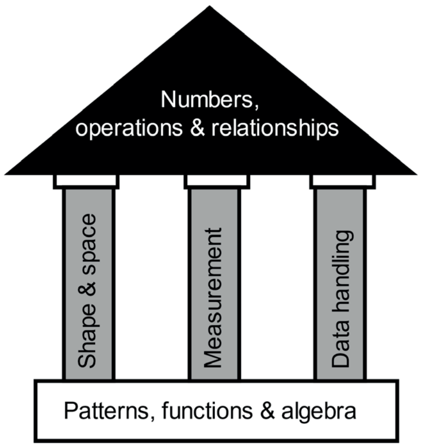
- The **small counting song** with a counting unit of 'one': 1 ; 2 ; 3 ; 4 ; ...
- The **medium counting song** with a counting unit of 'ten': 1 ten, 2 tens, 3 tens, 4 tens, ...
- The **large counting song** with a counting unit of 'hundred': 1 hundred, 2 hundred, 3 hundred, ...

Diagram: Small, medium and large counting songs for discrete objects



Children follow the same 6 stages in the counting learning trajectory for the different counting units.

Diagram: How the content areas fit together



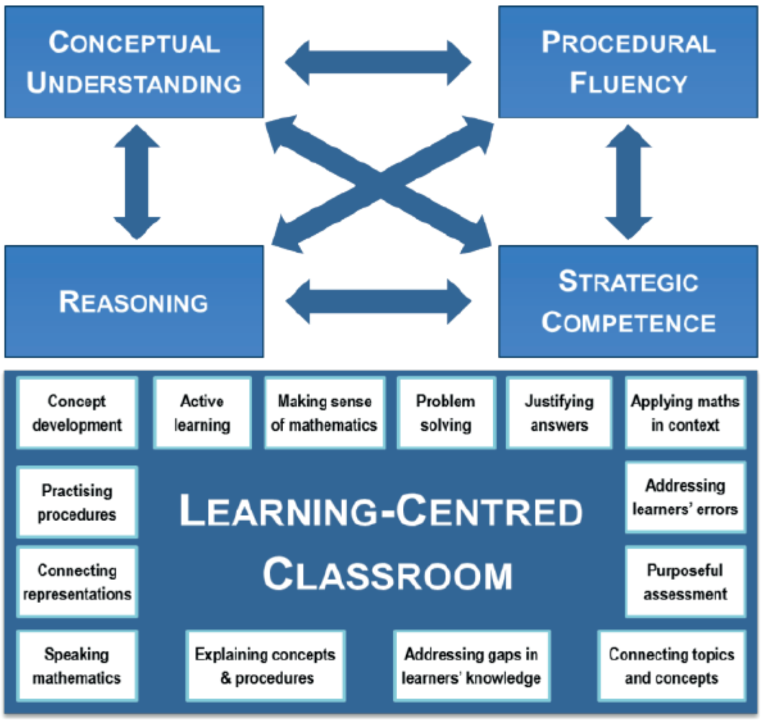
When discussing the counting pathway, we always also use an underlying early algebra approach. This is supports **'patterns, functions and algebra'** in CAPS, which is a foundation for learning about numbers, operations in relationships in 3 difference contexts.

We structure numbers to **identify, describe and generalise** the patterns, functions and algebra for groups of 2s, 5s, 10s and 100s.

We do this by using the **8 mathematical powers** to conjecture and convince; organise and classify; imagine and express; specialise and generalise.

In South Africa we have a curriculum that specifies which mathematics content is covered in each grade. In primary school mathematics there are **five content areas**. The main content area is **number, operations** and **relationships.** There are three contexts for learning about number operations and relationships: **shape** and **space, measurement,** and **data handling**. The underlying foundation is the **patterns, functions** and **algebra**.

In South Africa, we also have a framework – called **Teaching Mathematics for Understanding** (TMU). This framework describes the mathematics processes.



Source: Department of Basic Education (2018) Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics For Understanding. The ideas draw on Kilpatrick, Swafford & Findell (2001)

- **Conceptual understanding** enables learners to see mathematics as a connected web of concepts. They should be able to explain the relationships between different concepts and make links between concepts and related procedures. Conceptual knowledge enables learners to apply ideas and justify their thinking.
- **Procedural fluencies** are the processes through which mathematics is done. Learners need to perform mathematical procedures accurately and efficiently. They also need to know when to use a particular procedure.
- **Strategic competence** Learners should be able to identify and use appropriate strategies and to devise their own strategies to solve mathematical problems.
- **Reasoning** includes justifying and explaining one's mathematical ideas, and communicating them using mathematical language and symbols. Mathematical reasoning includes deductive and inductive reasoning processes.
- **Learning-centred classrooms** focus on learning – where the teacher designs learning experiences to help learners learn mathematics, using whatever teaching and learning strategies s/he thinks are most suitable for the specific lesson that will be taught.

UMTHAMO 2.1

Discrete quantities

Introduction

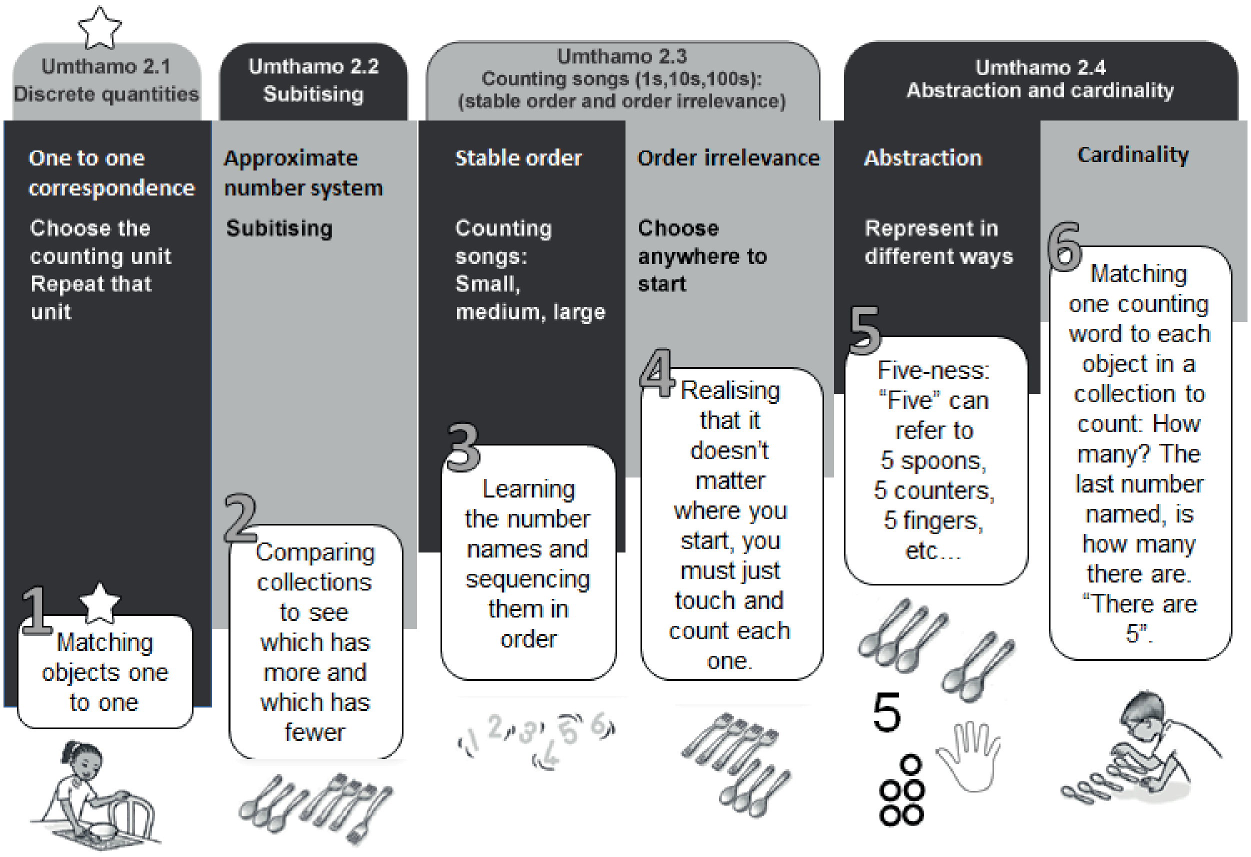
Key Ideas

The key ideas in umthamo 2.1 are:

1. **Stage 1 of the counting trajectory is choosing a counting unit.** What are we counting?
2. **In Stage 1 the chosen counting unit is repeated, and discrete objects are matched one-to-one** (a finger is matched to an object, forks are matched to knives, a counting word is matched to 1 discrete object
3. **A mental picture for a number of discrete objects is a container** (tomatoes on plates)
    - We can express and imagine whole numbers as discrete objects and events, using **many different representations**
    - We use whole numbers in many **different contexts:** measurement, shape & space, and data handling

6. **Maths does not make sense if your only mental picture is counting and structuring in ones.** To understand the base-ten number system of place value, you need to:
    - **Count and structure in 1s, 10s and 100s.**
    - You use the **small, medium and large counting songs**. Count in 1s, 10s and 100s from any number
    - **Add and subtract 1s, 10s and 100s** from any number

The counting trajectory



Stage 1a: Choose what to count (the counting unit)

When children count how many discrete objects they are in a set, they must first decide what should be in the set. They use their mathematical powers to **organise and classify.** They choose which objects to count. They choose an object, and count how many times that object is repeated.

Activity for learners

Number talk

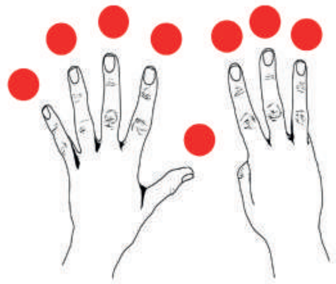
There are many **"things" or units** to count. Some children may count the **number of plant tubs**. Some children may count the **number of plants** or the **number of leaves.** There is no right answer. What you count, depends on what you decide is the unit.

In every collection, there is **a unit** which we count to see how many **of "somethings"** is in that collection or set or group. In any collection you should define the unit: What are you counting or structuring?

Stage 1b: Repeat the unit and match discrete objects one-to-one

Once children have decided what they are focusing on (the counting unit), then they can match discrete objects one-to-one. Even before they can say the number song for counting, children can match objects one-to-one.

Diagram: Matching one finger to one dot



The child can put one finger on each sticker. A child could put one sticker on each finger. To do this, they don't need to know that there are 'eight' stickers, or 'eight' fingers. They can just match the discrete objects one-to-one. Children can also match discrete events one-to-one.

Diagram: Matching on action/event to one sound

Jump/skip each time I clap.

Clap	Skip
Clap	Skip
Clap	Skip
Clap	Skip
Clap	Skip

The child can jump each time they hear a clap. The don't need to be able to count. They match discrete events one-to-one.

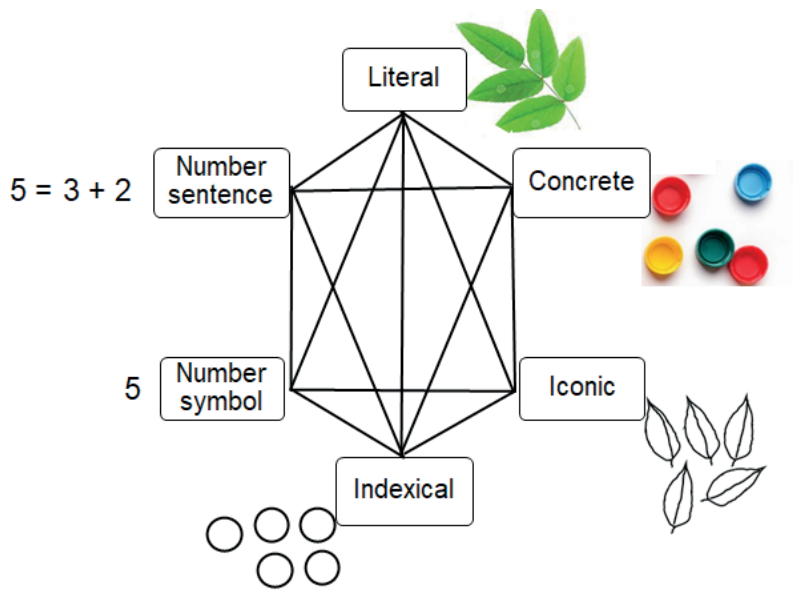
Children use one-to-one correspondence, even before they can talk.

A mental plcture for discrete objects:

We **imagine and express** discrete objects in many different ways.

Children move between **representations** of numbers: literal, concrete, iconic, indexical, and number symbols.

Diagram: Types of representations



Note:
As this is a book/ presented online there are no "literal" objects. We use a photograph to depict the "literal" objects.

Concrete	**Literal objects** are real physical objects in their surroundings: sticks, stones, forks, bowls, leaves, hats, shoes **Concrete objects** are physical objects which represent the literal objects such as bottle tops, or unifix cubes. They are also called manipulatives. They are concrete as they can be touched, joined and arranged.
Pictorial	**Iconic pictures:** Children also use drawings or picture of collections objects. They can draw a lock or key or hat, or a few stones or a pair of shoes. **Indexical picture:** Overtime, they start to draw with less detail. Instead of drawing 4 leaves, they just draw 4 circles to show how many there are.
Abstract	**Number symbols:** Only in Grade R and 1 do they start to recognise and learn to write number symbols. **Number sentences (symbolic syntactical):** Later they start to write number sentences (using symbols like + and – and = in Grade 1, and only using × and ÷ by Grade 3). A number sentence describes the relationship between collections of discrete objects.

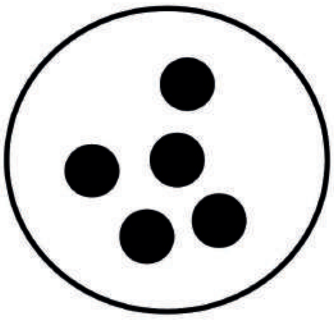
Researchers used to think that children **must always start** with the concrete, move to the pictural and then move to the abstract (Concrete→Pictorial→Abstract). We now know this is not true. Children need to work in both directions: Concrete→Pictorial→Abstract and Abstract→Pictorial→Concrete. Children need to have a **strong mental picture** of what the number symbols mean and of how different numbers fit together.

A key representation for the counting pathway are **sets of discrete objects** (like apples or marbles, or people) which can be counted in ones. The discrete objects are contained or encircled into a **set**. We use a **'container' metaphor** for the number of discrete objects contained in a set (Lakoff & Núnes 2001).

Diagram: A number picture for discrete objects

One way that we think about a number is using the idea of **a container.**

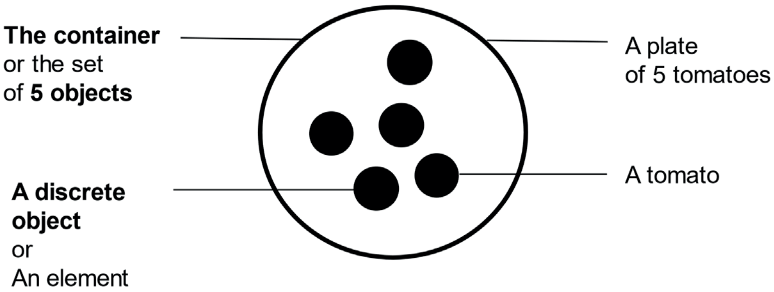
5 is a collection of 5 discrete objects,



The discrete objects are the elements of the set. The elements are the units we use to count how many there are.

Diagram: A number picture – the container metaphor

The number 5 is a collection of 5 discrete objects

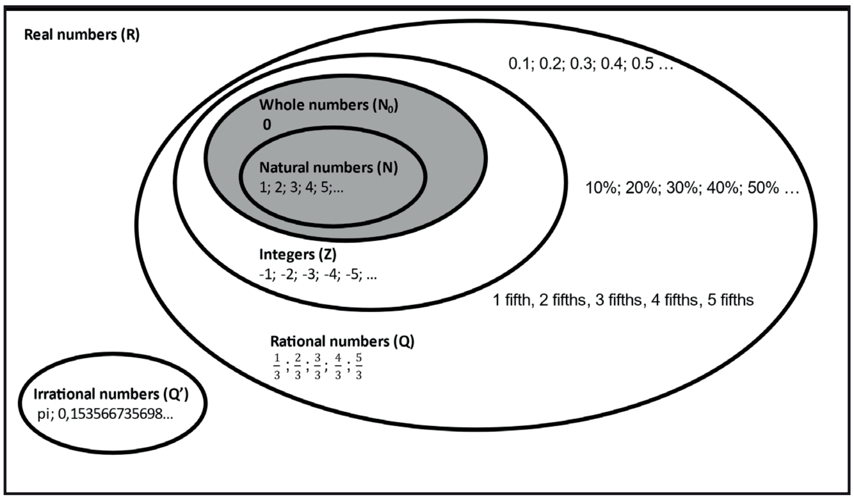


Thinking about the number 5, we imagine 5 objects in a set or collection. The 5 discrete objects are grouped together into a container. This is the **cardinal** use of number. There are five discrete objects in the set – so the set has a cardinality (size) of 5.

To count **discrete objects**, or to count **discrete events**, we use the set of **Natural numbers.**

As a teacher, you should know that the Natural numbers are part of a much bigger set of numbers, which children will slowly learn about. The symbol for natural numbers is N. Sometimes we use the symbol: N . N {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,...}

Diagram: Number systems



Can you see the difference between the set of Natural numbers and the set of Whole numbers?

PrimTEd NA2

Knowledge of number systems

Young children don't need to know about all the other types of numbers. As a teacher, you need to know that children learn to count first with the **natural numbers and then with whole numbers.** You should be able to draw Venn diagrams, using the container metaphor for sets.

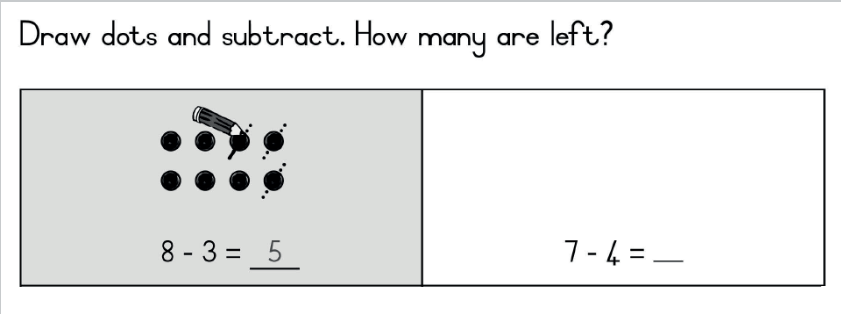
Maths does not make sense if your only mental picture is counting and structuring in ones

We know that there are many problems in the way children are taught maths in school in South Africa. We think a big reason why maths does not make sense to many children is that they don't have a mental picture of how the **powerful mathematics ideas** fit together. We think that many children have not been supported to see how the content areas in maths fit together, They have not used their 8 mathematical powers to conjecture and convince; organise and classify; imagine and express; specialise and generalise.

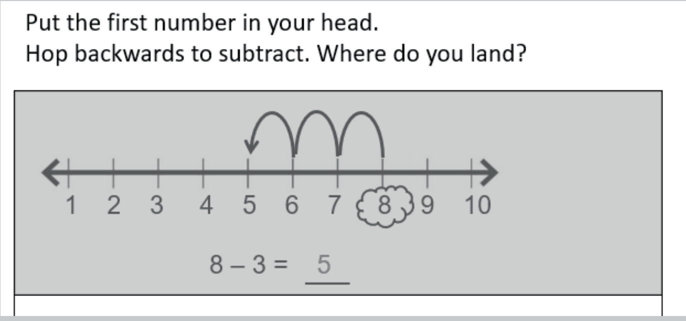
In particular we know that thinking about numbers as "a collection of ones" is a major problem. The counting unit of one is a special case. Working only in 1s means we don't generalise to use the same processes for other (bigger) counting units. Counting in 1s is a very inefficient way of working. It makes working with larger numbers very difficult.

Activity for learners

This example for subtraction uses a **number picture (discrete objects)**. To subtract, the learner draws the number of dots for the first number. Then they cross out the number that is being taken away or removed. They find the answer by counting the remaining dots.



Activity for learners

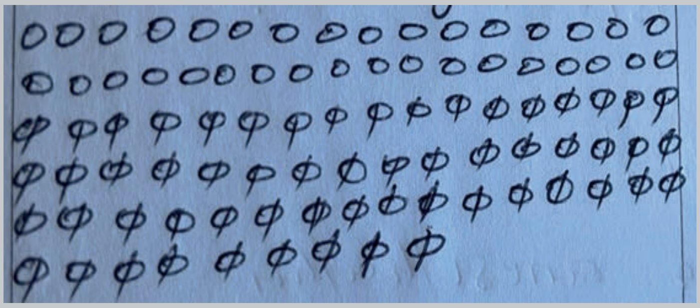


This example for subtraction uses a **number line (continuous length)**. To subtract, the learner puts the first number in their head. They draw around the first number. Then they hop backwards (to the left) the number that is being taken away or removed. They find the answer where they land.

Drawing discrete objects (a number picture) works well as an introduction to the operations. Using a structured number line in ones also works well in the beginning, when children are first learning the operations. BUT as the numbers get bigger, this becomes very inefficient. We see many South African children using 'counting in ones' strategies where they draw discrete objects for multi-digit mathematical calculations.

Examples

This is an example of how a teacher calculated 92 – 58 = ...



Notice that the teacher uses a very **inefficient method** of drawing 92 discrete objects. They cross out 58 discrete objects. They do not structure their drawing into 5s or 10s. It is very hard to quickly see how many dots are drawn and it is hard to keep track.

As teachers we therefore need to help South African children to **move away from counting in ones, unstructured drawings of discrete objects and inefficient calculation methods**.
It is therefore important that we don't ONLY count and structure in 1s, but ALSO:

- count and structure in 1s, 10s and 100s
- count and structure in 2s, and 5s,

SOME IDEAS FOR YOUR CLASSROOM

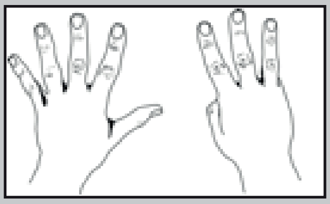
Counting and structuring in 1s

Maths teachers help children to **organise and classify** discrete objects and events. We help children to **imagine and express** numbers. You can ask children to "show me eight". They can show you eight objects, or eight events. They can draw eight using **different representations** – literal, iconic, indexical, symbolic, symbolic syntactical.

As adults we may think about calculations just using number symbols. But children need to know the meaning of the symbols. They work with literal objects, and concrete materials. They draw iconic and indexical representations of numbers.

Examples

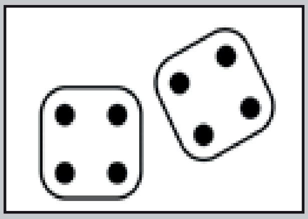
A collection of 8 fingers



The unit we are counting is fingers.
We are not counting hands.
We are not counting knuckles.

There are 8 fingers.
They are structured as 5 fingers on one hand, and 3 fingers on the other hand. Together the hands show 8 fingers.
5 + 3 = 8

A collection of 8 dots



The unit we are counting is dots.
We are not counting the di.

There are 8 dots.
They are structured as 4 dots on each dice.
4 + 4 = 8.

Moving between different types of representations

Teachers should give young children concrete materials to work with. But they should also give them number symbols and ask then to fetch that number of objects or draw a picture the number.

Concrete→Pictorial→Abstract	Abstract→Concrete→Pictorial
A teacher could ask children: **1.** Here are some counters (**concrete**) **2.** Draw one dot for each counter (**pictorial**) **3.** Write the number symbol to show how many counters you have (**abstract**)	A teacher could ask children: **1.** This is the symbol for eight: 8 (**abstract**) **2.** Show me 8 counters (**concrete**) **3.** Draw eight circles to make a picture of the 8 counters (**pictorial**)

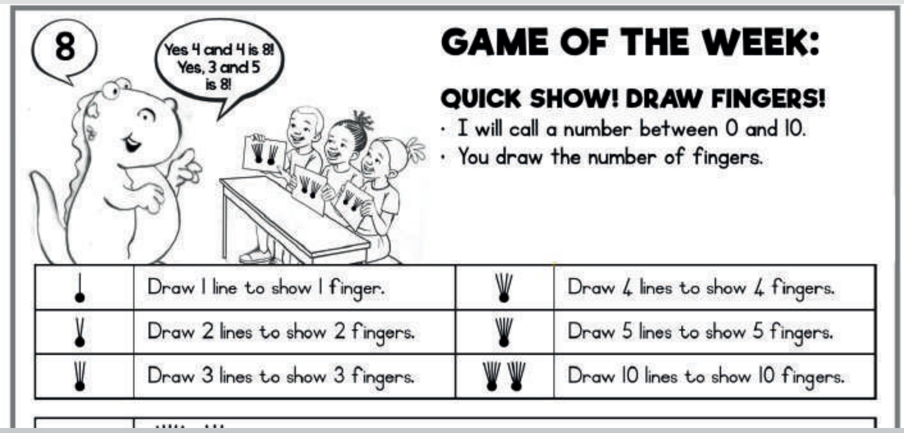
Counting and structuring in 2s, 5s and 10s

Because we have 5 fingers on each hand we structure our number system using 5s and 10s (we have a base-ten place value system). **Learning mathematics is embodied**. Our mathematics and our number systems would be different if our bodies were different. Imagine if we had paws each with 4 claws. Do you think we would still use a base 10 number system?

In the structural approach teachers should emphasise relationships and structure:

- In 2s (odds, evens, doubling and halving)
- In 10s (bonds of 10, counting in 10s from any number and using 10 fingers on 2 hands).
- In 5s (bonds of 5, counting in 5s from any number and using 5 fingers on a hand)

Activity for learners



GAME OF THE WEEK:

QUICK SHOW! DRAW FINGERS!

- I will call a number between 0 and 10.
- You draw the number of fingers.

	Draw 1 line to show 1 finger.		Draw 4 lines to show 4 fingers.
	Draw 2 lines to show 2 fingers.		Draw 5 lines to show 5 fingers.
	Draw 3 lines to show 3 fingers.		Draw 10 lines to show 10 fingers.

PrimTEd NA1

Knowledge and use of emergent number awareness of learners

Examples

Number talk – How many?
Choose what you will count: Children, eyes, hands, fingers, ...girls/boys, etc?
How many are there? How did you work that out?

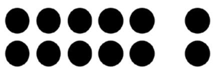
If you count how many eyes, ears or hands, you **structure in 2s**. Let's count the eyes. Each child has 2 eyes. Say a "counting in 2s" number song (2; 4 ; 6 ; 8 ; etc)

- Match each word in the counting in 2s number song to one child (one to one correspondence)
- Notice where you start and end (so don't count the same child twice)
- Know that the last number they say in the counting song is how many eyes are in the set (cardinality).

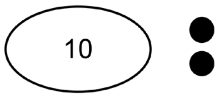
If you count how many fingers, by focusing on each hand, you **structure in 5s.** Each hand has 5 fingers. Each child has 10 fingers. Say a "counting in 5s" number song (5 ; 10; 15; 20 ; ...)

The same process is used if you count how many fingers, by focusing on each child, you **structure in 10s**. Each child has 10 fingers. Say a "counting in 10s" number song (10 ; 20; 30; 40 ; ...). This is the medium number song.

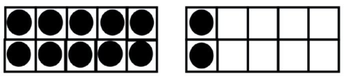
Diagram: Structured diagrams in 2s, 5s and 10s



Discrete objects:
Structured in 2s and 5s



Number picture:
Structured in 10s and 1s



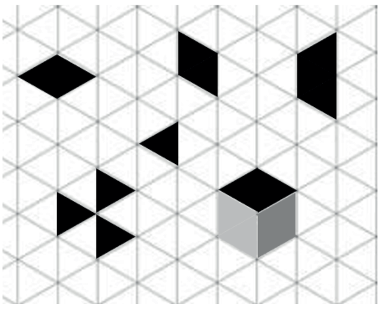
Discrete objects:
Structured in 10-frames

Number line:
Structured in 10s and 5s

Tasks: Conceptual understanding

Task 2.1.1: Writing and drawing whole numbers

To quantify “things”, we first need to choose a counting unit. We first decide, what are we counting. Then we repeat and count that unit.



On the left, there are many shapes that we could choose to count.

1. Choose ONE shape that you can see in the pattern. Your shape is your **unit**. It could be a triangle, a quadrilateral, and cube, a hexagon etc. Which **unit** will you choose to **repeat**? Copy your shape onto the pattern paper.

2. Now **repeat** your shape to make a pattern. You can use colours.

Mathematical acting and thinking

Express and imagine whole numbers as discrete objects and events, using many different representations

Mathematical concept

One pathway that children take to learn about number is by counting discrete objects. A metaphor for discrete objects is a container (tomatoes on plates)

Every pattern has a unit of repetition. First choose a unit. Then repeat that unit.

Every pattern has a unit of repetition.

Choose a shape to be your **unit of repetition**. Then **repeat** your unit **to make a pattern**.

Mathematical acting the thinking

Express and imagine 10 as involving numbers, operations and relationships in different contexts: measurement, shape & space, and data handling.

2. Write (and draw) 10 in as many different ways as you can.

Remember there are many ways to represent the number 10. Show as many ways as you can.

3. Choose one of these labels: literal, concrete, iconic, indexical, number symbol, number sentence

a.

8

____

b.

____

c. Drawing of leaves

____

d. An actual leaf

____

4. Complete this table giving examples of discrete objects. Discuss with a partner

Discrete objects or substances **You can count each one.**	**Not discrete (continuous) objects or substances** **You can't count each one.**
Leaves	Water

Mathematical acting and thinking

First choose a unit. Then repeat that unit to count how many there are. Classify and sort shapes by size and orientation Work systematically to exhaust all possible options

Task 2.1.2 Investigation - How many rectangles in a window?

How many rectangles?

This window is made of 12 panes of glass. There are many rectangles.

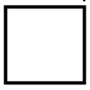
When counting the rectangles you first define which size of rectangle is your counting unit. You find one. Then you repeat that unit to see how many there.

What is a rectangle?

A rectangle is a quadrilateral with 4 right angles.
Remember that a square is a special kind of rectangle.

1. **How many rectangles with only 1 pane, are in the window?**

The squares with only 1 pane of glass, are special rectangles.

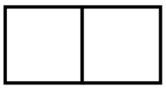


Find one square, with only 1-pane in the window. Now repeat it. Find more.

a. How many 1-pane squares are in the window? ___________.

2. **How many rectangles that have 2 panes, are in the window?**

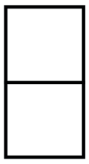
A rectangle with 2 panes can either be horizontal or vertical.
This is a horizontal rectangle, with 2 panes.



Find one horizontal rectangle with 2 panes in the window. Now repeat it. Find more.

a. How many horizontal rectangles with 2 panes, are in the window? ___________.

This is a vertical rectangle, with 2 panes.



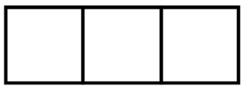
Find one vertical rectangle with 2 panes in the window. Now repeat it. Find more.

b. How many vertical rectangles with 2 panes, are in the window? ___________.

c. So, how many rectangles (vertical and horizontal) with 2 panes, are in the window? ___________

3. **How many rectangles that have 3 panes, are in the window?**

A rectangle with 3 panes can either be horizontal or vertical.
This is a horizontal rectangle, with 3 panes.



PrimTEd MAT 1

Playful engagement to search for and develop mathematical insight

Mathematical concept

A rectangle is a quadrilateral with 4 right angles
A square is a spacial type of a rectangle

Find one horizontal rectangle with 3 panes, in the window. Now repeat it. Find more.

a. How many horizontal rectangles with 3 panes, are in the window? ___________.

b. Draw a vertical rectangle, with 3 panes.

Find one vertical rectangle with 3 panes, in the window. Now repeat it. Find more.

c. How many vertical rectangles with 3 panes, are in the window? ___________.

d. So, how many 3-panes rectangles (vertical and horizontal) are in the window? ___________

4. How many rectangles that have 4 panes, are in the window?

a. Draw and label all the options for rectangles with 4-panes.

Find one rectangle with 4 panes, in the window. Now repeat it. Find more.

b. How many horizontal rectangles with 4 panes, are in the window? ___________.

c. How many vertical rectangles with 4 panes, are in the window? ___________.

d. How many squares with 4 panes, are in the window? ___________.

e. So, how many rectangles with 4 panes, are in the window? ___________

5. How many rectangles that have 5 panes, are in the window?

6. Now what about rectangles that have 6 panes, 7 panes, 8 panes...? Investigate and record your working.

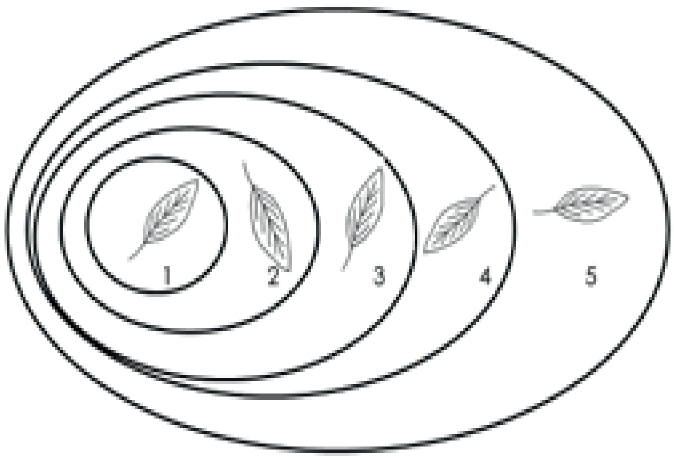
7. How many rectangles, in total, are in the window?

In total there are ________ rectangles in the window.

Task 2.1.3 Number pictures – A container metaphor for discrete objects

Sets of leaves

In the counting song for ones (small number song), each number is within the next number. They are a **nested set.**



Mathematical acting and thinking

Express and imagine discrete objects and events in many different ways
A metaphor for discrete objects is a container (tomatoes on plates)
Smaller numbers are contained within bigger numbers – they form nested sets

1. Continue the nesting pattern:

1 is inside the set of 2
2 is inside the set of 3
_______ is inside the set of ______
_______ is inside the set of ______
_______ is inside the set of _____

The big idea here is that 1; 2; 3; and 4 are all within the set of 5.

2. Choose a discrete object to draw (eg. an apple, a pencil, a hand, a brick...). Draw 12 of these discrete objects. Draw a circle around the ten objects to show the set of 10.

3. Now draw the 12 discrete objects using an **index/symbol** (e.g. a circle or a triangle, or line) for each object. Draw a circle around the ten object to show the set of 10.

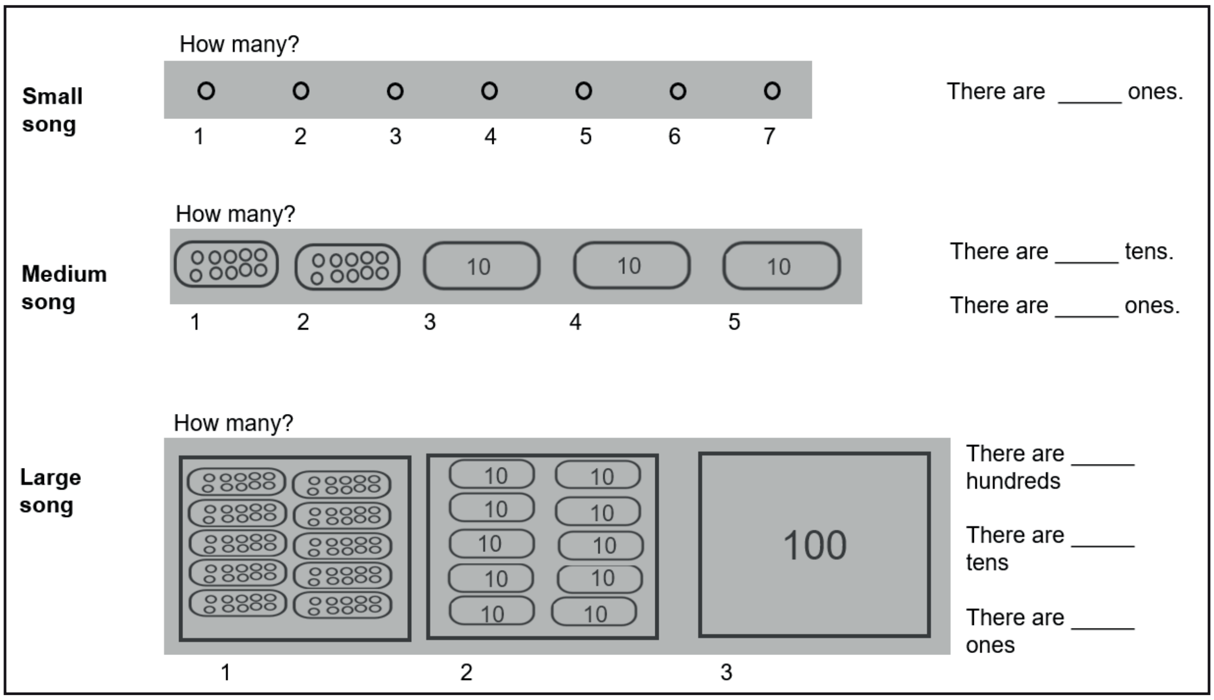
Structuring representations

Number pictures: A container metaphor for a collection of discrete objects

PrimTEd N1

Emergent number awareness of learners

4.



As teachers we ALWAYS draw by structuring in 100s, 10s and 1s
Children will start drawing with 1s.
But, as teachers, we help children to structure in 100s, 10s and 1s

Fluencies

Bonds of 5
Bonds of 10

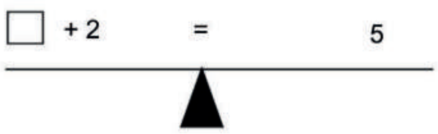
Task 2.1.4 Two minute tango

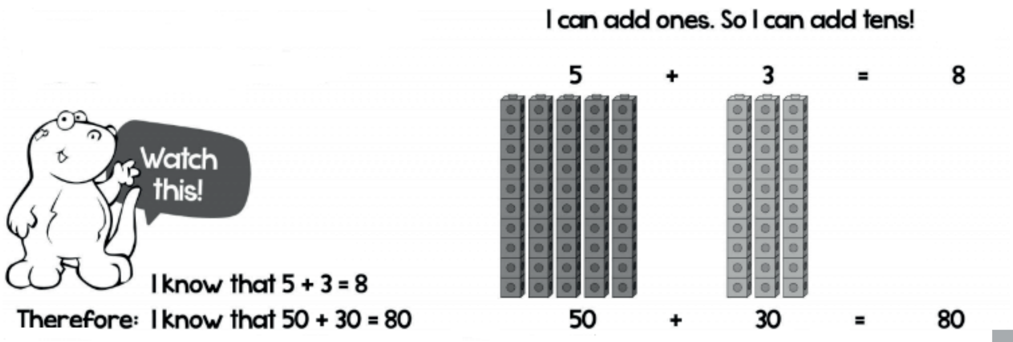
1	2 + _______ = 5	11	5 - 5 = _______
2	20 + _______ = 50	12	75 - 5 = _______
3	200 + _______ = 500	13	375 - 5 = _______
4	_______ + 1 = 5	14	_______ + 7 = 10
5	_______ + 10 = 50	15	70 + _______ = 100
6	_______ + 100 = 500	16	_______ + 700 = 1000
7	4 + _______ = 25	17	4 + _______ = 10
8	_______ + 4 = 25	18	4 + _______ = 20
9	250 - _______ = 40	19	14 + _______ = 20
10	250 - 40 = _______	20	_______ + 4 = 20

Equals means **is the same as**.
Equals DOES NOT mean “makes” or “find the answer”.
My number plus 2 **is the same as** 5. What’s my number? or
Something add 2 **is the same as** 5. What’s my number?

Representations

Equals means “**is the same as**”.





Number facts connect in **1s, 10s and 100s.**

- If I know that **5** + **3** = **8**, then I know that **5**00 + **3**00 = **8**00
- If I know that **5** + **3** = **8**, then I know that **5** 000 + **3** 000 = **8** 000

Make connections

If you know one number fact, then use the **small, medium & large numer songs** to work out new facts

SUMMARY

Name and draw the different **types of representations** children use.
Have you completed the **5 steps for success**?
Draw a **number picture** (tomatoes on plates) for **discrete objects**.
Is your drawing **indexical or iconic**?
What **concepts** to do you think you will be tested on in Quiz 2.1?
What **mental fluencies** will you be tested in in Quiz 2.1?

…….

PrimTEd MAT4

Reflect for action
Reflect on the process to make decisions for further action.

UMTHAMO 2.2

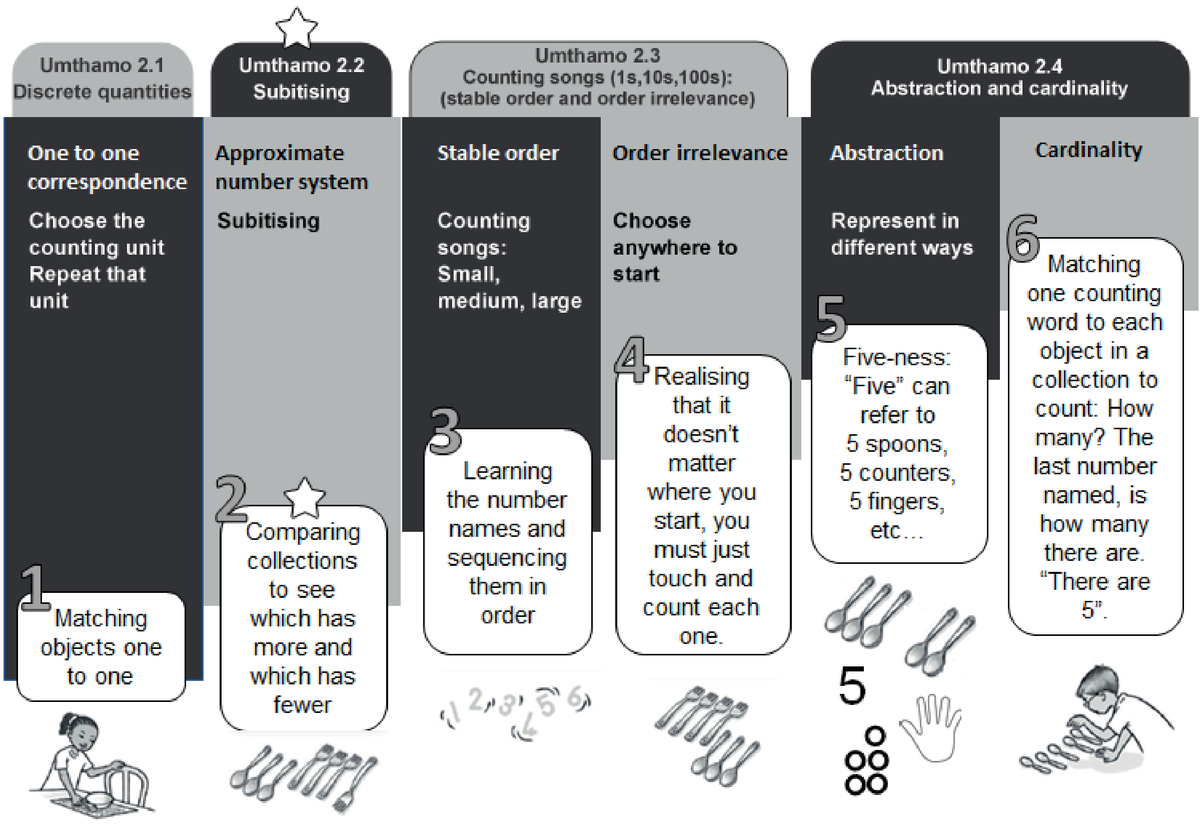
Subitising

Introduction

Key Ideas

The key ideas in umthamo 2.2 are:

1. **There are 6 principles which children must apply to count with meaning:** one-to-one correspondence, comparison, stable order, order irrelevance, abstraction and cardinality.
2. **Stage 2 of the counting trajectory uses the approximate number system to compare** which has more or fewer.
3. **In Stage 2 children subitise** (quickly see without counting)
   - First they use **perceptual subitising** (for small sets less than 5 or 6)
   - Then they use **conceptual subitising** (bigger sets are split up)
   - Next they use **conceptual subitising with place value and skip counting** (using the small, medium and large counting songs)
   - Finally they use **conceptual subitising with place value and multiplication.**
4. Maths does not make sense if a child's only mental picture is counting and structuring in ones. Children can:
   - Use **number pictures** that are structured into 1s, 10s and 100s.
   - Structure number in **2s and in 5s**
   - Be fluent with **bonds of 10, and counting in 5s.**



With plenty of hands-on activities and guidance from the teacher, children begin to understand and apply the following counting principles:

One-to-one correspondence principle: Matching discrete objects and discrete events one-to-one. Children match discrete objects by touching and arranging them. Children match discrete events with an action. They can track events on their fingers or by moving stones or by drawing a mark.

Compare using the approximate number system: Children use the Approximate Number System (ANS) to compare small sets of objects and identify which set has more and which has fewer. They use perceptual subitising to compare two small sets.

Stable order principle: Children know the small number song. Number names are always arranged in the same fixed order, e.g. one is followed by two, two is followed by three, three is followed by four, and so on.

Order-irrelevance principle: The order of counting the objects in a collection does not matter. Children need to understand that however we arrange the objects, the total number of objects in the collection remains the same.

Abstraction principle: Children understand that even if groups with the same number of objects look very different (e.g. five grapes, five people, five houses) they have the same numerosity, i.e. ‘fiveness’. They realise that counting can be applied to objects, pictures, colours, shapes, or even actions or sounds.

Source: Kuhne, et al. (2017) R-Maths Concept Guide

Cardinality principle: The last number word said when counting a collection represents the total number in the collection. When a child is able to answer: ‘How many are there?’, they have reached an important counting milestone. They can now count with meaning. They realise that the last number that they say in the number song name, is also how many there are in the collection. They have found the cardinality of the set.

In this umthamo, we focus on comparing discrete quantities using the ANS.
We learn about subitising

Humans are born with an OBJECT TRACKING SYSTEM (OTS). This allows a baby to track the movement of up to 4 objects (or people).

Humans can reason about quantities BEFORE THEY CAN TALK

Humans are born with an APPROXIMATE NUMBER SYSTEM (ANS). This allows a baby notice which has more or less, or when an object is missing (up to 5 or 6)

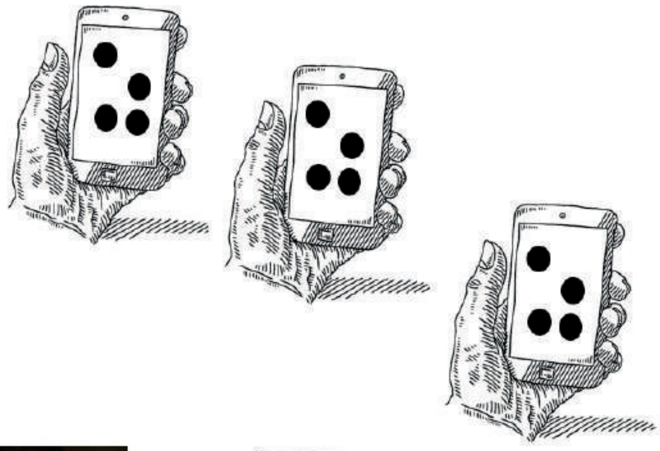
The approximate number system

At a very early stage children use their **approximate number system**. Babies as young as three months notice when a discrete object is missing.

If you show a baby a screen with 4 dots,

and then
show them the same screen with 4 dots,

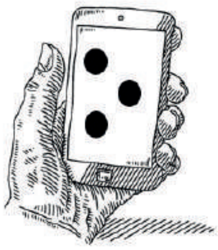
and then
show them the same screen with 4 dots,
they will get bored.



But then if you show them a screen with 3 dots, the baby will look puzzled,

They will look longer and wait to see what you do next.

They look for the missing dot.



Children can compare two small collections and identify which collection has more. They can show you which set has more and which set has less.

Activity for learners

Black dots

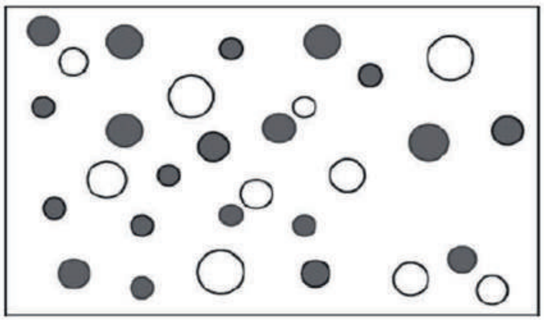
Grey dots

Are there more black dots or grey dots?

If this activity is too difficult try the same task with a small sets of objects. The bigger the difference in size between the two groups, the easier it is to compare.

You use your approximate number system to compare sets.

Are there more white dots or grey dots?



Early on (before and during Grade R) children use perceptual subitising. Soon there after (Grade R and Grade 1) they use conceptual subitising (in 1s). As they move beyond a group of ten, they use conceptual subitising to link this with place value. First they use skip counting (of the small and medium counting songs), later (Grade 2 and 3) they link place value to multiplication.

SOME IDEAS FOR YOUR CLASSROOM

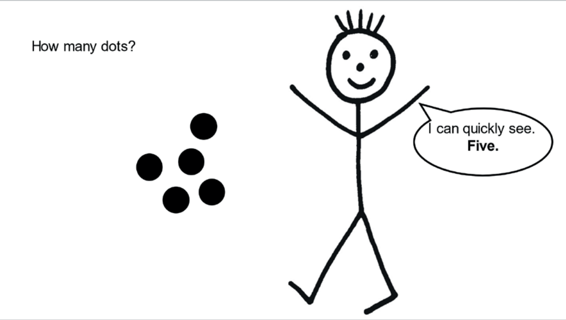
Teachers can design tasks that are the right level for their children. Here are examples of activities which move from perceptual subitising (Grade R and before) to conceptual subitising with place value and multiplication (Grade 3 and after).

Perceptual subitising

Perceptual subitising is for small sets of objects (up to 5 or 6). It is instantly recognizing and naming quantities without counting.

Activity for learners

How many dots?



This child can subitise up to 5 or 6. They use perceptual subitising.

Children use their approximate number system to compare two groups. They can see who has more and who has less (without counting).

Children subitise to play **1,2,3 Show**! or **1,2,3 Veza!**

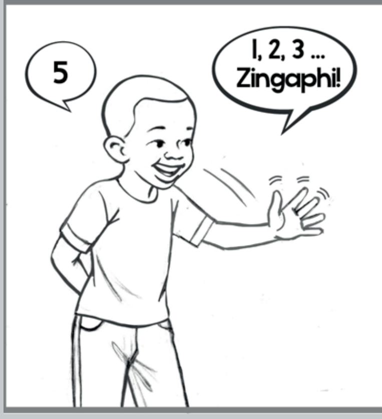
1. Start with children using only one hand. They use **conceptual subitising** to play
   - Who has more?
   - Who has less?
   - How many? Zingaphi?
   - How many more? How many less?
2. Later, children can use **both hands**. They use **perceptual subitising** to play
   - Who has more?
   - Who has less?
   - How may? Zingaphi?
   - How many more? How many less?

PrimTEd GPM3

Engage learners productively with mathematics.
Home language - develop tasks to enhance mathematical language use.

Use perceptual subitising to play 1,2,3 Show! with only one hand.

Game



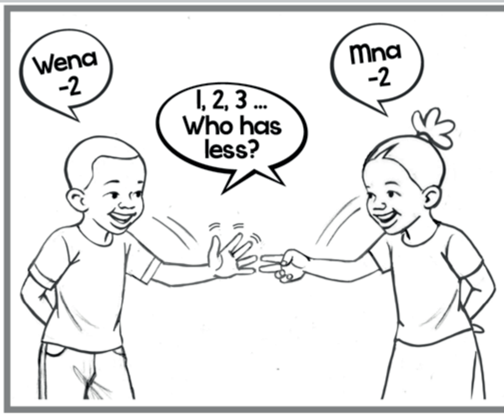
GAME OF THE WEEK:

123! How many? Zingaphi?

- Play in pairs.
- Put hands behind back.
- Say together, "1, 2, 3! Zingaphi?
- One child shows fingers on their one hand.
- The other child says how many they see, quickly.
- Do again! Faster!

To find **How many?**, children can subitise - they don't need to count.
They can use their approximate number system to find
Who has more? and **Who has less?**

Game



GAME OF THE WEEK:

123! Who has less?

- Play in pairs.
- Put hands behind back.
- Say together, "1, 2, 3! Who has less?
- Both children show fingers on both hands.
- Who has less? Ngubani onokuncinci?
- Do again! Faster!

Parents or siblings can play 1,2,3 Veza at home.
The child uses perceptual subitising to quickly say **How many?**

Game

1-2-3 VEZA: ONE HAND

- You and your child put one hand behind your back.
- Say together, "1, 2, 3, Veza!"
- Both of you show fingers on one hand.
- Your child says how many they see, as quickly as possible.

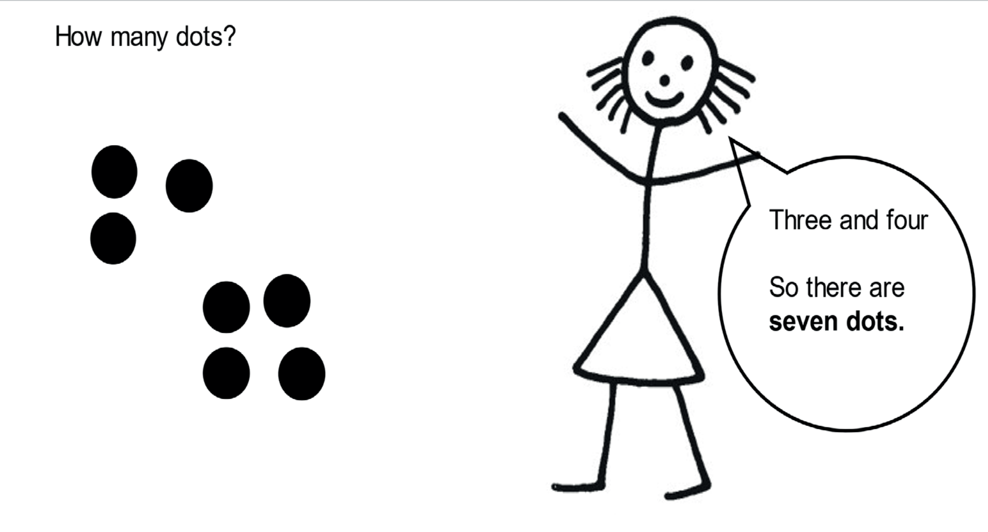
Conceptual subitising

Conceptual subitising is for bigger sets of objects (more than 7). Subsets are created and then combined.

Children **subitise** and use their fingers to **structure numbers** by breaking up bigger numbers into smaller parts. When children break up a bigger collection into smaller sub-groups, this is called **conceptual subitising.**

Activity for learners

How many dots?



This child can subitise beyond 5 or 6. They use conceptual subitising.

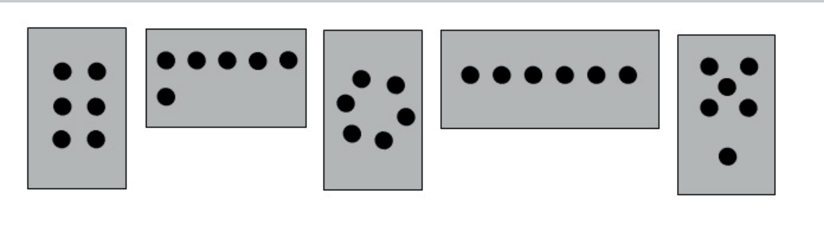
Conceptual subitising by structuring in 5s

Because we have 5 fingers on each hand we structure our number system using 5s and 10s (we have a base-ten place value system).

Activity for learners

How many?

Which one is easiest to quickly see how many, without counting?
Which one does not belong? Why?



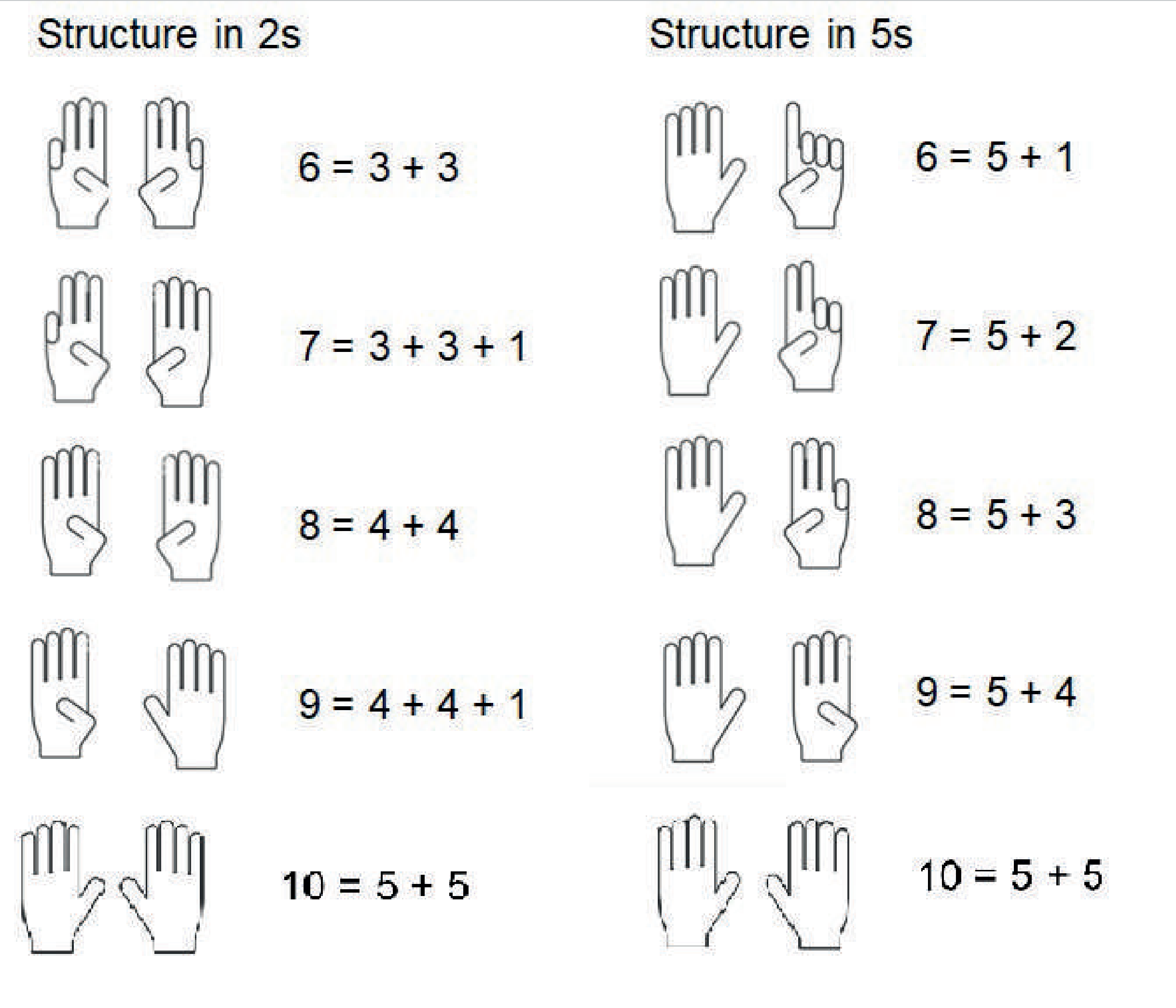
PrimTEd NA1

Recognise learners ability to subitise, approximate, order and compare numbers.

Children can subitise! They know they have 5 fingers on each hand. To show 5, they show all their fingers. There is no need to count each time. Beyond 5 they start using 2 hands.

✂--Activity for learners

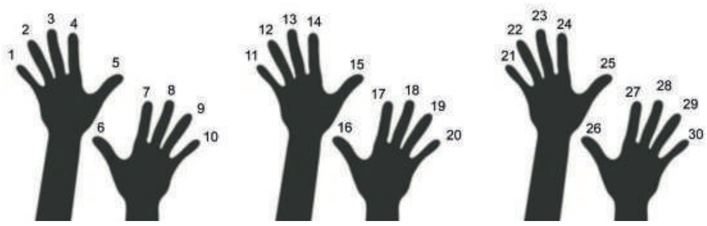
Show me a number.



To **structure in 5s**, children first use their one hand, then add on the additional fingers from their other hand. Learning to use the base-five number structure is supported by the 'five-ness' embodied in our fingers.

To **structure in 2s**, children use both hands and use symmetry. For even numbers they show the same amount of fingers on each hand. For odd numbers they show the same amount of fingers on each hand and raise one more finger

Our number system uses 10 digits:
1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.
10 means 1 group of 10 and 0 ones
11 means 1 group of 10 and
1 one



Game

123! How many? Zingaphi?

- Play in pairs.
- Put hands behind back.
- Say together, "l, 2, 3! Zingaphi?
- One child shows fingers on their hands
- The other child says how many they see, quickly.
- Do again! Faster!

Chidlren use conceptual subitising to show **How many?**, using two hands

Game

Game 1: Quick Show

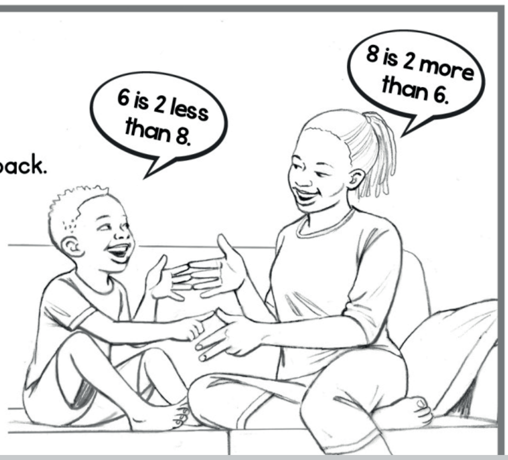
- You flash fingers on two hands.
- Your child says number as quickly as possible.
- You repeat back, "Yes, 5 fingers and 3 fingers is 8 fingers!"
- Do it again! Try to go faster!

Children structure their fingers in 2s (using doubles, 4 and 4 is 8) or they structure thier fingers in 5s (3 and 5 is 8).
Children can use two hands to find **Who has more?** and **Who has less?**
Children can match-one-to-one to compare and find **How many more?** and **How many less?**
All of these games - with two hands - **use conceptual subitising**.

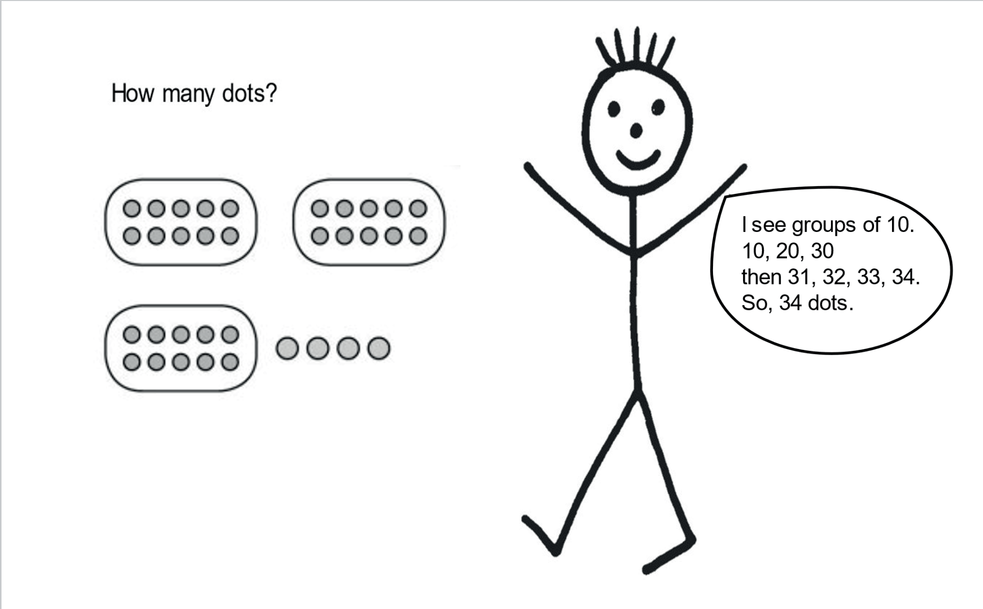
Game

1-2-3 VEZA: COMPARE!

- You and your child put both hands behind your back.
- Together with your child say, "l, 2, 3, Veza!"
- Show fingers on one or two hands. Compare.
- You say, "I have 8 fingers, you have 6. 8 is 2 more than 6!"
- Your child says, "I have 6 fingers, you have 8. 6 is 2 less than 8!"
- When your child is ready, let your child start!



✂--Activity for learners

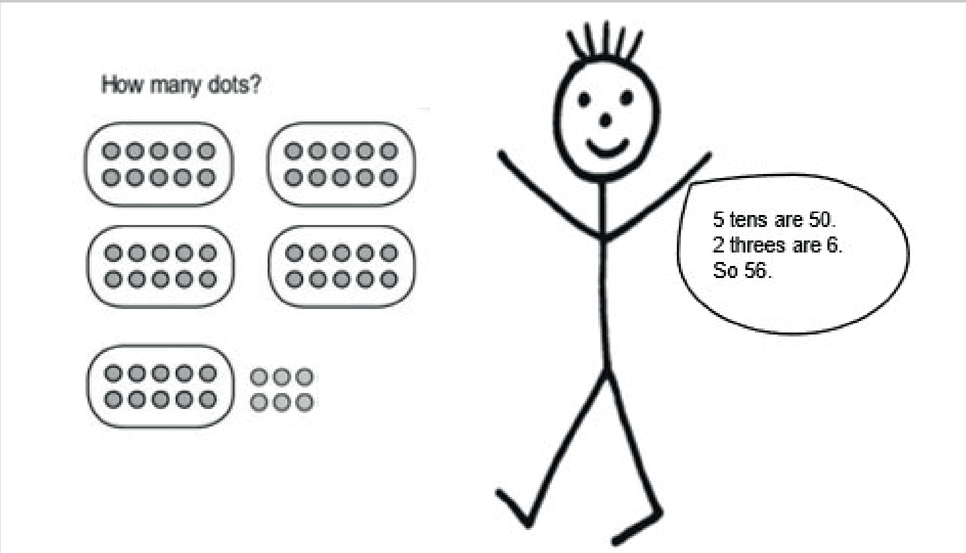


This child uses conceptual subitising (for the 3 groups of tens, and the 4 ones). They skip count in tens (the **medium number song**) and then switch to count in ones (the **small number song**).

Conceptual subitising using multiplication

This drawing is done by a child.
A teacher would show '10' and not draw 10 ones

✂--Activity for learners



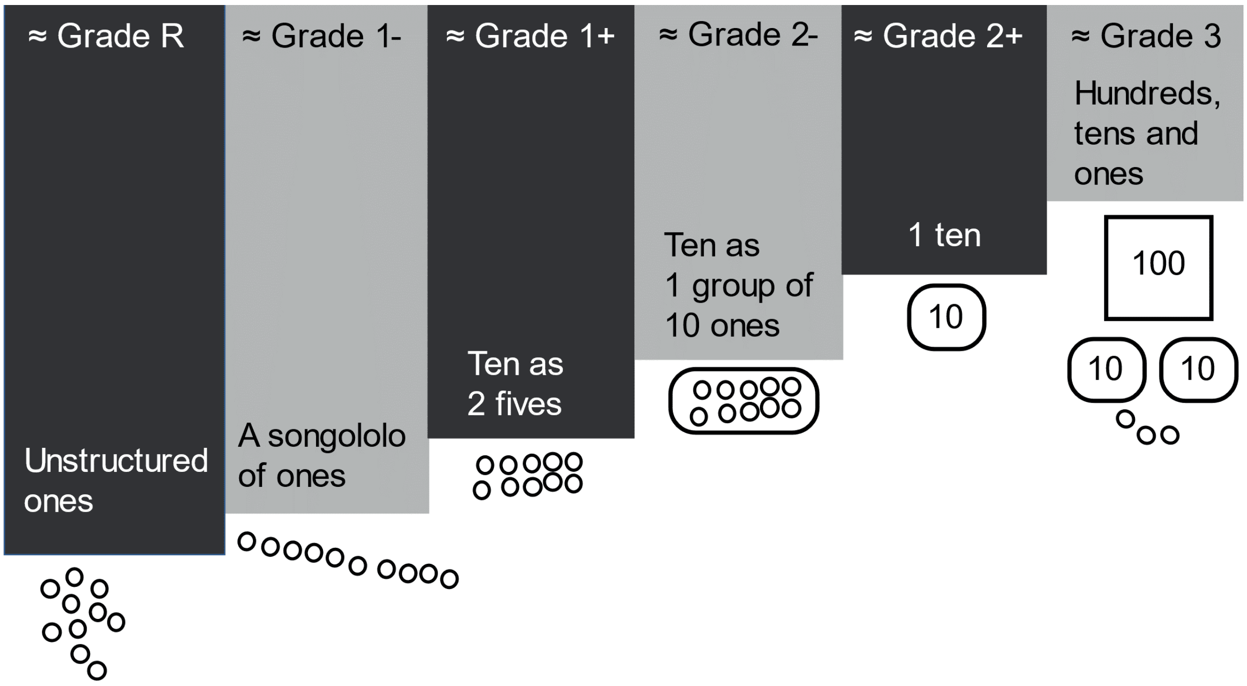
The child uses conceptual subisiting with place value and multiplication
10 repeated 5 times is 50. (10 x 5 = 50)
3 repeated 2 times is 6. (3 x 2 = 6)
50 + 6 = 56
So it it 56.

Number pictures that are structured into 1s, 10s and 100s

Children need to structure collections of discrete objects. When working with concrete materials they should arrange the objects into groups. When drawing, they slowly learn to structure bigger groups of objects using the base-ten place value system.

It is helpful to have a mental picture of the time it usually takes for children to structure numbers and work with number pictures.

Diagram: A learning trajectory for number pictures



Number pictures use the base-ten place value system.
A **one** is a small circle.
A **ten** is big circles with 10 written inside.
A **hundred** is a square with 100 written inside.
Children need to fluent with **bonds of 10**. They re-order calculations to **find the 10s**.

Activity for learners

As teachers we ALWAYS draw by structuring in **100s, 10s** and **1s.**
Children will start drawing with **1s**.
But, as teachers, we help children to structure in **100s**, **10s** and **1s.**

Find the 5. Then add. Draw a line to find each 5.

$\overbrace{1 + 3 + 4}^{5}$ = ____	4 + 2 + 1 = ____	2 + 3 + 3 = ____
2 + 3 + 4 = ____	4 + 10 + 1 = ____	1 + 2 + 4 = ____

PRACTICE

Tasks: Conceptual understanding

Emergent number sense

Subitising is quickly seeing how many there are without counting.
Perceptual subitising is for small groups of discrete objects (less than 5)
Conceptual subitising is for bigger groups of discrete objects (more than 5)
Children can subitise before they can count.

Task 2.2.1 What is subitizing?

Kitso Maragelo has studied subitising in South Africa. Her work focuses on ECD practitioners. She would like teachers to use children's subitising skills rather than always asking children to count. Children don't need to always start at one and count in ones. They can quickly see how many there are, without counting.

Reading 2

Maragelo (2021) A literature on subitising in mathematics: *A South African perspective*
in Z. Jojo & M. Du Plooy (Eds) Proceedings of the 26th *Annual National Congress of the Association for Mathematics Education of South Africa,* 14th– 16th July 2021, Muckleneuk Campus, University of South Africa: Pretoria. pp. 502 – 512. ISBN: 978-0-620-94779-4

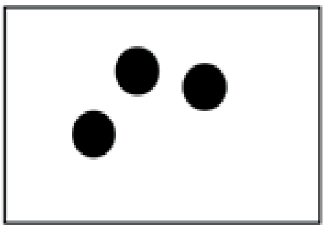
What is subitizing?

The word subitizing was first coined by researchers during the 1940s, who defined the concept as the fast and highly accurate way of quantifying collections of up to six items or less, without having to count each item (Kaufman Lord, Reese, & Volkmann, 1949; Taves, 1941). Derived from a Latin word meaning suddenly, subitizing is the direct perceptual apprehension and identification of the numerosity of a small group of items (Clements & Sarama, 2009).

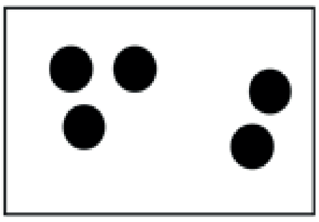
There are two prominent types of subitizing: perceptual and conceptual subitizing (Clements & Sarama, 2014).

Perceptual subitising is instantly recognizing and naming quantities without counting. Perceptual subitizing is viewed as an innate ability to discriminate quantities and emerges among young children as early as three to five months of age (Clements & Sarama, 2009; MacDonald & Wilkins, 2015). For example, when a child looks at three apples, the child will intuitively know there are three apples without counting or using any mathematical strategies (Clements 1999).

A typical activity that can be used to encourage perceptual subitizing is using a flashcard with three dots and briefly showing the kids and place it face down, and then invite the children to say how many dots they saw. The figure below is an example of a dot card for perceptual subitizing



Conceptual subitizing combines perceptually subitized quantities to determine the total sum. Conceptual subitizing is applying the perceptual subitizing processes repeatedly and quickly uniting the numbers (Trick & Pylyshyn, 1994). Children with the ability to conceptually subitize, group the number of items to count. For example, when presented with seven objects, they will see a group of five and just count the other two to get a seven (Feza, 2015).



A typical conceptual subitizing task will be flashing a dot card for few seconds and asking the children how many dots they saw and how did they see it. Figure 2 below is an example of a dot card used to develop conceptual subitizing in children.

Developing young children's conceptual subitizing helps children to begin to see part-whole strategies. Children can also see that 5 is made up of groupings of 3 and 2.

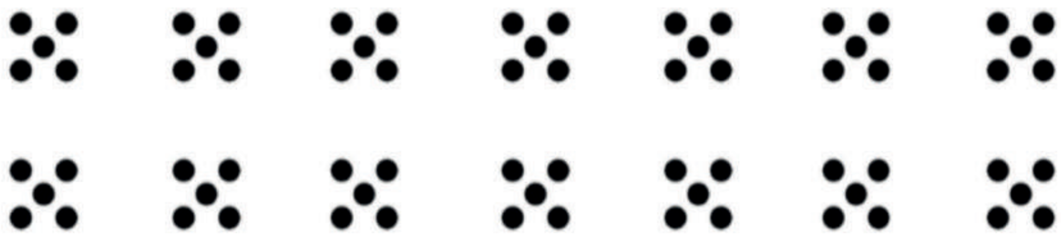
1. Who wrote this article on subisiting?
2. When was the article published?
3. What is the title of the journal article?
4. Where was the article published (in which journal or conference proceedings)?
5. Explain the concept of **perceptual subisiting**.

6. **Using your own words,** explain what **conceptual subitising** is.

   Maragelo (2019) explains that conceptual subitising is

7. Draw an example where a child can use **conceptual subitizing.**

8. **Number talk:** How many dots?



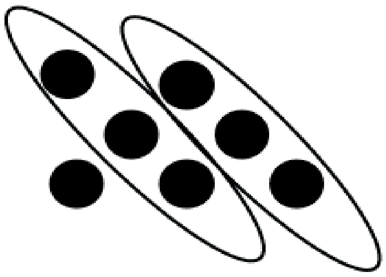
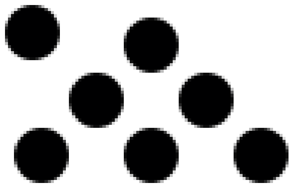
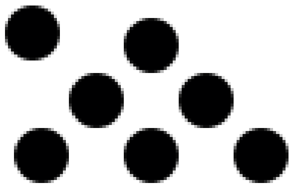
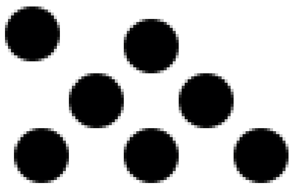
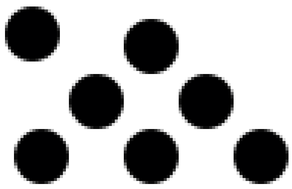
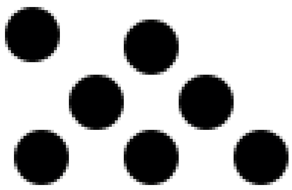
Mathematical acting and thinking

Express and imagine discrete objects and events in many different ways
A metaphor for discrete objects is a container (tomatoes on plates)

Smaller numbers are contained with bigger numbers – they form nested sets

Task 2.2.2 Designing subitising tasks for children

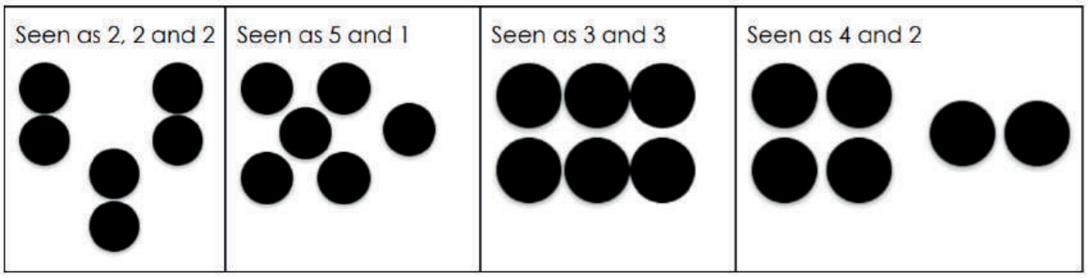
1. How many different ways can you see how many dots there are in total?

	7 = 1 + 3 + 3
	
	
	
	
	

Notice that the structure of the picture, stays the same.
Each picture has 7 dots. The structure is preserved. What changes is how the elements (or units) are arranged or grouped.

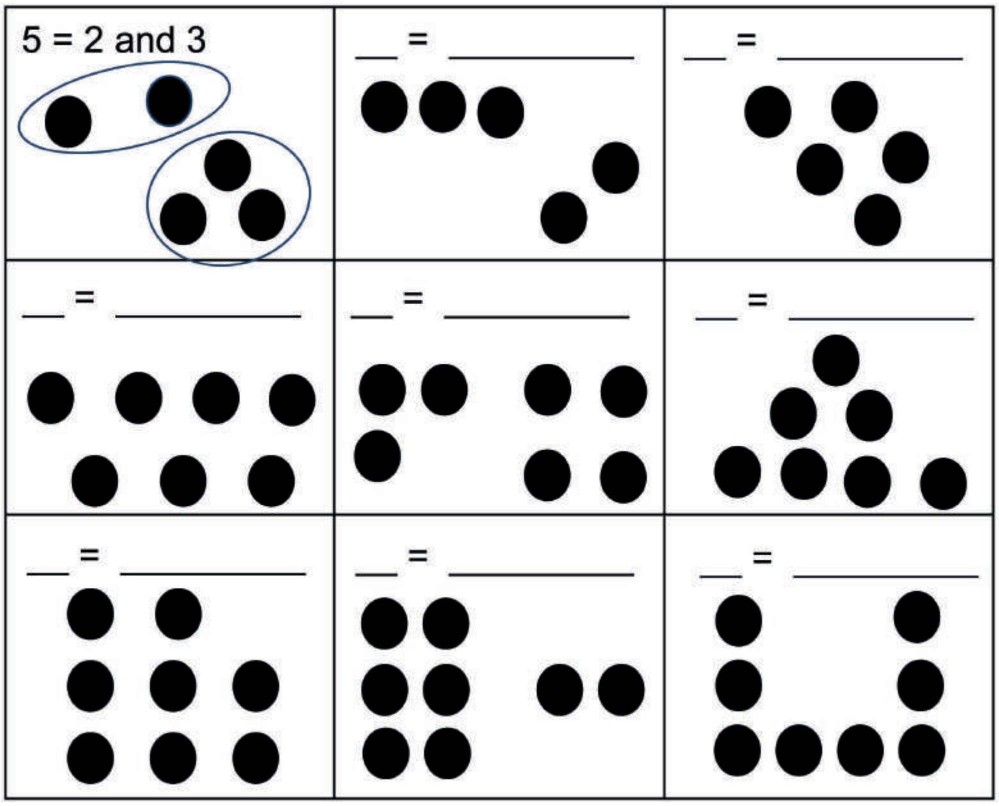
The South African Numeracy Chair project has produced a guide for grade R teachers on subisiting. Here is an extract from their guide:

> In this handbook, we refer to *regular* and *alternate* dot patterns.
>
> *Regular* dot patterns, especially from 1 to 6, are the easiest to recognise. These are those most commonly seen on dice, dominoes and playing cards.
>
> *Alternate* dot patterns work with other dot arrangements and are intended to help the learners to 'see' patterns in different configurations. This is because different arrangements of dots lead to different decompositions of that number.
>
> For example, here 6 dots can be seen in different ways:



Source: Rhodes University South African Numeracy Chair (SANC)
https://www.ru.ac.za/media/rhodesuniversity/content/sanc/documents/Early_Number_Fun_Programme_-_session_seven_handbook.pdf

2. Write how you see these dots. You can circle to show your groups.



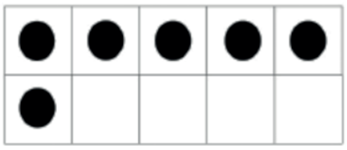
Tasks: Fluencies and structuring (in 2s, 5s and 10s)

Structuring representations

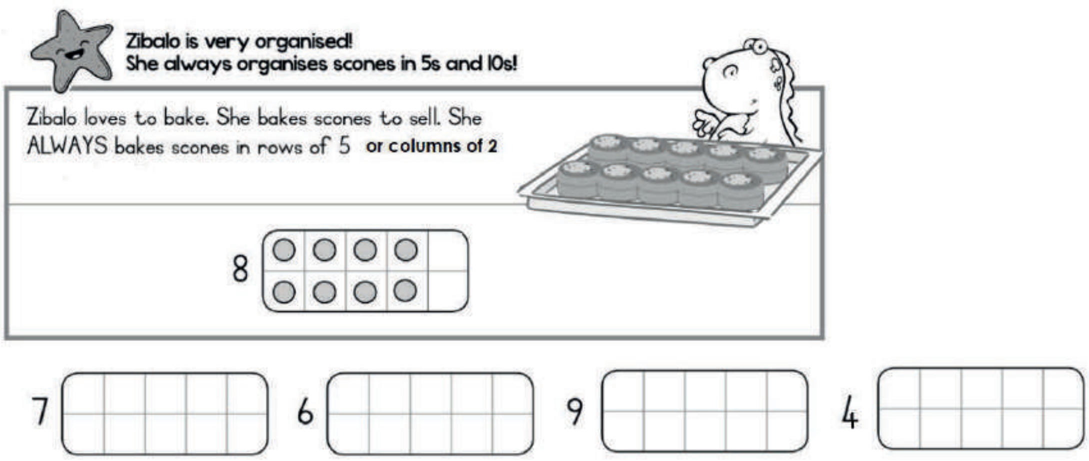
Number pictures: Using a ten-frame
Number pictures: Structuring by 2s and 5s .

Task 2.2.3 Fill up a ten frame

We can structure numbers by filling up a ten frame. This helps children to work with the base-ten place value. As children can subitise up to five, we use a ten-frame to picture numbers. Here is a ten-frame showing 6.

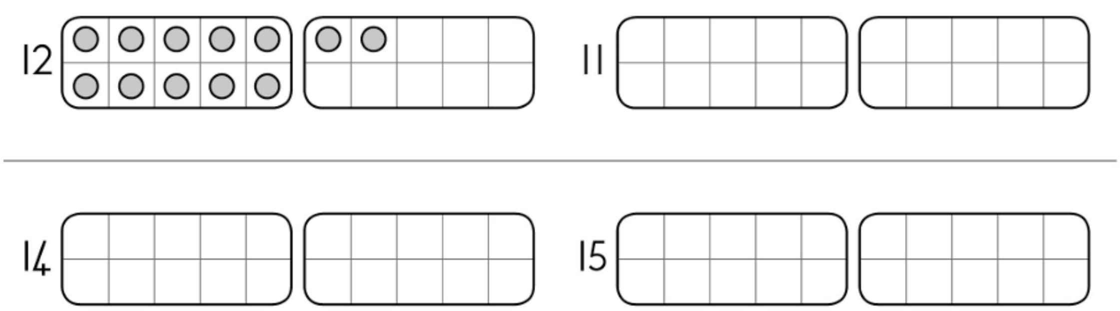
To structure in 5s, we make groups of 5. Each row has space for 5 dots.  Start on the left and fill up the top row then the bottom row.	**To structure in 2s**, we make groups of 2. Each column has space for 2 dots.  Start on the left and fill in pairs – one on the top, one of the bottom.

1. Structure in 2s



2. Structure in 5s

Help Zibalo place scones on baking trays. Organise each tray into rows of 5. Do not place more than 10 on one tray!

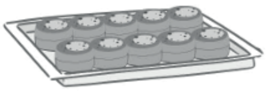


Children will start drawing with 1s.
As teachers we ALWAYS draw by structuring in 100s, 10s and 1s

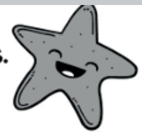
Game

1,2,3 draw! (Structure by 5s)

3. Draw using 1s, 10s and 100s

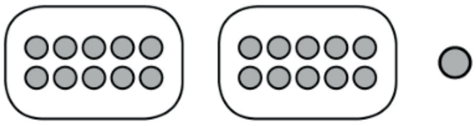


Zibalo just baked more scones.
Help her draw the numbers.



Draw the numbers.

21



8

13

53

153

653

As teachers we ALWAYS structure in **100s, 10s** and **1s**

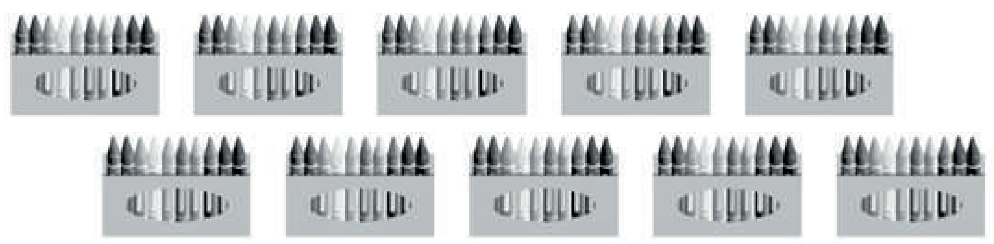
- For 10, draw a circle and write 10 inside it.
- For 100, draw a square and write 100 inside it.

Task 2.2.4 Investigate crayons in boxes

Crayons are discrete objects. They are not structured. We can structure them into boxes.
There are 10 crayons in each box.

> **Mathematical acting and thinking**
>
> First choose a unit (1 crayon and 1 box). Then repeat that unit to count how many there are.
> Connect skip counting in 10s to multiplication by 10.
> Generalise: Every box has ... crayons

1.
First specialise. How many crayons for particular numbers of boxes.



How many boxes?	How many crayons?

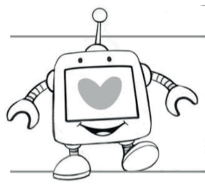
1 box: ___10___ crayons. 10 × 1 = ___10___

2 boxes: ________ crayons. 10 × 2 = ________

3 boxes: ________ crayons. 10 × 3 = ________

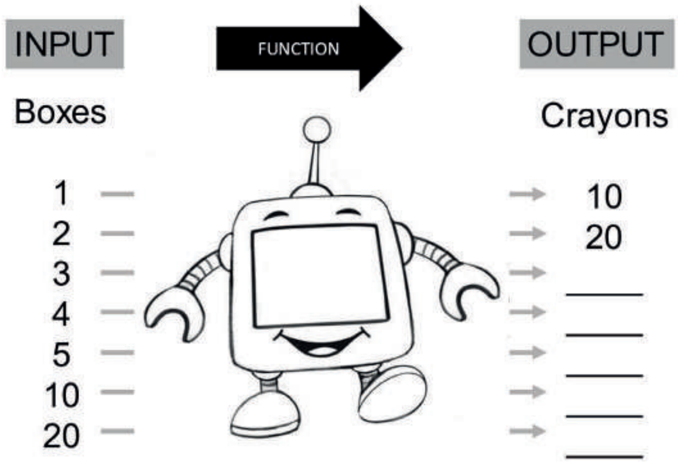
4 boxes: ________ crayons. 10 × 4 = ________

2.



This is a friend of Zibalo called, Function. He is a Maths Robot. His friends call him Funky. He can be programmed to produce different math functions.

Next organise (use a function machine) to look for a pattern.



> **Cinga**
>
> A function is special kind of relationship in mathematics: For each input there is only one output.
> An example of a function is **add two**: input + 2 = output.
> Another example of a function is **double**: input × 2 = output.

PrimTEd MAT3

Reason mathematically.
Functional thinking
- Express the relationship between crayons and boxes .

Now generalise

3. I have “a number” of full boxes of crayons.
Explain how I can work out how many crayons I have.

4. I have 35 crayons. How many boxes do I need? (Specialise again for a particular case)

5. If there are 120 crayons, how many boxes do I need? (Specialise for this particular case)

6. Write a rule that relates the number of crayons to the number of boxes (Generalise):

No. of crayons =
c =
c =

7. I have “some” crayons. Explain how I can work out how many boxes I need? (Generalise):

8. I have some crayons. I pack them into 5 boxes. The 5^{th} box is only half full. How many crayons do I have? (Specialise)

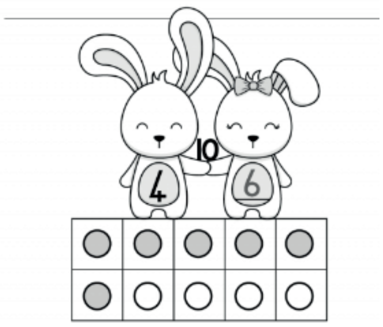
Fluencies

Make connections with bonds of 10

Task 2.2.5: Two minute tango

1	_______ + 2 = 10	11	100 - 80 = _______
2	_______ + 20 = 100	12	100 - _______ = 40
3	10 - 6 = _______	12	10 - _______ = 4
4	100 - 60 = _______	14	_______ + 7 = 10
5	8 + _______ = 100	15	20 + _______ = 100
6	100 - 8 = _______	16	_______ + 200 = 1 000
7	100 - 3 = _______	17	3 + _______ = 10
8	100 - 30 = _______	18	13 + _____ = 20
9	_______ + 20 = 100	19	23 + ____ = 30
10	_______ + 80 = 100	20	_______ + 3 = 20

Bonds of 10



Representations

Bonds of 10 are “friends in love”
Ten frames are trays of 10 scones.
Structure by 2s - with 2 in each column or structure by 5s - with 5 in each row.

4 + **6** = **10**. 4 and 6 are best friends. Together they add up to make a full baking tray of 10 scones.

You can use your bonds of 10 for many other calculations.

- If **4 + 6 = 10**, then **4**0 + **6**0 = **10**0
- If **4 + 6 = 10**, then **4**00 + **6**00 = **1 0**00, etc.

If you know that **4 + 6 = 10**, then you can **visit the next multiple of ten**:

- If **4 + 6 = 10**, then 1**4** + **6** = 10 + **10** = 20
- If **4 + 6 = 10**, then 2**4** + **6** = 20 + **10** = 30
- If **4 + 6 = 10**, then 3**4** + **6** = 30 + **10** = 40, etc.

Make connections

If you know one number fact (eg. a bond of 10), then use the **small, medium & large number songs**, to work out new facts

SUMMARY

Explain the difference between **conceptual and perceptual subitising** and give examples.
Give an example of a **mistake** that you made, and what you learnt from it.
What **concepts** to do you think you will be tested on in Quiz 2.2?
Design some questions which you think will be asked in the **mental fluencies** section of Quiz 2.2?

UMTHAMO 2.3

Counting songs

Remember the key ideas in unit 2 were:

1. Following the **counting pathway** to learn about numbers, children **count discrete objects** or **discrete events**.
2. Children learn to 'count with meaning' by moving along **a counting trajectory**.
3. Children first learn to count in 1s. But to understand number names beyond ten, they need to **count and structure in 1s, 10s and 100s**. They also count and structure in **2s and 5s.**

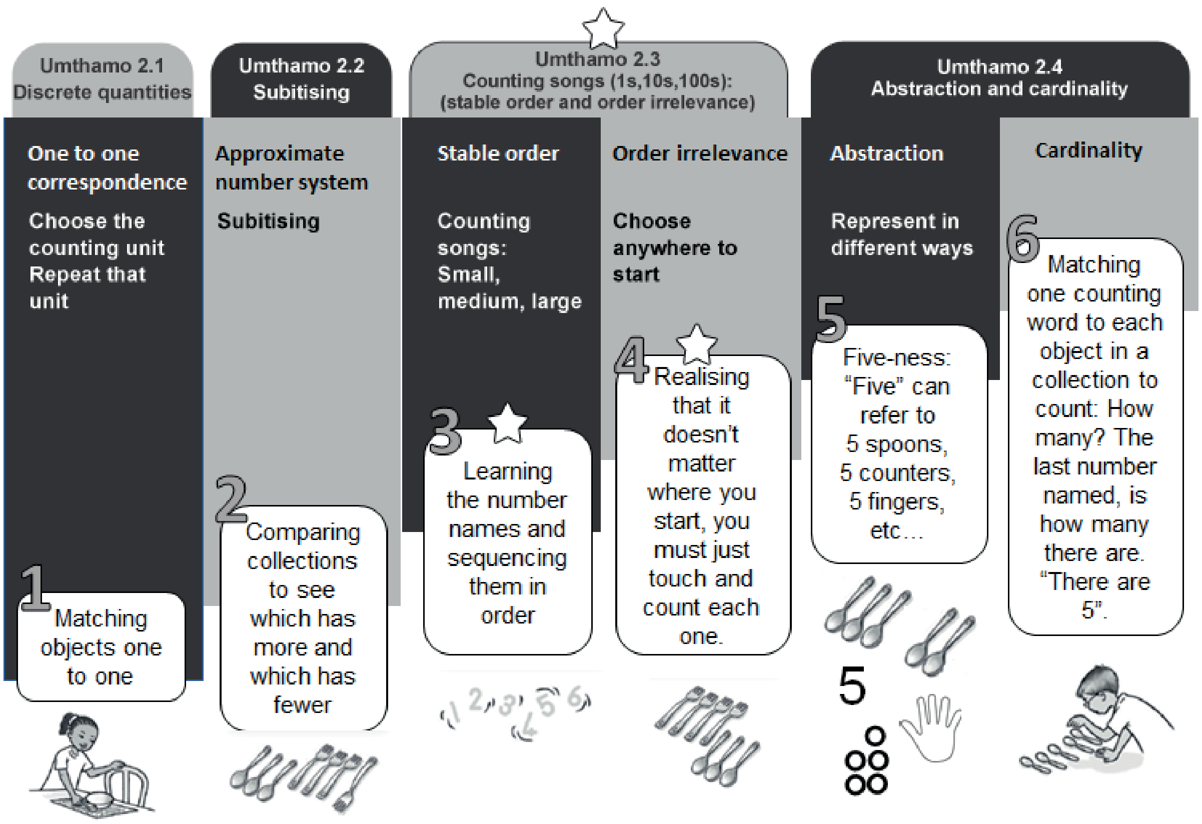
Introduction

Key Ideas

The key ideas in umthamo 2.3 are:

1. **Stage 3 of the counting trajectory is learning the small number song (in 1s)**
2. The number song is different depending on the language used. The numer names in **some languages are more transparent and regular** that other languages.
3. In **Stage 4 of the counting trajectory** children realise that no matter where they start when counting a set, they get the same amount **(order does not matter**). They touch each object as they say each word of the small counting song.
4. Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s,10s and 100s.**

There is a counting trajectory of different stages which children follow when learning to count.



In this umthamo we focus on learning the number song. When children start to match each number word to a particular discrete object or event, they then apply the order irrelevance principle.

Stage 3: Learn the number song – stable order

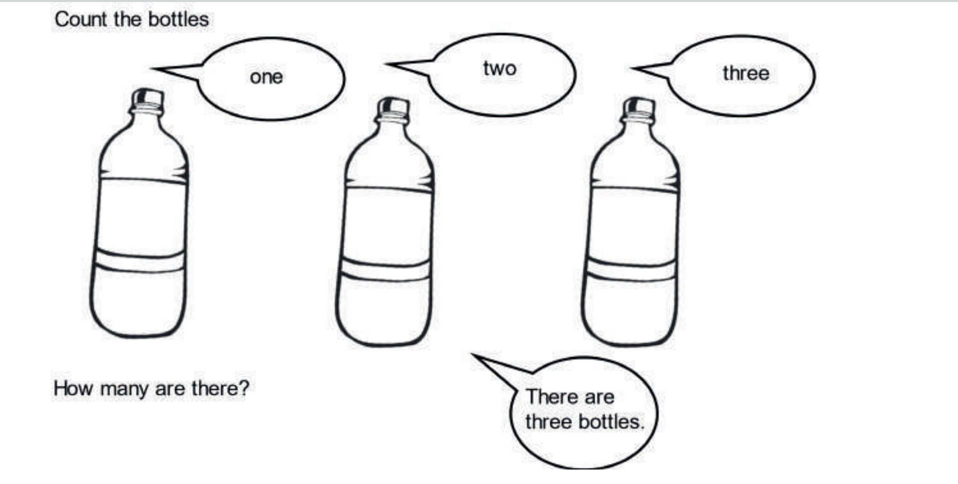
Children have reached stage 3 of the counting trajectory when they can apply the:
Stable order principle: Children know the small number song. Number names are always arranged in the same fixed order, e.g. one is followed by two, two is followed by three, three is followed by four, and so on.

To **count in 1s with meaning,** children need to;

- Know the **number song** (say the number words in sequence) starting at 1 (like a rhyme or a song).
- Match the number word with each action or object. This is o**ne-to one correspondence**.
- Know that the last number name that they say, is how many there are in the collection (**cardinality** of the collection).

Activity for learners

How many bottles?



Some children **will not match** each word to each bottle. They might say "one-two-three-four", but they don't match each number word to each bottle. They know the forward counting song in ones, but they have not yet matched each number word to each object. Touching each bottle, and saying each word may assist.

Some children **will match** each word to each bottle. They might say "one-two-three". But when you ask them: So, how many bottle are there? They start counting again from one. They say "one-two-three" again. This child does not yet know that the last number they say, is the number of the set of bottles. To see if they can apply the cardinality principle for four, you can also ask them: Show me 4 claps, or click 4 times or fetch me 4 pencils.

If a child cannot do this for four, then try the same activity for smaller numbers. They develop the cardinality for sets of 1,2 and 3 objects and events, before they develop their sense of cardinality for bigger numbers.

Stage 4: Match the number word to each object – order irrelevance

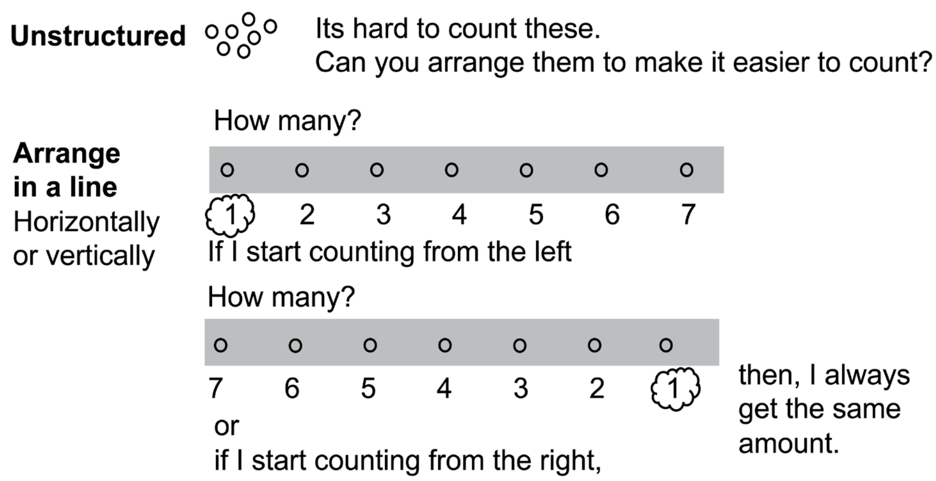
By the time children reach stage 4, they can **classify and sort** to choose what they count. They can **match objects one-to-one** (stage 1). They can tell you **which group has more, and which has less**. They know when two small groups are not the same (stage 2). They **know the number song** (possibly in more than one language) (stage 3), and they say the number words for the number song **in order** – forwards and backwards.

At stage 4 they now connect one-to-one matching with the number song. The say each number word as they touch each object AND, they realise that the order that they use to touch each object does not matter.

At first children don't structure groups of objects. Counting is then more difficult as they lose track of which objects they have touched and counted. They may count an object twice. They may miss out an object and not count it.

With help. children start to **arrange objects into patterns** so they can keep track of which objects they have counted. They may arrange the concrete counters in a long line **(vertically or horizontally)**.

<u>Diagram: From unstructured to arranging in lines or patterns</u>



When children realise that it does not matter which side they start to count from (left or right, top or bottom), they apply the **order irrelevance principle**. They know that however they arrange the objects, they will always get the same amount.

Order-irrelevance principle: The order of counting the objects in a collection does not matter. Children need to understand that however we arrange the objects, the total number of objects in the collection remains the same.

The number song is different depending on the language used.

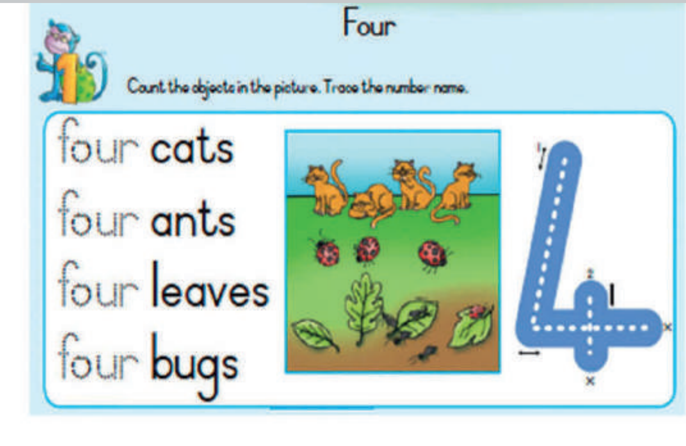
Children must learn to **say the small number song (counting in 1s) forwards and backwards**. When they get to school they start to write the numbers words (eg "two") and number symbols (eg. 2).

To **write** all numbers we only need 10 number symbols:
1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 and 0.
But, to say all numbers, we don't only use 10 number words.

In most South African classrooms the number song is learnt in two languages: a home language (like isiXhosa) and English.
Depending on the language we speak, we can have far more than ten words. We also have different ways of combining the words to denote the counting unit, and rules for how we say numbers that are bigger than ten.

Activity for learners

Count the objects in the picture. Trace the number names



Source: DBE rainbow book Grade 1

Notice that the English children write: four, four, four, four.
In English the number words always stay the same. It does not matter what unit you are counting. If there are 4 things, you always say and write: four

Activity for learners

Bala izinto ezisemfanekisweni. Khupela igama lenani

Source: DBE rainbow book Grade 1

In isiXhosa, the number words change. Which number word you use, depends on what unit you are counting. What you write, and say, depends on the noun class of the objects you are counting. If there are 4, you could write zi**ne** or ezi**ne** of ama**ne**.

Children first learn the **small counting song** (forwards and backwards) in ones.

1 ; 2 ; 3 ; 4 ; 5

By changing the counting unit they can count anything. They choose the what to count (this is the counting unit). They can count chickens, or pens, or people or fingers, etc. The counting units can be a group.

The counting unit for the **medium counting song** is a group of one ten (tens).

1 ten ; 2 tens ; 3 tens; 4 tens ; 5 tens

The counting unit for the **large counting song** is a group of one hundred (hundreds).

1 hundred ; 2 hundreds ; 3 hundreds; 4 hundreds ; 5 hundreds

SOME IDEAS FOR YOUR CLASSROOM

Making tens (structure within 1 ten)

The bonds of ten are key facts which children should know fluently. If children learn the bonds of 10, they can use them for many different calculations:

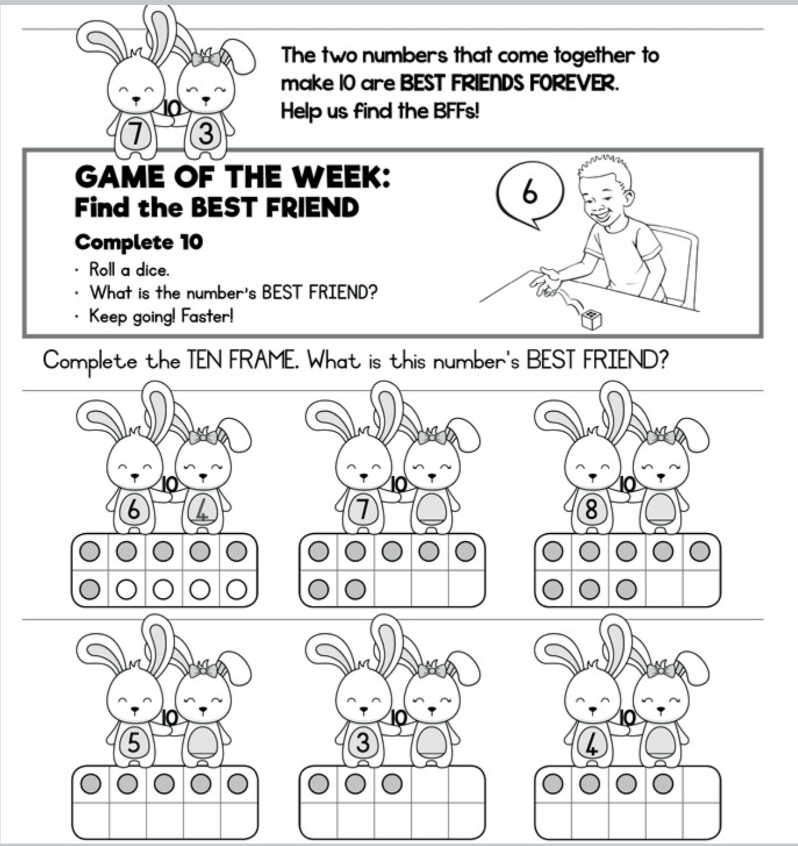
- 7 + 3 = 10
- 70 + 30 + 100
- 27 + 3 = 30, etc.

Activity for learners

Structure in 10s

Structuring builds on our ability to subitise.

- Once children are secure counting in 1s, they start counting in 2s, 5s and 10s
- When children **circle ten dots** to make a set of 1 ten, they use the idea of **discrete objects contained in a set**
- Bonds of 10 make a full tray of ten scones. 14 + ___ = 20 14 is 1 full tray and 4 scones. So I need 6 scones to fill the second tray. Bonds of 10 fill up the ten frame.



Source: Porteus, et al. (2021) Magic Classroom Collective learner workbook, Nelson Mandela Institute

Game

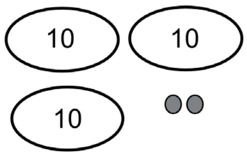
1,2,3 Draw!

Draw 12

So I can quickly see.

Draw 32

So I can quickly see.



Children will start drawing in 1s. As teachers, we help them to draw quickly by structuring in 100s, 10s and 1s

Game

Counting forwards and backwards in 10s

Game 1: Counting

COUNT FORWARDS IN 10s

- Count FORWARDS in 10s past 200.
- Start at any number from 0 to 10.

3, 13, 23, 33, 43, 53, 63, 73, 83, 93, 103, 113, 123, 133, 143, 153, 163, 173, 183, 193, 203

Game 1: Counting

COUNT BACKWARDS IN 10s

- Count BACKWARDS in 10s from 210.
- Start at any number from 200 to 210.

206, 196, 186, 176, 166, 156, 146, 136, 126, 116, 106, 96, 86, 76, 66, 56, 46, 36, 26, 16, 6

PrimTEd NA1

Emergent number awareness

1.4 Distinguish between lists of number words, lists of numerals, sequences of number words, sequences of numerals and counting proper.

Activity for learners

Complete the following patterns:

65	75	85							

135	136	137							

92	82						

Medium song (forwards)

Small song (forwards)

Medium song (backwards)

PRACTICE

Tasks: Conceptual understanding

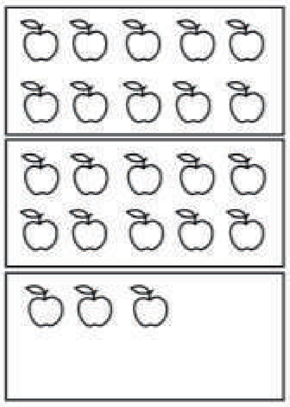
Task 2.3.1 Learning the number song in English and an African language (transparency and regularity).

Multi-lingual children take longer to learn the counting songs. They have to make 5 associations to count with meaning, and these differ in three different languages: English, isiXhosa and Sesotho.

Emergent number sense

Children must learn the small number song (in 1s) **forward and backwards, and in more than one language**

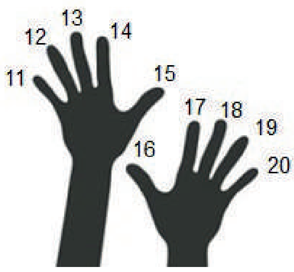
Children apply the **stable order** principle of the counting trajectory.

Stimulus	English	isiXhosa	Sesotho
A visual stimulus depicting a number of objects. This could be imagined from situation or a story problem, or shown visually as concrete objects, iconic pictures of a set of objects, or a more indexical depiction of the objects.			
A chain of sounds which makes up the number word	twen-ty-three	a-ma-shu-mi-a-ma-bi-ni-a-nan-ta-thu	ma-sho-me-a-ma-be-li-a-me-tso-e-me-ra-ro
The 10-wise place value structure implicit in the number name, number symbol	2 tens and 3 units (Transparent in number words for isiXhosa and Sesotho, but not transparent for English)		
The number name written using letters of the alphabet and pronounced using a chain of sounds.	Twenty-three	amashumi amabini anantathu	mashome a mabeli a metso e meraro
A multi-digit number symbol making use of only 10 digits (0, 1, 2..., 8, 9)	23		

A number naming system is **regular** if the rules for creating big numbers are consistent (there are no exceptions). **A number** naming system is **transparent** if you can hear the counting units (in tens and hundreds)

If English was **regular,** we would say:

One-ty one;
One-ty two;
One-ty three;
One-ty four; ...



If English was **transparent,** we would say:

One-ten and one
One-ten and two;
One-ten and three;
One-ten and four; ...

Let's talk maths

Transparency and regularity

Number names are **transparent** if we name the counting units. The counting unit is usually 'tens'. In English, the number names are **NOT transparent** (we do not say counting units- which is 'tens').
11 = 1 ten-and-1 = 'eleven'
12 = 1 ten-and-2 = 'twelve'
13 = 1 ten-and-3 = 'thirteen'
23 = 2 tens-and-3 = 'twen-ty-three'
This makes learning the counting song in ones for English, more difficult.

In isiXhosa, the number names for numbers bigger than ten are **transparent** (we say the counting units, which is 'ishumi').
11 = l**ishumi** elina**nye** = l**ishumi** elina**nye**
12 = i**shumi** unya **bini** = l**ishumi** elines**bini**
13 = i**shumi** elinesi**thathu** = li**shumi** elinesi**thathu**

This makes learning the counting song in ones in isiXhosa easier. Be sure to pronounce each syllable and make the tens structure explicit.

Number names are **regular** if the rules for creating numbers bigger than ten are consistent. Transparency and regularity are important for teaching in multilingual classrooms in South Africa.

In urban contexts you will probably have children in your class who do not have the same language as you. When they tell you how to count in their language, you should think about regularity and transparency.

The extract below discussed transparency and regularity of number names in English and in isXhosa

Reading 4
Mostert, I. (2019) Number Names: Do They Count?, African Journal of Research in Mathematics, Science and Technology Education, 23:1, 64-74, DOI:10.1080/18117295.2019.1589038
To link to this article: https://doi.org/10.1080/18117295.2019.1589038

Transparency
The transparency or explicitness of a number naming system refers to the extent to which spoken numbers explicitly state the units that are implied in the written place-value system. These units (ones, tens, hundreds, etc.) are referred to as numeration units (Houdement & Tempier,2015).

The English number naming system is not entirely transparent for numbers 11–99 as the numeration units for 'ones' and 'tens' are not stated explicitly. For example, from the name 'thirty-seven' it is not possible to infer that there are 3 tens and 7 ones.

In isiXhosa, when naming numbers bigger than 10, first the tens are qualified (i.e. the number of tens is given) and then the units are added using the connective concord a-(ana + isixhenxe = anesixhenxe) which can be translated as 'with'. For example, thirty-seven is written as:

ama<u>shumi</u>	**ama**<u>thathu</u>	**ane**<u>sixhenxe</u>
<u>tens</u>	that are <u>three</u>	that are with seven

In this example the three (-thathu) is being used adjectivally to qualify the numerosity of the tens (ama-shumi), a plural noun in class 6.

The close link between isiXhosa number names (spoken numbers) and the written place-value system (written numbers) could be leveraged to help children understand the conventions of the place-value system. However, observations of isiXhosa teachers confirm the observations of Sepedi teachers made by, namely that teachers do not generally make use of this linguistic feature of African languages.

One way in which this feature could be leveraged, as is done in Chinese mathematics education, is to use the idea that adding two things of the same unit gives an answer of the same unit, e.g.
5 dogs and 2 dogs are the same as 7 dogs.
5 tens and 2 tens are the same as 7 tens
(Sun & Bartolini Bussi,2018).
[insert: Notice the counting units change, but the numbers stay the same]

In English this kind of leveraging is not possible for tens (as, say, two tens is spoken as 'twenty') but is possible for hundreds (e.g. five hundreds and two hundreds is the same as seven hundreds)

Regularity

A number naming system is regular to the extent that the same 'rules' are used to name all numbers bigger than 10 (for a base-10 number system). Transparent number naming systems such as those discussed in the previous section (Chinese, Japanese, Korean) are typically also completely regular(which might explain why the terms transparent and regular are often used interchangeably). It is, however, possible for an opaque number naming system to be (partly) regular. For example number names in French for the numbers 80–99 are not transparent (the number of tens is not stated explicitly) but they are regular (the same rules are used to determine the names of numbers).English, which is partly opaque (see previous section), also contains a number of irregularities due to influences of Old Saxon on the names eleven and twelve, as well as due to phonemic modifications as listed by Mark and Dowker (2015):

An analysis of isiXhosa number names reveals a number of irregularities such as the difference between
five (zin**tlanu**) and fifteen (ishumi elinesi**hlanu**)
and between
fourteen (**i**shumi **eli**nesine) and twenty-four (**ama**shumi amabini anesine).

However, a closer look at isiXhosa grammar rules reveals that all these apparent irregularities are instances of changes that occur as a result of the system of concordial agreement or as a result of regular phonetic mutations (when two phonemes, occurring next to each other because of agglutination, influence each other).This difference in the form of the number word used (and consequently the difference in spelling of related number names) is not emphasised in Grade 2 and 3 texts, even though CAPS states that learners should write number names for 1 to 100 in Grade 2 and 1 to 1000 in Grade 3 (Department of BasicEducation,2011). Further research is required to establish to what extent these different forms (and spellings) impact on learner performance and whether there are significant differences across languages when requiring young learners to write and correctly spell number names.

SCAN ME

Number names: Do they count? A paper about how we name numbers in isiXhosa and English.

Mostert (2019)

Note

Number names in African languages are more regular and more transparent than English

1. Who wrote this article on number names in isiXhosa and English?

2. When was the article published?

3. What is the title of the journal article?

4. Where was the article published (in which journal or conference proceedings)?

5. Using your own words, explain what a transparent number naming system is.

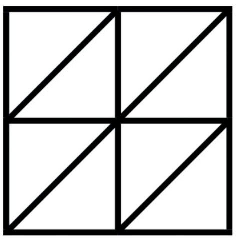
   Mostert (2020) argues that a number naming system is transparent if ...

6. Give an example of a South African language, where the number names are transparent. Include a few examples of particular number names from this language.

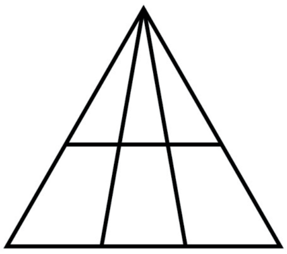
7. How many triangles? ☐

   How many squares? ☐

   How many rectangles? ☐



   How many triangles? ☐



8. Choose the correct underlined words in the next two sentences. Each sentence gives an example to show if a particular language has transparent number names or not:

For example, English is transparent/ not transparent for numbers bigger than 10, because the counting unit ("ten") is /is not said.

However, isiXhosa is transparent/ not transparent for numbers bigger than 10, because the counting unit ("shumi") is/is not said.

9. Complete the sentence and give some examples.

A number naming system is regular...

Note

In academic writing (needed for university), you often need to carefully define the concept. Then you give some examples to show that you understand how the apply the concept to a particular situation.

Many number names in English are irregular. For example, ...

10.

Count backwards in 10s:

When we count BACKWARDS in tens, we subtract tens.

943							
725							
846							

Medium song (backwards)

Task 2.3.2 Design a number display in your language

A number display is a poster showing different represents of a number.

A number display includes

- The number symbol;
- The number name (in English and an African language);

Representations of the number (literal, concrete, iconic, indexical, symbolic).

Design a number display for the number 7 or 8, which is not in English.

PrimTEd GPM3

Engage learners productively with mathematics

Let's talk maths

If you were teaching in isiXhosa and you made a number display for your classroom, would you write ne, or zine? Why?. Discuss. What happens in other languages?

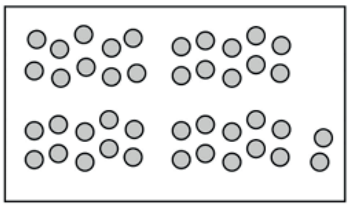
Tasks: Fluencies and structuring (1s, 10s and 100s)

Structuring representations

Number pictures: Structuring by 1s, 10s and 100s.

Task 2.3.3: Number pictures with 1s, 10s and 100s

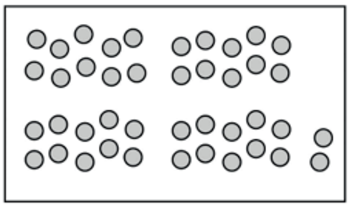
1. Circle the tens



How many 10s? ______

How many 1s? ______

Children will start drawing with 1s.



How many 10s? ______

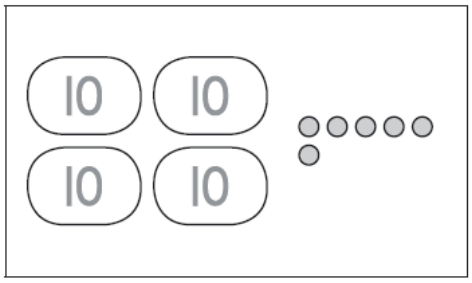
How many 1s? ______

As teachers we help children to circle 10 ones, and then to write 10.
As teachers we ALWAYS draw by structuring in 100s, 10s and 1s

2. Quickly see and write down how many there are:

What is the number?

"10s:" means how many tens?
"1s:" means how many ones?

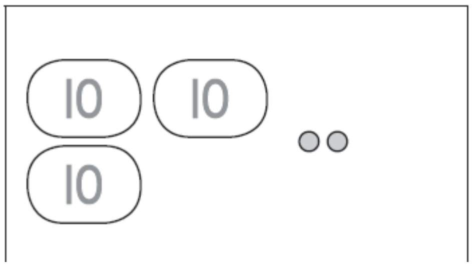


10s:	1s:
4	6

46

46 = 10 + 10 + 10 + 10 + 6

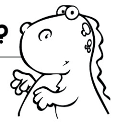
46 = 40 + 6



10s:	1s:

Circle the 10s. How much money?

3.



Draw each number. Draw ● to show one. Draw (10) to show ten. Draw $\boxed{100}$ to show 100.

Number	Drawing	100s?	10s?	1's?
342	$\boxed{100}$ $\boxed{100}$ $\boxed{100}$ (10) (10) (10) (10) ● ●	3	4	2
215				
305				
142				

4.

Count forward by 100s:

large song (forwards)

5							
23							

Count backwards by 100s:

large songs (backwards)

982							
905							

Task 2.3.4: Explore and then generalise a hashtag

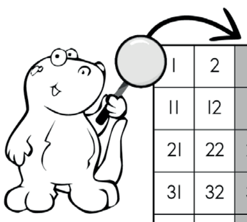
We can use the **medium number song** adding or subtracting tens.

Let's ADD TENS using a number chart!

A column is vertical. We move up and down a column.

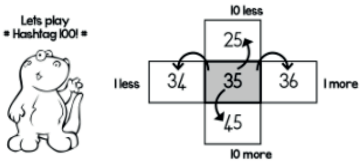
Look at this column. Count down along this column starting at 3.

What happens when we jump down one row?
What happens when we jump down two rows?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Can you see when I move down one row, I add 10!



1. Practice adding on 10 each time.

Circle the number on the chart. Write 10 more. Write 20 more.

13	10 more on the chart!	43	27	35	50	29
23						
33	10 more on the chart!					

2. Fill in the missing words. We **hop** in ones. We do bigger **jumps** (in tens)

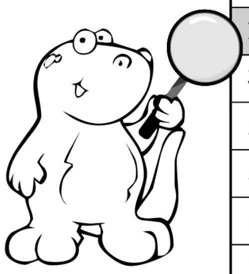
Let's Talk Math in English!

NgesiXhosa sithi:	In English we say:
Itshathi yamanani	Number chart
Imiqolo isuka ekhohlo iye ekunene	Rows go from left to ______
Xhumela phambili ka-1. U-1 ngaphezu.	Hop forward 1. 1 more
______ ngasemva. U-1 ngaphantsi.	____ backward. 1 ____
Iikholamu zisuka phezulu ukuya ezantsi.	Columns go from ____ to bottom
_____ EZANTSI umqolo omnye. I-10 ngaphezu.	Jump DOWN one row. 10 more.
Tsibela PHEZULU umqolo omnye. I-10 ngaphantsi.	_____ UP one row. 10 less.

We use the small number song for adding and subtracting in ones.
A row is horizontal. It goes from left to right.

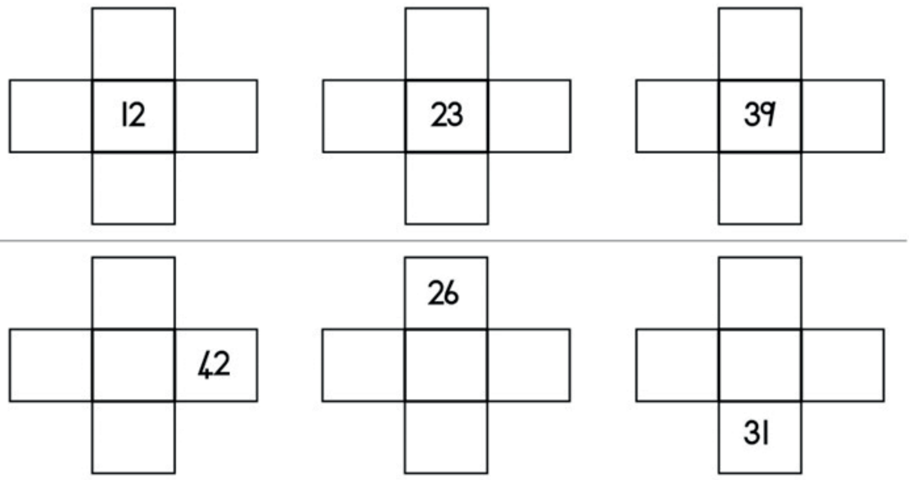
Let's SUBTRACT small numbers using a number chart!

When we subtract, numbers get smaller. When we subtract one, we hop backwards one!



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. Make a hash tag. Complete the missing numbers.

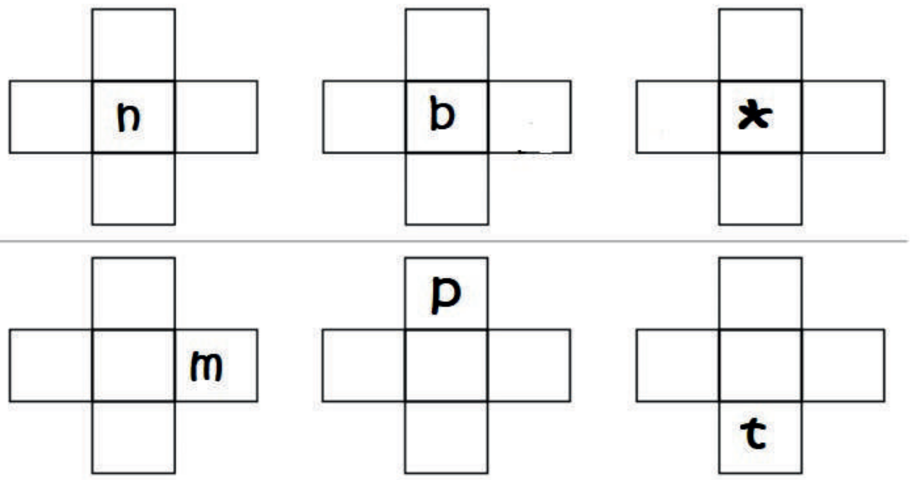


PrimTEd NA8

Early algebra

NA 8.1 Use the notions of structure and generality to explain the algebraic treatment of arithmetic

Now generalise. Complete the hashtags

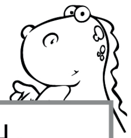


Note

In a hashtag we use 2 counting units. For across (left to right), we count in ones.For up and down, we count in tens over, or 1 half.

4. We can use the medium number song for adding tens.

We can add big numbers by breaking down the second number. Let's draw number pictures!



Ta' Jola drove 45 kilomentres. Then he drove 30 kilometres more. How many kilometres did he drive altogether?

45km + 30km = 70km

45 + 10 + 10 + 10

55 65 **75**

I write the first number. I put it in my head.

I DRAW the second number. 30 is 3 tens! I ADD each 10!

45 + 20 =

63 + 20 =

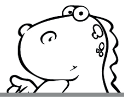
57 + 30 =

Put the first number in your head. Count forward by 10s!

35 + 10 = ___	59 + 30 = ___	37 + 40 = ___	27 + 50 = ___
61 + 20 = ___	38 + 40 = ___	64 + 30 = ___	37 + 50 = ___

5. We can use number pictures and the medium number song for subtracting tens..

We can subtract big numbers by breaking down the second number. Let's draw number pictures!

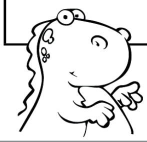


Usenathi had R75. She bought a book for R30. How much Rand does she have now?

R75 - R30 = R45

75 - 10 - 10 - 10

65 55 45



I write the first number.
I put it in my head.

I DRAW the second number.
30 is 3 tens!
I TAKE AWAY each 10!

63 - 20 =

54 - 30 =

87 - 30 =

Solve by breaking down the second number.

65 - 30 = 65 - 10 - 10 - 10 = 35

59 - 20 = 59 - ________________ = ______

91 - 40 = 91 - ________________ = ______

Task 2.3.5: Two minute tango

1	33 + ________ = 40	11	50 - 10 = ________
2	________ + 3 = 60	12	75 - 2 = ________
3	________ + 7 = 20	12	100 - ________ = 90
4	64 + ________ = 70	14	9 + ________ = 100
5	97 + ________ = 100	15	Half of 14 = ________
6	________ + 9 = 100	16	140 ÷ 2 = ________
7	83 - 3 = ________	17	Double 90 = ________
8	84 - ________ = 80	18	98 - ________ = 90
9	65 - ________ = 60	19	________ + 2 = 40
10	16 - ________ = 10	20	________ + 9 = 100

Fluencies

Use bonds of 10.
Add to visit the next ten.
Subtract to visit the previous ten

Representations

Number lines to visit the next and previous 10 (or multiple of 10).

When adding we use number lines, and visit the next ten.

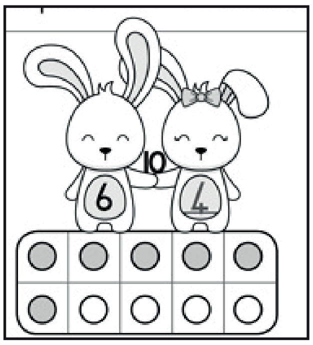
Zibalo LOVES 10!
When she adds she asks two questions.
What is the next 10?
How far to the next 10?

6 + ____ = 10

So, for any number, how far is it the next 10? This uses bonds of 10.

Next 10:	20
How far:	3

Another mental map or picture for visiting the next 10, is filling up the 10 frame. You can ask: How many to fill up the ten?



If I know that **7 + 3 = 10**, then I know that 17 + 3 = 10 + **7 + 3** = 20

Make connections.

If you know one fact (eg a bond of 10), use the medium song to add 10 to both sides, and work out a new fact (eg. a bond of 20).

SUMMARY

Explain what is meant by a **transparent** number naming system. Give examples.
Given an example of where you have used your mathematical power to be able to **imagine and express.** Draw a hashtag with 57 in the centre
What **concepts** to do you think you will be tested on in Quiz 2.3?
What **mental fluencies** will you be tested in in Quiz 2.3?
What are **the key ideas** which have learnt so far?

Cardinality and Abstraction

Remember, the key ideas across unit 2 were:

1. Following the **counting pathway** to learn about numbers, children **count discrete objects** or **discrete events**.
2. Children learn to 'count with meaning' by moving along **a counting trajectory**.
3. Children first learn to count in 1s. But to understand number names beyond ten, they need to **count and structure in 1s, 10s and 100s**. They also count and structure in **2s and 5s.**

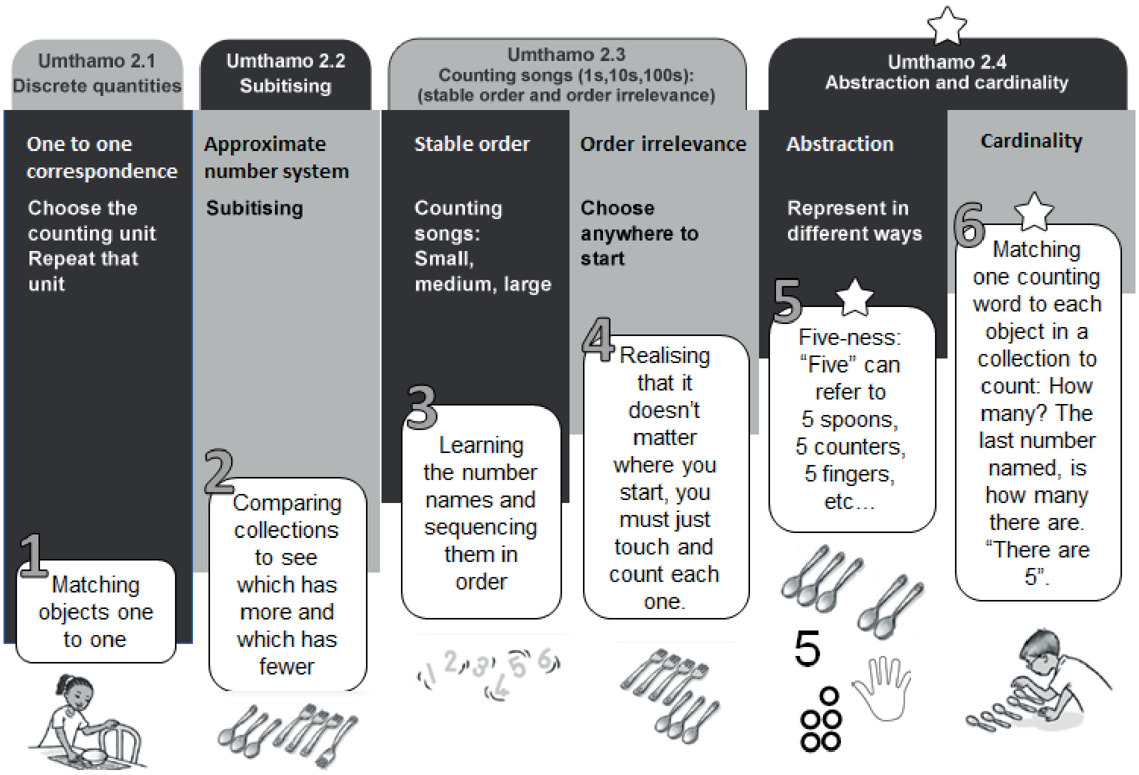
Introduction

Key Ideas

The key ideas in umthamo 2.3 are:

1. **Stage 5 of the counting trajectory is abstraction.** Children move between different representations of a number.
2. In **Stage 6 of the counting trajectory is cardinality.** Children know that they last number they name in
   the counting song, is how many there are.
3. Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s,10s and 100s.**
4. You also need to be **fluent with structuring and counting in 2s.** So there are tasks and ideas for your classroom which involve:
   - The number song in 2s
   - Odd and even numbers
   - Doubling and halving
   - Investigate a number **pattern** from counting in 2s. Explore the doubling and halving **functions** and express the function using **algebra**.

Diagram: A learning trajectory for counting



In this umthamo we focus on **abstraction** and **cardinality**.

Stage 5: **Abstract** the number-ness of a number. Know that five can refer to five objects or five pictures or five events or 5 steps etc.

Stage 6: Know how many there are. Match one counting word to each object in the collection and realise that the last number named is how many there are in the set (**cardinality**).

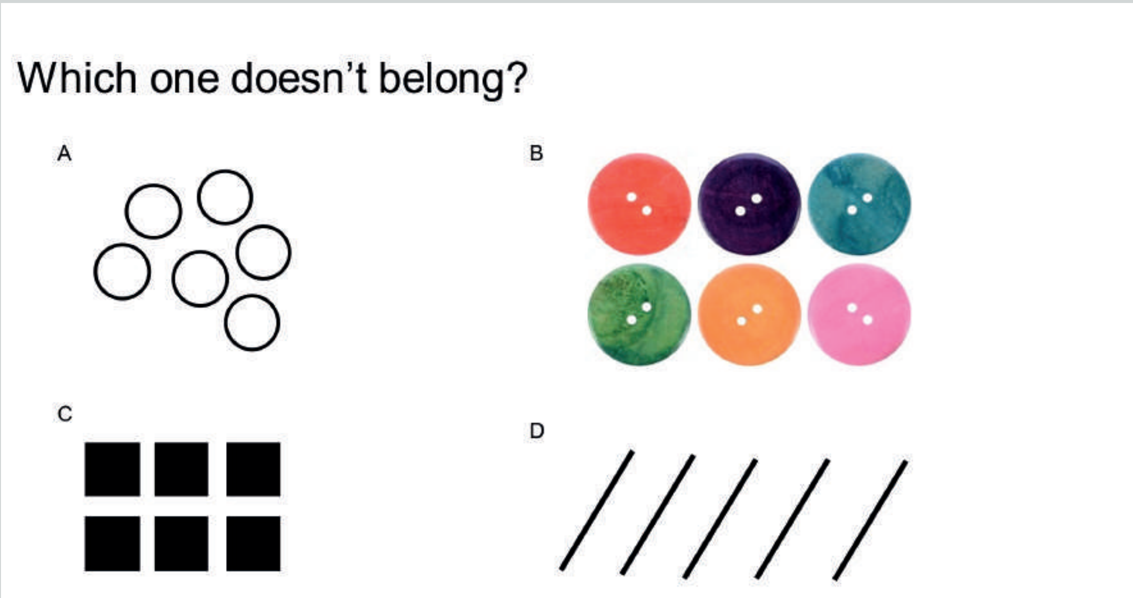
Stage 5: Abstracting

ABSTRACTION principle: Learners understand that even if groups with the same number of objects look very different (e.g. five grapes, five people, five houses) they have the same numerosity, i.e. 'fiveness'. They realise that counting can be applied to objects, pictures, colours, shapes, or even actions or sounds.

Source: Kuhne, et al. (2017) R-Maths Concept Guide

Teachers can see whether children can abstract by offering different sets, and seeing if the child can see which one does not belong.

Activity for learners



In this activity children move between different indexical representations (A, C and D) and the literal representation (B).
They need to realise that A B and C all represent 6, but that there are only 5 lines in D.

Children develop their sense of number sequentially. If I child cannot abstract the concept of 5, then try with smaller numbers. Show them 3 fingers. Ask them to fetch 3 stones. Then they could draw 3 dots.

If a child can't see which set does not belong for 5 objects, then try the same activity but with only 3 objects.

Stage 6: Cardinality

In mathematics, **cardinality** means the size of a set. The cardinality of a set is how many elements are in the set. When we think of numbers as being collections of objects which are in a container, we are using the **cardinal meaning** of number.

Counting objects and saying how many there are, is also called **rational or resultative counting**. This means that objects or events are matched with a number name. To count 'how many', learners need to realise that each object in a collection gets a number name ('one, two, three, four ...') and that you count each object only once.

Cardinality principle: The last number word said when counting a collection represents the total number in the collection. When a child is able to answer: 'How many are there?', they have reached an important counting milestone. They can now count with meaning. They realise that the last number name that they say in the number song, is also how many there are in the collection. They have found the **cardinality of the set**.

SOME IDEAS FOR YOUR CLASSROOM

Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s,10s and 100s, as well as in 2s and 5s**

Children move through the 6 stages along the counting trajectory for 1s. Children also build on their knowledge of counting in 1s and start to:

- Count and structure numbers into 2s and 5s
- Count and structure numbers in 1s, 10s and 100s

Later the counting trajectory is extended as they start to use counting for the four operations (add & subtract, multiply & divide).

Counting and structuring in 2s

Remember that **structuring** is as important as counting. Structuring builds on our ability to subitise.

Counting in 2s
Once children are secure with the number song in ones, they can learn to count in twos.

They need to realise that counting in twos is saying every second number in the counting song. The whisper the first number word, then say the second number word loudly.

Whisper; **loud**, whisper, **loud**, whisper, **loud,** whisper, loud, whisper,

One, **TWO**, three, **FOUR**, five, **SIX**, seven, **EIGHT**, nine, **TEN**, ...

1 **2** 3 **4** 5 **6** 7 **8** 9 **10**

Then learners need to stop whispering and just pause:
pause **2** pause **4** pause **6** pause **8** pause **10** pause **12** pause,...

They learn the counting in 2s song:
Starting at 2: **2, 4, 6, 8, 10, 12, 14,** ... and
Starting at 1: **1, 3, 5, 7, 9, 11, 13,** ...

PrimTEd NA1

Emergent number awareness

Demonstrate awareness of the learning trajectories in counting.

Just as with counting in ones with meaning, children then need to match their counting in 2s song with pairs of discrete objects (cardinal meaning) , or as jumps along a number line (ordinal meaning).

Structuring in 2s
Give children situations that involve pairs or jumping in twos to practicing matching each word to each pair or jump of 2.

Activity for learners

Counting in 2s ...
How many socks are shown?

It is more efficient to count in 2s than to count in ones. The same counting principles apply, but now the counting song in twos is used.

- Say a "counting in 2s" number song
- Match each word in the counting in 2s number song to one pair of socks (one to one correspondence)
- Notice where they start and end (so they don't count the same pair of socks twice)
- Know that the last number they say in the counting song is how many socks are in the set (cardinality).

A doubling function

Once children can fluently count in twos forwards and backwards, they can link this to doubling. To double, you choose a number and repeat it. You take 2 groups of that number. The idea of a mirror line, or line of symmetry can help children picture what happens when you double.

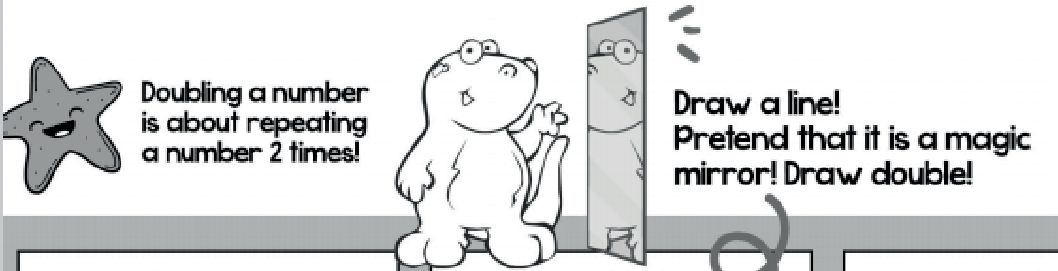
Start with small numbers (less than 5), then increase the size of the numbers for children to practice doubling. Notice that numbers bigger than ten a structured in 10s. the number picture shows groups of tens – and not ones.

Activity for learners

GAME OF THE WEEK:

Double!

- Draw a line as a magic mirror.
- Your teacher will call a number (0 to 100)
- Draw the number on the left of the mirror
- Draw the number on the right of the mirror.



Double 4	Double 12	Double 32
○ ○ ○ ○ | ○ ○ ○ ○	10 ○○ | 10 ○○	10 10 10 ○○ | 10 10 10 ○○
Double 4 is 8	Double 12 is 24	Double 32 is 64
4 + 4 = ____	12 + 12 = 24	32 + 32 = 64
There are two 4s in 8.	There are two 12s in 24.	There are two 32s in 64.

Once children have a written method or picture for doubling, they can practice doubling mentally.

Game

GAME OF THE WEEK:
Fast Maths with Cards

DOUBLE

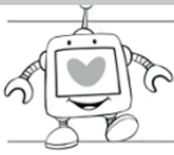
- Place number cards 0 to 5 in a pile. Mix them up.
- Flip over one card.
- DOUBLE!
- Do it again! Faster!

When you get very fast, add number cards 6 to 10. Then add 11 to 15! Can you get up to 20?

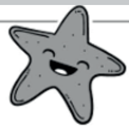
If children can double, you can help them think about a doubling function.

Doubling as a x2 function

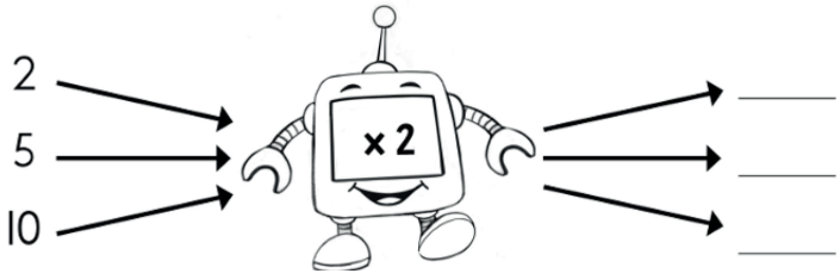
Activity for learners



This is a friend of Zibalo called, Function. He is a Maths Robot. His friends call him Funky. He can be programmed to produce different math functions.



Function is now programmed to multiply 2s. When you put a 3 in to Funky, it produces 6! What will happen when we put each of these numbers into Funky?

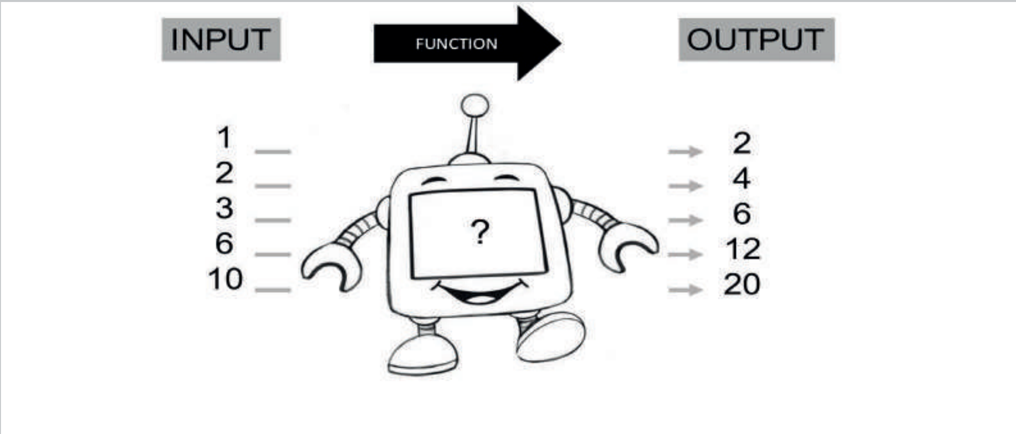


Children can play: I am thinking of a number.

- I am thinking of a number...Double my number is 6. What's my number?
- I am thinking of a number...Double my number is 10. What's my number?
- I am thinking of a number...Double my number is 4. What's my number?
- I am thinking of a number...Double my number is 12. What's my number?

Activity for learners

They are then ready to play: **What's my function.**



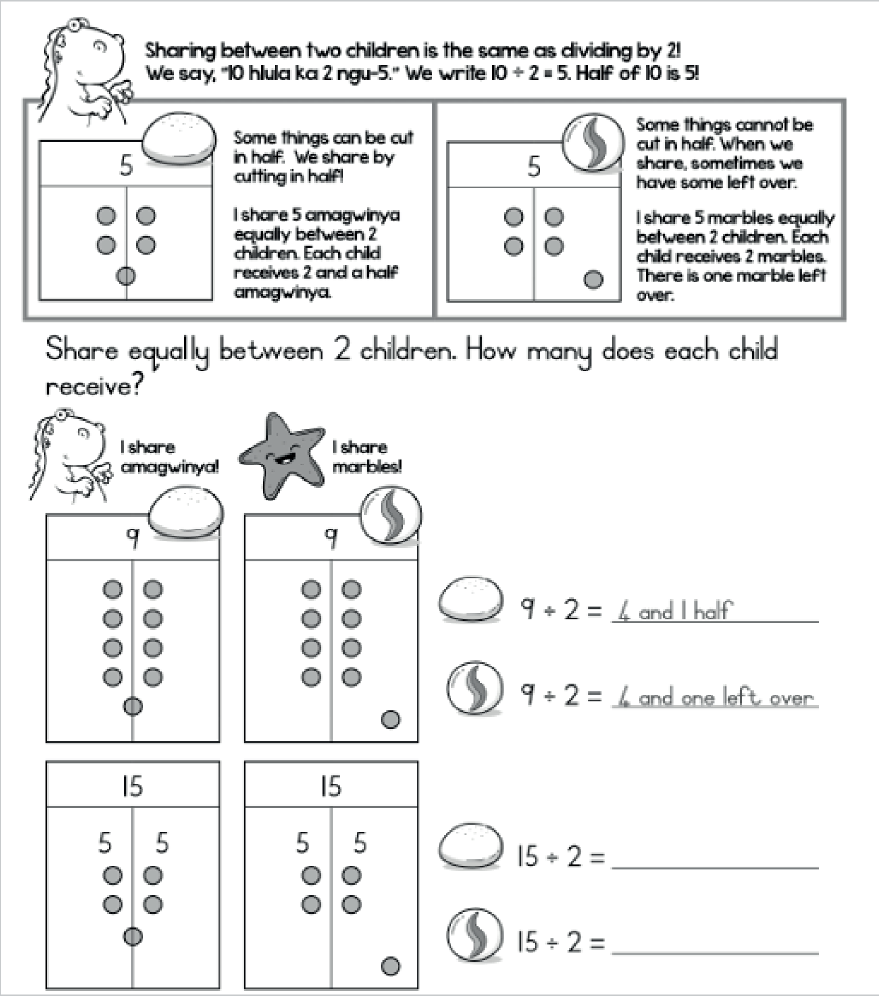
I have been programmed to be a function (like funky). Every time you give me a number (an input), I change that number in the same way.

- If the input is 2, then the output is 4
- If the input is 6, then the output is 12, etc

You can ask children to give you any input. You double it to give the output.

The inverse function of doubling is halving.

Game



Note

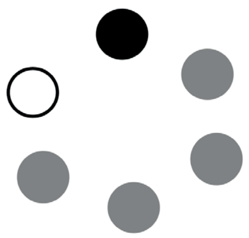
Halving is the same as sharing equally between 2. There are two ways to halve an odd number. Depending on what we are halving we get 1 left over, or 1 half.

Tasks: Conceptual understanding

Task 2.4.1: Cardinality for 'counting with meaning'

A teacher wants to see whether her grade R children can count with meaning (rational counting). She wants to see if they can apply the cardinality principle.

She puts some counters onto the mat (or desk):



Prudence starts at a grey circle. She counts: 1, ,2 ,3 ,4 ,5, 6, and carries on going round the circle...7, 8, 9, 10.

Chantal starts at the black circle.
She counts: 1, 2, 3, 4, 5, 6 as she touches each circle.
When the teacher asks: "So how many are there?" She says: "6".

Evan starts at the black circle.
He counts 1, 2, 3, 4, 5, 6 as he touches each circle. He stops.
Then he starts at the white circle and counts again, matching each word to each circle 1, 2, 3, 4, 5, 6.

Faith touches the black circle. She says: 1.
She touches the white circle. She says: 2,3.
She touches each grey circle. She says: 4, 5,... 6, 7, 8.,... 9, ...10.

1. Which child does NOT YET apply the **order irrelevance** principle?

a. Prudence
b. Chantal
c. Evan
d. Faith

2. Which child does NOT YET apply the **one-to-one correspondence** principle?

a. Prudence
b. Chantal
c. Evan
d. Faith

3. Which child clearly shows they can apply the **cardinality** principle?

a. Prudence
b. Chantal
c. Evan
d. Faith

Mathematical acting and thinking

Early algebra
- Generalise: A bicycle always has … wheels
- Connect counting in twos to doubling
- Functional thinking for doubling (x 2) and halving (÷ 2)
- Express the relationship between bicycles and wheels as an equation.

Task 2.4.2: Investigate bicycles and wheels.

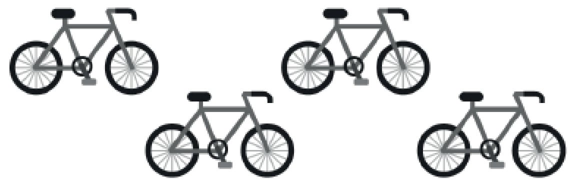
This investigation uses the ideas discussed to far in this course: discrete objects, choosing a counting unit, repeating that unit to count with meaning. But it goes further – building on what is expected in patterns, functions and algebra, CAPS – to explore a structured relationship between two different counting units (bicycles and wheels).

What is the relationship between bicycles and wheels? Can we describe and quantify the relationship?

- If we know how many bicycles can we work out how many wheels?
- If we know how many wheels, can we work out how many bicycles?
- How be we get from the number of bicycles to the number of bicycles? What is the function?

1. Bicycles are discrete objects. A bicycle is structured: Each bicycle has 2 wheels.

First **specialise**, by working with particular cases. Then **organise** (in a table of function machine) to look for a pattern.

	How many bicycles?	
	How many wheels?	

										
bicycle	1	2	3	4	5	6	7	8	9	10
wheels	2	4								

3 bicycles, how many wheels?		6 bicycles, how many wheels?	
5 bicycles, how many wheels?		10 bicycles, how many wheels?	

2. I count 20 bicycles. How many wheels? _______________

3. If there are:
100 bicycles, how many wheels? _______________

1000 bicycles, how many wheels? _______________

10 000 bicycles, how many wheels? _______________

Note

This is an example of exploring a mathematical function.

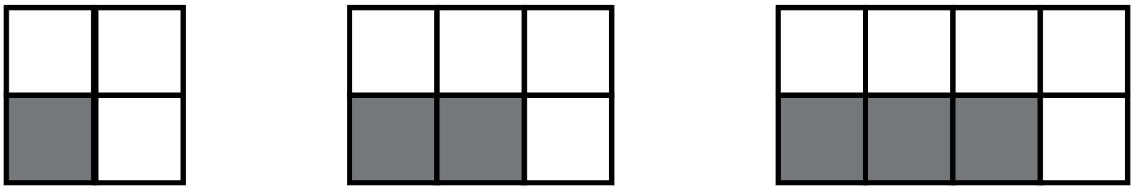
Mathematical acting and thinking

Connect shape patterns to number patterns.
Functional thinking for an increasing pattern.
Generalise: For every grey tile there are … white tiles
Express the relationship grey and white tiles as an equation.

Task 2.4.3 Investigation – Grey and white tiles

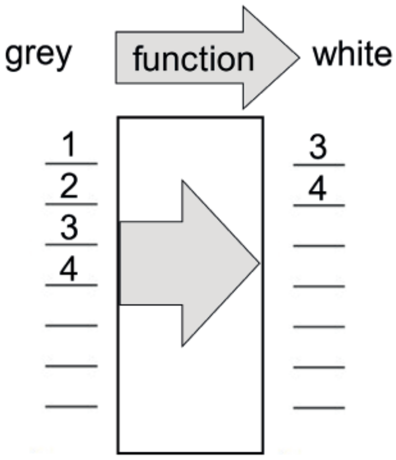
(Adapted from Jooste, et al. (2012). *Study & Master Mathematics*. Cambridge University Press.)

Explore the arrangement of the grey and white tiles in this tiling pattern.



1. Draw the next arrangement in the pattern.

2. We can represent the number of tiles in a **function machine** as below.



PrimTEd MAT1

Reason mathematically
specialise -generalise - conjecture - justify/refute - prove

3. Complete the number of white tiles in the function machine. What if there are 5 grey tiles? Or 10? Or 100?

4. How do you calculate the number of white tiles if you know the number of grey tiles?

5. What is the rule to calculate the number of white tiles if you have any number (*n*) of grey tiles?

6. How do you calculate the number of grey tiles if the number of white tiles are given?

7. What is the rule to calculate the number of grey tiles if you have any number (*m*) of white tiles?

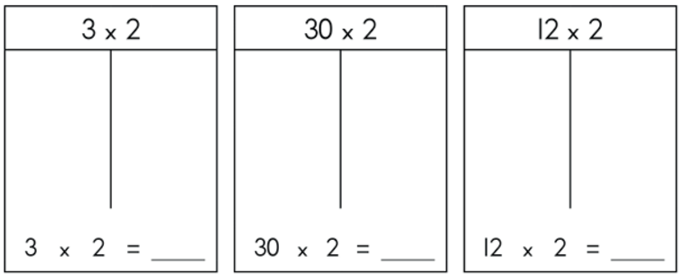
8. What you have learnt about the relationships between the tiles and numbers.

Task 2.4.4: Doubling and halving

Structuring representations

Number pictures: Using a mirror line for doubling and halving

1. Practice doubling, and show your working using structured number pictures.



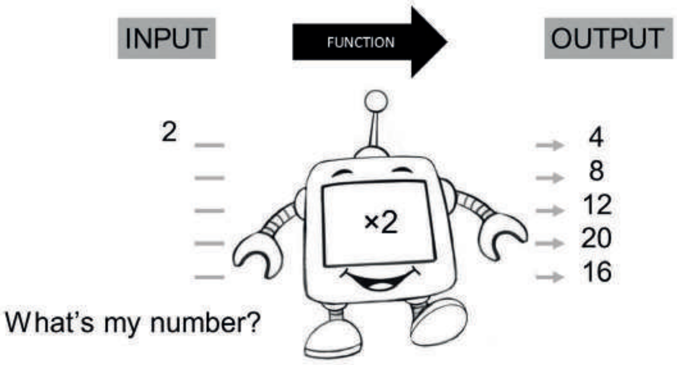
2. What is my number?

   a. I am thinking of a number. Double my number is 4.
   What's my number? ________

   b. Double "my number" is 8. My number is _________.

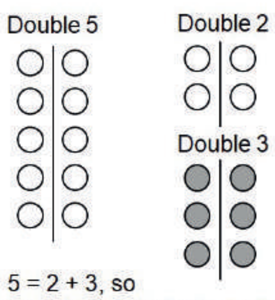
   c. Double "my number" is 12. My number is _________.

   d. Complete the doubling function diagram.



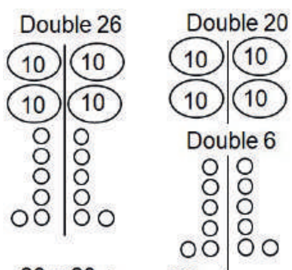
   e. What function is the inverse of doubling? _______________.
   Write the function on funky's screen

   f. Complete



5 = 2 + 3, so
Double 5 = Double 2 + Double ____
5 **× 2** = (___ **× 2**) + (3 **× 2**)

   g. Complete



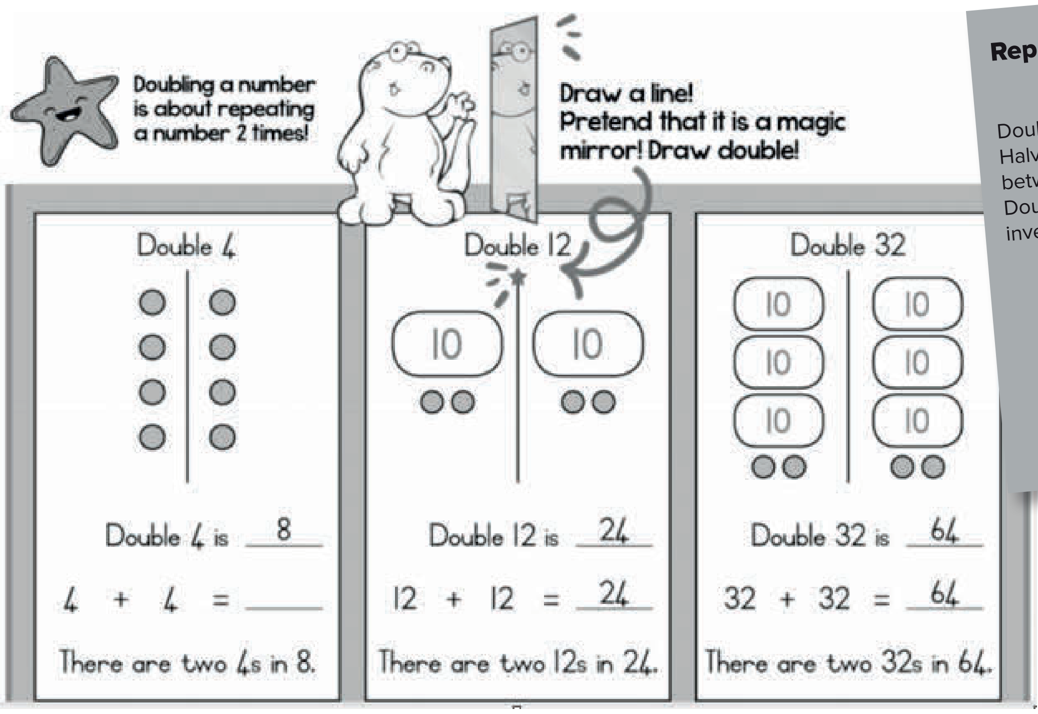
26 = 20 + ___, so
Double 26 = Double 20 + Double ____
____ **× 2** = (20 **× 2**) + (6 **× 2**)

Fluencies

Doubling and halving

Task 2.4.5: Two minute tango

1	Double 2 = ________	11	20 ÷ 2 = ________
2	3 × 2 = ________	12	Double 40 = ________
3	5 repeated 2 times = ____	12	30 × 2 = ________
4	Double 7 = ________	14	Half of 60 = ________
5	9 + 9 = ________	15	50 + ________ = 100
6	9 × 2 = ________	16	20 ÷ 2 = ________
7	6 × 2 = ________	17	Half of 160 = ________
8	Half of 8 = ________	18	________ + 400 = 1 000
9	4 ÷ 2 = ________	19	100 - ________ = 70
10	Half of 16 = ________	20	1 more than 300 is ________



Representations

Doubling uses a mirror line
Halving is sharing equally between 2.
Doubling and halving are inverse functions

If you know your **doubles,** you know your **halves** (they are inverses)

- If **double** 4 = 8, then **half** of 8 = 4
- If **double** 6 = 12, then **half** of 12 = 6, etc.

If you know your doubles, then you can use this for small (1s), medium (10s) and large (100s) songs:

- If double 7 = 14, then **double 7 tens** = **14 tens** = **14**0
- If double 7 = 14, then **double 7 hundred** = **14 hundred** = **14**00

Make connections

If you know one number fact (eg. a double), then use the **small, medium & large number songs**, to work out new facts

SUMMARY

Draw a mind map of the **counting trajectory**. Show all six stages.
Pose some questions that you think will be asked about **concepts** in Quiz 2.4.
What mental **fluencies** will you be tested in in Quiz 2.4?

UNIT 3: The measurement pathway

In unit 2 we focused on the counting pathway. Now, in unit 3, we focus on the other main route to develop children's number sense: the **measurement pathway**.

Key Ideas

The key ideas in Unit 3 are:

1. **Following the measurement pathway children compare continuous quantities that can be measured.**
2. Children learn to **measure by following the same 4 steps**.
3. Children first learn to measure 1s. But very soon they use **standard units using 1s, 10s, 100s and 1000s**

Following the measurement pathway children compare continuous quantities that can be measured

Any number always occurs in a sequence of other numbers. Children learn the counting song as a list of words which is always in the same **order**. They know the sequence "one-two-three-four-five". They start to learn that:

- three comes before four; and that
- three comes after two.

Children make lists and use the ordinal numbers to plan actions: First, second, third.

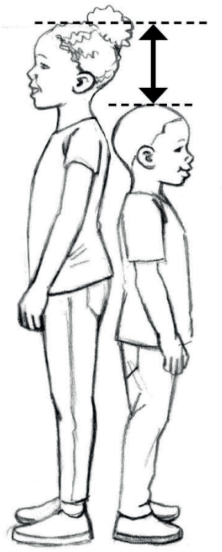
In the **measurement pathway** children reason about, and act on, **continuous quantities.** A continuous quantity is unbroken. Children **measure these quantities using attributes** such as length, height, mass, volume, capacity, time, and temperature. The measurement pathway into number starts with continuous quantities that can be compared directly and later measured.

To measure we introduce a unit (a standard or a non standard unit). The measurement pathway connects number work with spatial reasoning (shape and space, and measurement contexts) in early mathematics.

To compare continuous quantities you have a few options.

First, you could compare the objects directly.

To see which is heavier – an apple or a pencil – they can be com pared directly on a balance scale.



To compare continuous quantities you can compare them directly.

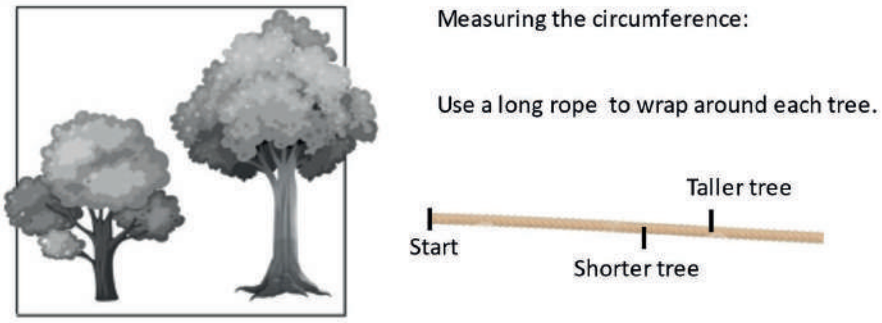
Children are able to order and compare objects or events by particular attributes:

- Tall, taller, tallest.
- Heavy, heavier, heaviest.
- Late, later latest

Height, time and mass are all examples of continuous measurement contexts.

To see who is taller, children stand back to back.

Second, you could choose a measurement unit that is bigger than the attribute.
To compare continuous quantities which cannot moved or lined up, you could use unit which is bigger than each quantity. So comparing the width of circumference of two tree trunks you can use a rope or a string, and mark off the circumference of each tree trunk.



To see whether a cupboard will fit into the space between the bed and wall, you could use a dressing gown cord or belt. Mark the width of the space on the cord and compare it to the width of the cupboard.

Third, you could choose a measurement unit that is smaller than the attribute. You repeat that unit to quantify the attribute**.**

The continuous quantity becomes countable by introducing a **unit of measurement.** At first this unit of measurement of repeated using counting or natural numbers. Very quickly they need to use parts of a unit, and rational numbers are needed.

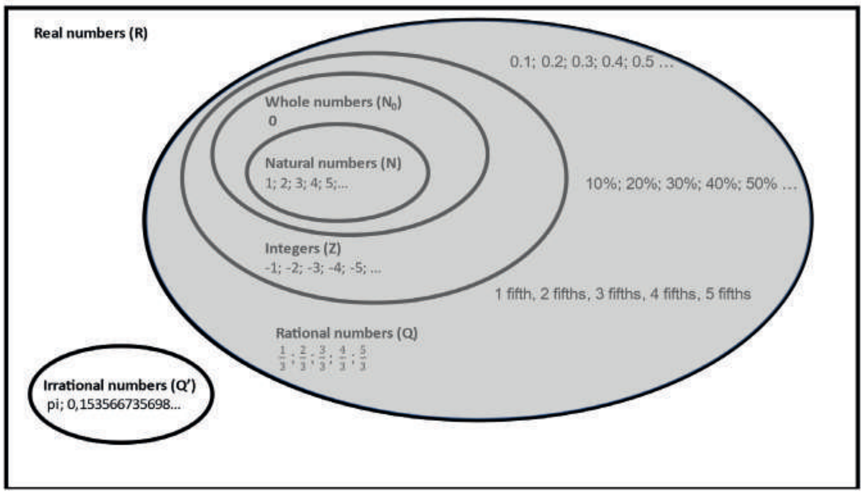
PrimTEd GM2

Knowledge of measurement
Recognise the attribute being measured - idenitfy a suitable unit

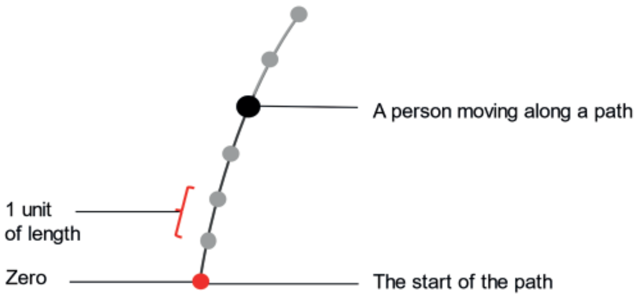
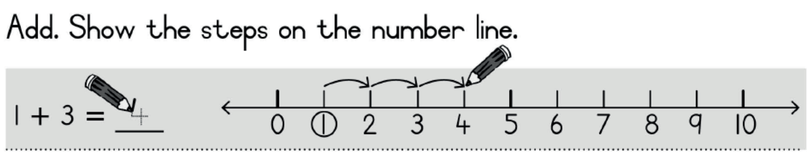
PrimTEd NA2

Knowledge of number systems

Diagram: Number systems



These are the main features of the measurement pathway.

Quantities	**Continuous quantities that can be measured**
Pathway into number sense	Measurement pathway
Unit	Unit of measurement (standard or non-standard)
Metaphor	Movement along a path 
Representations	Number lines 
Number meaning	**Ordinality** Numbers are in a sequence arranged from smallest (first) to biggest (last)

When children measure they always follow the same 4 steps.

Children learn to measure, by **following these steps:**

Diagram: 4 steps for measuring

Step 1
- Choose the attribute

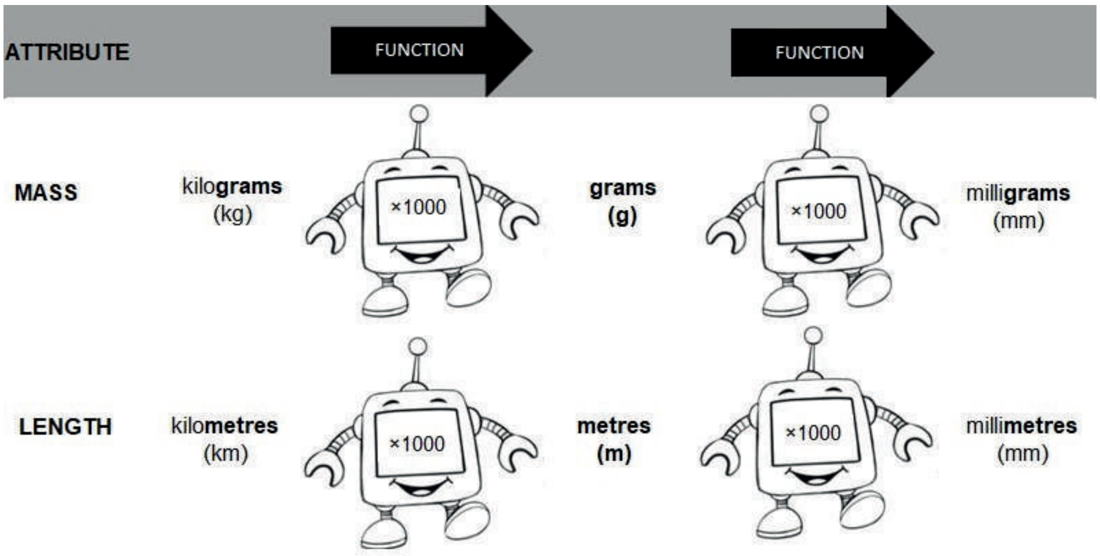
Step 2
- Identify the start and end of the attribute

Step 3
- Choose a measuring unit

Step 4
- Compare the attribute to the measuring units
  - Line up the start the unit with the start of the attribute
  - Repeat the unit
  - Don't leave any gaps or make any overlaps
  - Count how many measuring units are needed to reach the end of the attribute.

Children first learn to measure in 1s. But very soon they use standard units using 1s, 10s, 100s and 1000s

In the measurement pathway children reason about, and act on, **continuous quantities.** They **measure these quantities using attributes** such as length, height, mass, volume, capacity, time, and temperature.



UMTHAMO 3.1

CONTINUOUS MEASUREMENT

Key Ideas

The key ideas in Umthamo 3.1 are

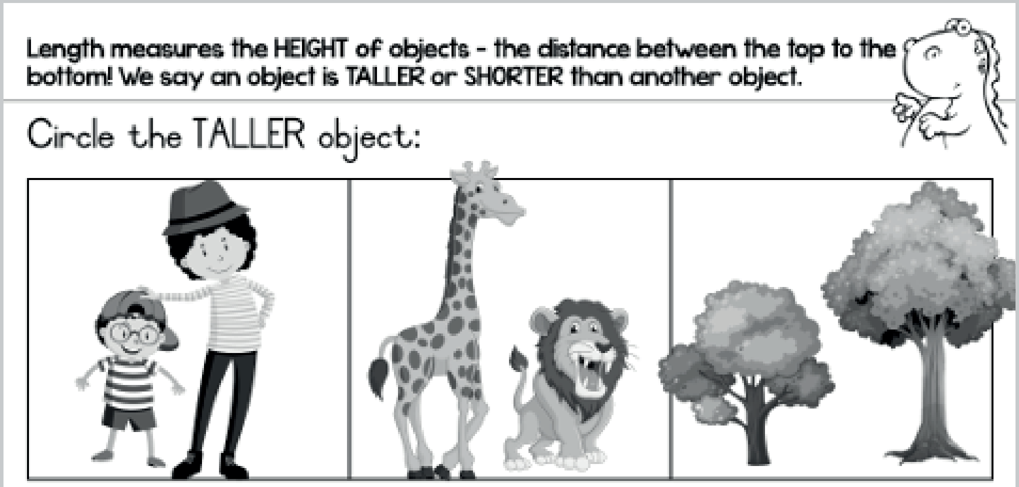
1. **Children experience number** – not just as discrete objects or events – but also **as continuous quantities**
2. **A mental picture for measing a continuous quantity (like length) is a person stepping along a path**
3. The ordinal meaning of number is having the numbers in a sequence
4. Whichever attribute children measure, they always follow the same 4 steps
5. We measure different attributes: **length; time; mass; capacity; volume; temperature**
6. **Maths does not make sense if your only mental picture is measuring and structuring in ones.** To understand measurement you need to understand the base-ten number system of place value and the Standard International (SI) system of measurement (in 1s,10s, 100s, 1000s). For time there is an unusual system of measuring units involving 5s, 12s and 60s.

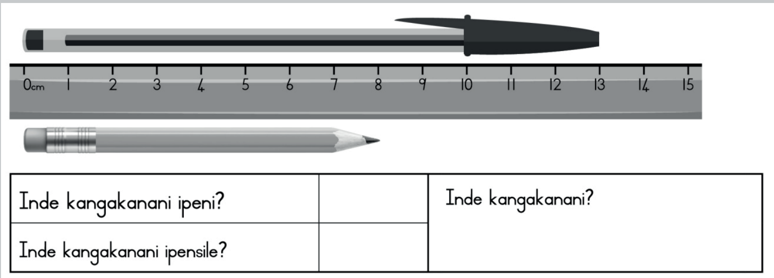
Continuous quantities

The **measurement pathway** into number starts with **continuous quantities** that can be compared directly and later measured. Only later are discrete objects introduced. To measure we introduce a unit (a standard or a non standard unit). The measurement pathway connects number work with spatial reasoning (shape and space, and measurement contexts) in early mathematics.

When children measure a continuous quantity, they must **first decide what attribute should be measured.** They must decide what are they measuring: they are measuring length, they are measuring mass. At first this is about direct comparison. Which object is taller? Which is shorter?.

Activity for learners





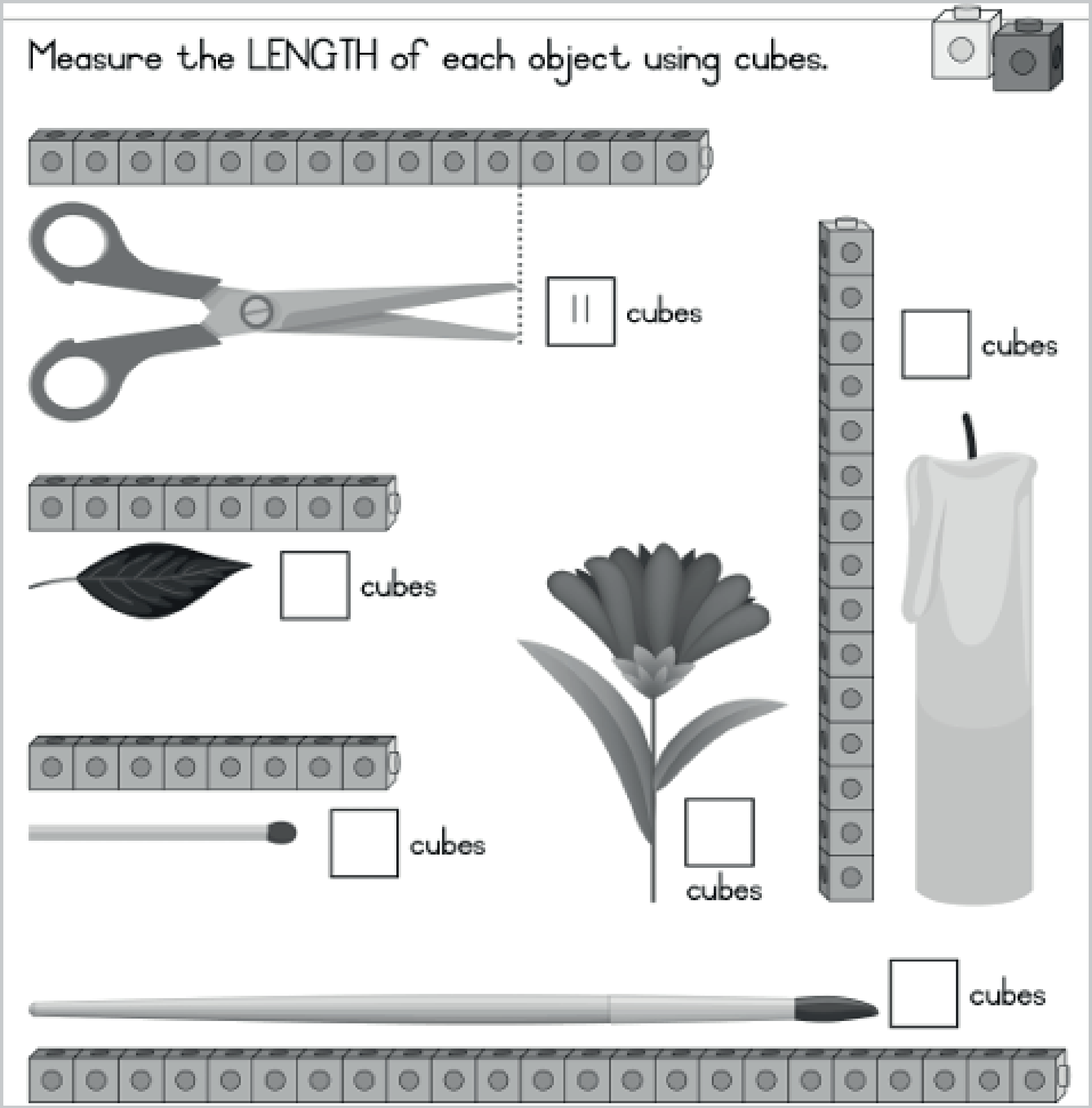
At first chilldren compare without measuring. At school they start using measuring instruments (like a ruler) to compare and measure.

To compare continuous quantities you can **compare them directly**. To see who is taller, children stand back to back.

Continuous quantities are unbroken. They can be measured using a measurement unit. The amount of time, the amount of ground and the amount of water, the volume and the mass are continuous. We can't count time, distance, water, or length. We need to measure using an agreed unit, as these are continuous extents.

To measure a continuous quantity children have to introduce a **measuring unit**. The unit could be standard or non-standard. Repeating the unit allows them to count the units of measurements.

Activity for learners



In this learner activity the measuring unit is a cube. Cubes are smaller than the length of the objects being measured.

A metaphor for continuous measurement: 'Walking along a path'

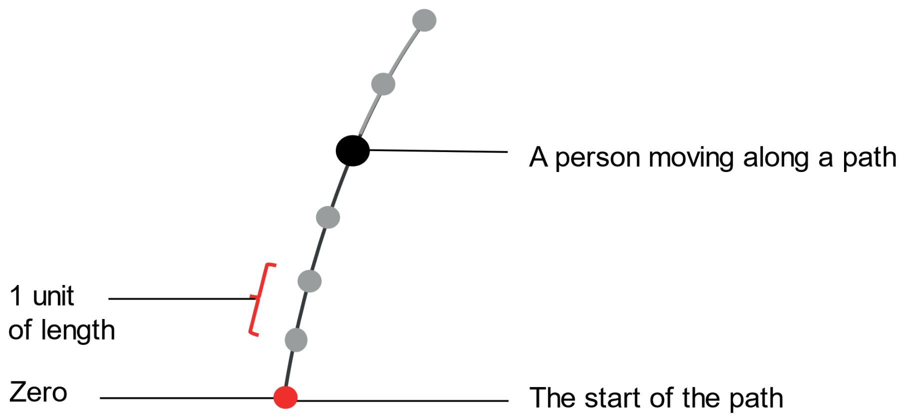
In the measurement pathway, the **metaphor for number as a movement of a point (or object) along a line (or path)** is discussed in detail by Lakoff and Núnes (2001) in their book *Where Mathematics Comes From*.

Diagram: The 'point moving along a path' metaphor for number

Another way that we think about a number, is using the idea of **a path.**

5 is a movement of 5 steps/sections along a line.

Diagram: Number line to show the number five is a continuous length (5 steps)



PrimTEd NA1

Emergent number awareness
Distinguish between types of quantity (discrete and continuous) and forms of quantification (counting and measure)

The ordinal meaning of number in a sequence

We take the 1st step, 2nd step, and then the 3rd step as we move along a path. There is a continuous length of 5 units (steps or metres) from the starting point (zero). The number 5 is not seen in isolation. It is thought about as having a **position (a place in the sequence of numbers).** Five denotes the 5th step or the 5th unit of length. Five comes after 4 and before 6.

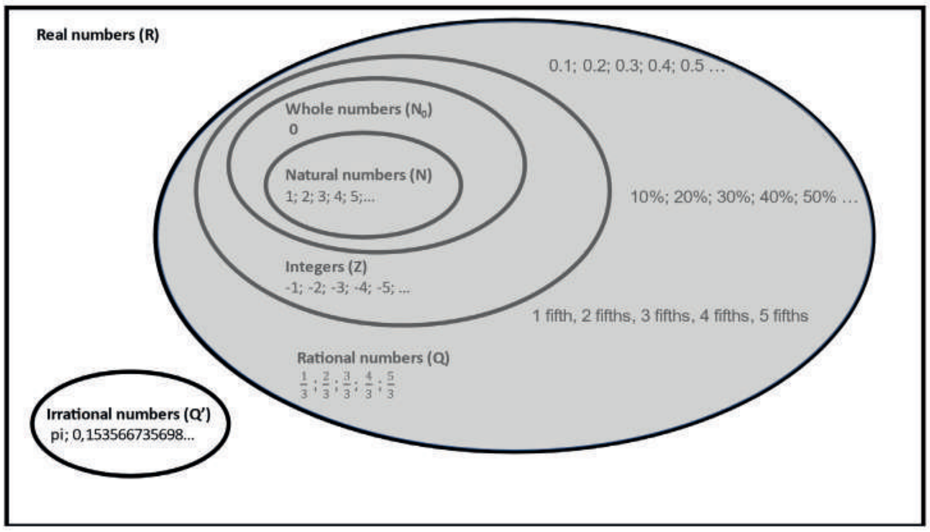
Together with developing language, **babies start to move through their environment:** first crawling or sliding, then walking. To move through the world, the baby uses their **spatial reasoning**. They need to know where they are, and plan a route to get to a new position. Humans learn how to walk and use paths to keep track of their **position relative to fixed points** (like their bed, their home, or the nearest shop). **People can imagine their own movement as if their body is a dot moving along a line**. This is a powerful mathematical idea, and is used in many communities and in further mathematics (such as dynamic geometry and calculus).

The **ordinal** use of number is also evident in the counting song. The number song is first learnt as a **sequence** or **order** of words. The words don't yet have meaning. The 1st word we say is 'one', the 2nd word is 'two', then 'three' and so on. So the number words in the counting song are in a sequence or an order. There is a 1st, 2nd , 3rd and 4th number. The number song is said within the continuous quantity of time, which we can measure. We can say it fast, or we can say it slowly. There is a rhythm or a beat to the number song.

We are using the ordinal use of number when we ask children: 'What number comes **before** 4?' or 'What number comes **after** 4?' So we think about a particular number (like 4) in relation to other numbers. **We think of the number as having a position or a place in the number sequence**. Five means the measurement (often a length) or the sequence of an event (a step along a path). Five comes after 4, and five happens before 6.

To measure we introduce a measurement unit, to make a continuous quantity countable. As we often get partial units, we use the set of **rational** numbers. The symbol for natural numbers is Q. A rational number can be represented as a **fraction**.

Diagram: Number systems



Which ever attribute children measure, they must follow the same 4 steps

Children learn to measure, by **following these steps:**

Diagram: 4 steps for measuring

Step 1
- Choose the attribute

Step 2
- Identify the start and end of the attribute

Step 3
- Choose a measuring unit

Step 4
- Compare the attribute to the measuring units
  - Line up the start the unit with the start of the attribute
  - Repeat the unit
  - Don't leave any gaps or make any overlaps
  - Count how many measuring units are needed to reach the end of the attribute.

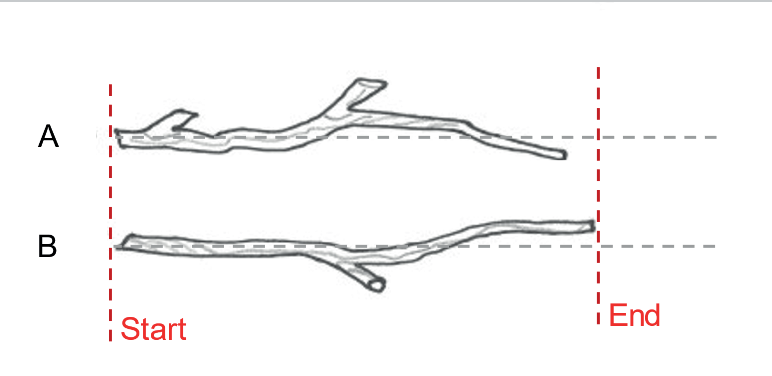
To compare two sticks the childrenhave to decide what aspect of the sticks they are comparing. They need to choose an **attribute** to compare.

- Some children might compare the lengths. Which stick is longer?
- Some children might compare the mass. Which stick is heavier?
- Some children might choose another attribute like – which one is straighter? Which one is smoother or more flexible?

Activity for learners

Let's agree that the children will compare the lengths of the sticks.

Compare the length of two sticks



PrimTEd GM2

Knowledge of measurement
Recognise the attribute being measured.

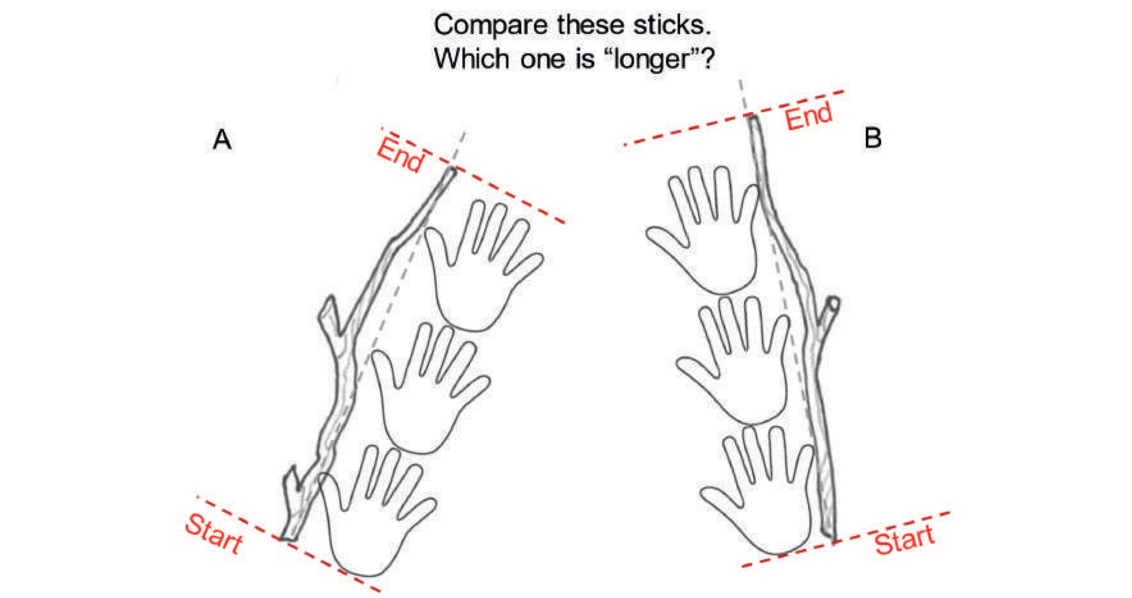
Even very young children can choose the longer stick. The attribute is length. The child must position both sticks at the **same start point**. The sticks must both be straight (so their lengths are parallel). They can see which is longer.

The children are not counting or using number as a collection of discrete objects. But they are **comparing lengths** (using their visual sketch pad). They have to **imagine the start and end lines**. They have to move the sticks into position, and orient the sticks next to each other. Then it is possible to compare lengths.

But what if they could not move the sticks? How could they compare the lengths?

Activity for learners

Compare the length of two sticks with informal units



Perhaps the sticks are on a tree, or stuck in the ground, or drawn on the blackboard. If children can't move the sticks, the children might **introduce a unit to measure with**. The unit might be simple – like **a piece of string that is longer than either stick**. They must agree on where to start measuring – and where the end of the stick is. They line up the string on one stick. Then take the string to the other stick to compare lengths.

They might use a body part – like their arm - to create a unit for length. They line up each stick from their finger tip and see how far the stick goes up their arm.

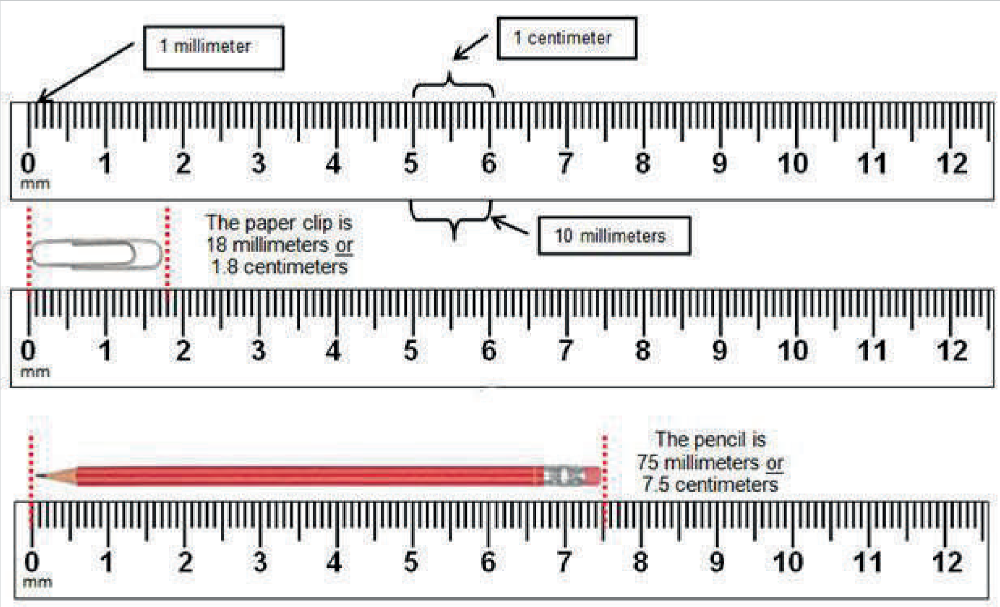
They might use a hand. They then have to carefully be sure to start at each end, and repeat the unit (their hand) with no gaps. They count how many hands each stick is. They might mark off on their hand for how much extra is needed.

Notice that now – when using a small unit like "a hand", **they start using numbers**. The sticks are both: "three-and-a-bit hands". Stick B has a bigger "bit" than Stick A. This is the beginning of fractions!

Measuring different attributes

When children compare and measure continuous quantities, they use the ordinal meaning of number. They choose a measurement unit and repeat it, counting in order. The measurement tools (instruments) use an **ordinal scale**.

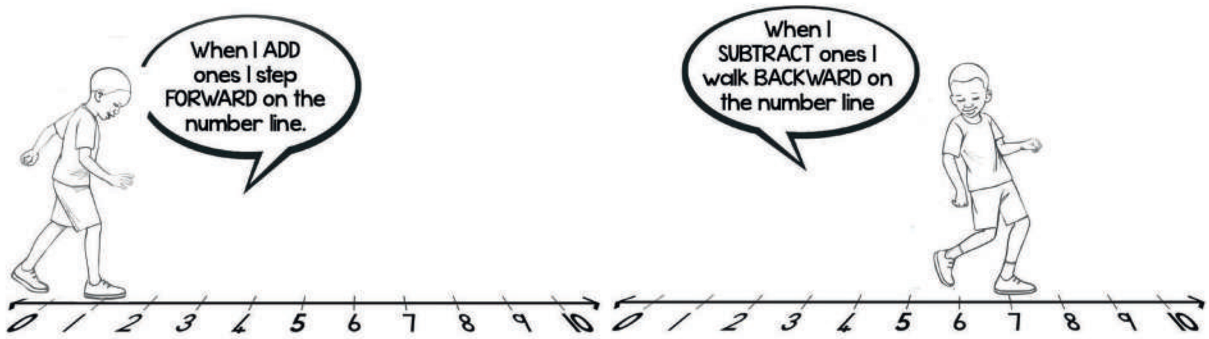
Examples



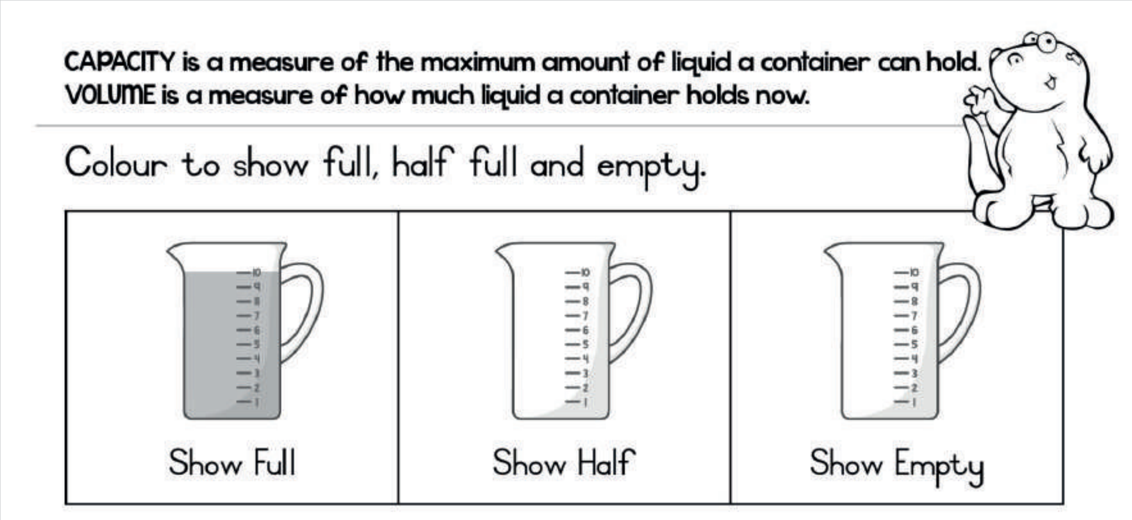
Length and distance are **continuous**. You can't count the length of a pencil. You need to agree on a unit to measure the length of the pencil. You could use an informal unit like a paper clip.

The standard unit for measuring length is a **metre.** A bigger unit of length is a kilometre. Smaller units of length are centimetres and millimetres. We can use a **ruler** to measure length. A ruler is ordinal. A ruler has a structured number line, where

- the numbers are equally spaced (the intervals are equal),
- the numbers are arranged from smallest to biggest,
- it starts at 0.
- the line is used vertically or horizontally.



Examples

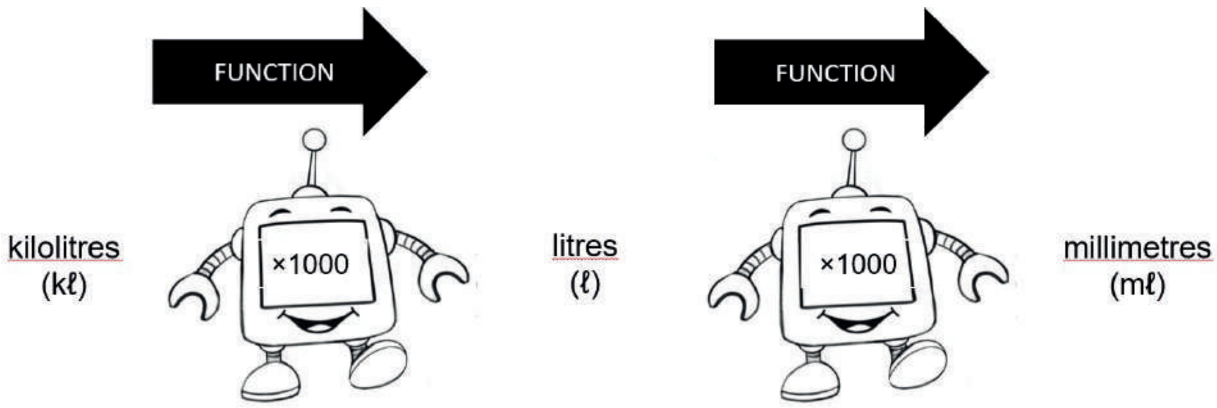


Capacity is how much a container can hold. **Volume** is the amount of liquid in a container. Both capacity and volume are **continuous quantities**.

We often use containers to measure the volume of a liquid. A liquid is continuous. To collect water from a river, you need a container. The amount you collect is measured by the size of container. To quantify the water you choose a unit - 1 **cup**, 1 **bottle** or 1 **bucket** – and repeat that unit. You count how many cups, bottles, or buckets you can fill.

The standard unit for measuring capacity is a **litre**. Bigger units for volume are kilolitres. Smaller units for volume are centilitres and millilitres. We can use a **measuring cylinder or measuring jug** to measure an amount of liquid. A measuring jug has an ordinal scale. A measuring jug has a structured number line, where

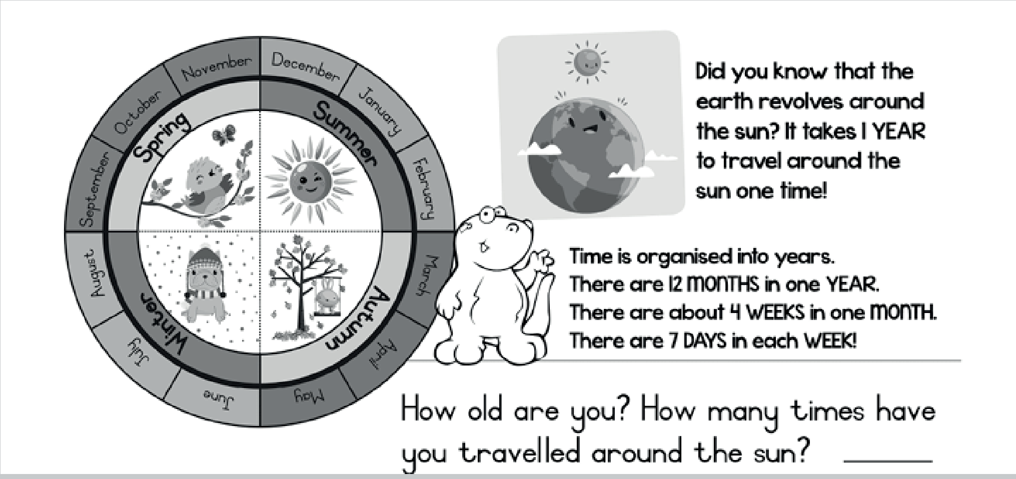
- the numbers are equally spaced (the intervals are equal),
- the numbers are arranged from smallest to biggest,
- it starts at 0; and
- the line is used vertically.



Examples

Time

Time is continuous. We use different measuring units to measure time: seconds, minutes, hours, days, months, years, decades, centuries, millenia...



A calendar is a kind of structured number track where

- the numbers are equally spaced (the days are equal),
- the numbers are arranged from smallest to biggest (the first, second, third of the month),
- the days are placed in rows of 7.
- A number track starts at 1. A number line starts at 0.

January

M	T	W	T	F	S	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

February

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28						

March

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Analogue clocks have a kind of structured number line where

- the hours our equally spaced,
- the hours are arranged from smallest to biggest
- it starts at 0 and ends at 12. It does not label the start, but just the end.
- the line is curved into a circle.

2 o'clock

PrimTEd GM2

Knowledge of measurement
Recognise the attribute -
Identify a suitable unit -
iterate units

SOME IDEAS FOR YOUR CLASSROOM

Use discrete objects and continuous contexts
Remember that measurement is an important context for number work. So when creating learner activities with numbers that involve discrete objects, ask yourself:

- How can I explain this same idea using continuous measurement?
- How can I use length, time, mass, capacity, volume, temperature, or money as the context for this concept, or in this problem?

Ordering numbers on a number line

Children need to be able to **write and say the small number song**. They also need to be able to **order the number symbols**.

The ordinal meaning of number is needed when children answer questions like:

- what number comes **before** 6?
- what number comes **after** 6?

They learn to sequencing numbers with **smallest on the left and biggest on the right**.

Children learn about numbers - not just as collections or sets, but also as having an order. The numbers are in a sequence: smallest on the left and biggest on the right.

Children slowly learnt that:

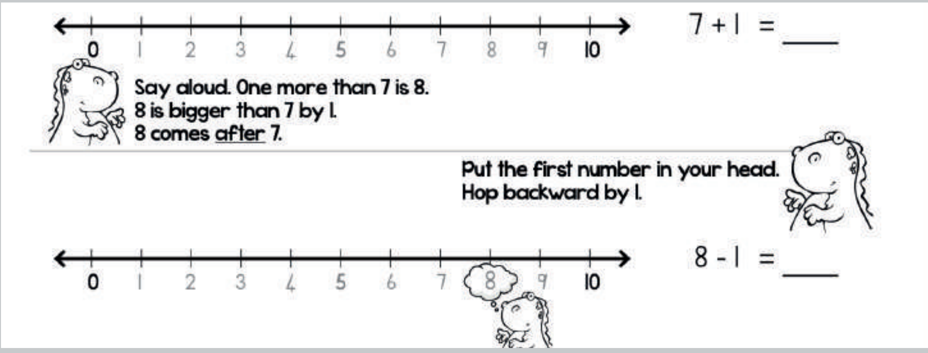
- Getting bigger, is a movement along a line going up.
- Getting bigger is imagined as a movement to right.

Next, they learn that

- Getting smaller, or less as a movement, is a movement going down
- Getting smaller is a movement along a line going to the left.

Activity for learners

Say 1 more than / Say 1 less than



Source: Porteus, et al. (2021) Magic Classroom Collective learner workbook, Nelson

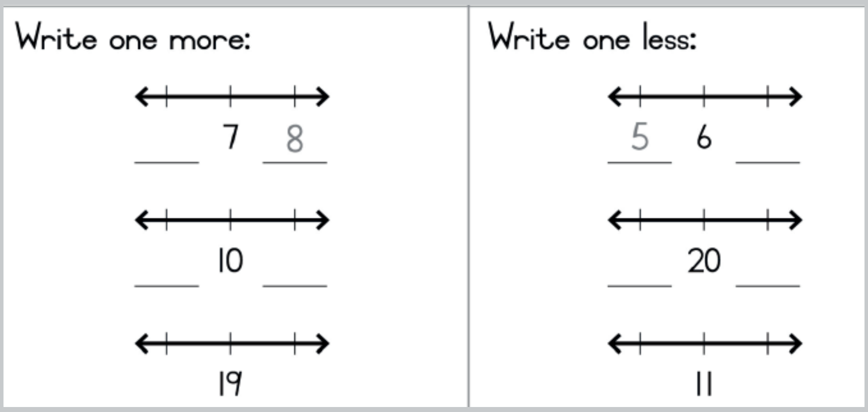
Mandela Institute

Children use the small number song to know which numbers come just before, and which come just after. This is the beginning of addition:
1 more than 7 is 8... because 8 comes after 7 in the number song.
Start at 7. Hop one forwards (to the right)

1 less than 10 is 9 ... because 9 comes before 10 in the number song.
Start at 10. Hop one backwards (to the left).

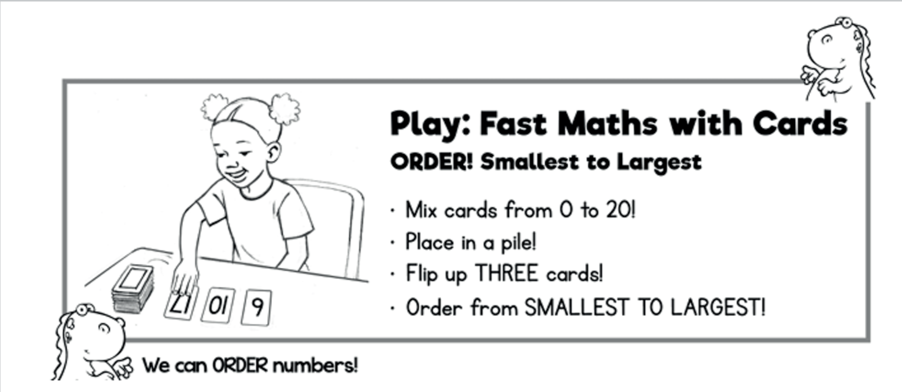
Activity for learners

Write one more / Write one less



Source: Porteus, et al. (2021) Magic Classroom Collective learner workbook, Nelson Mandela Institute

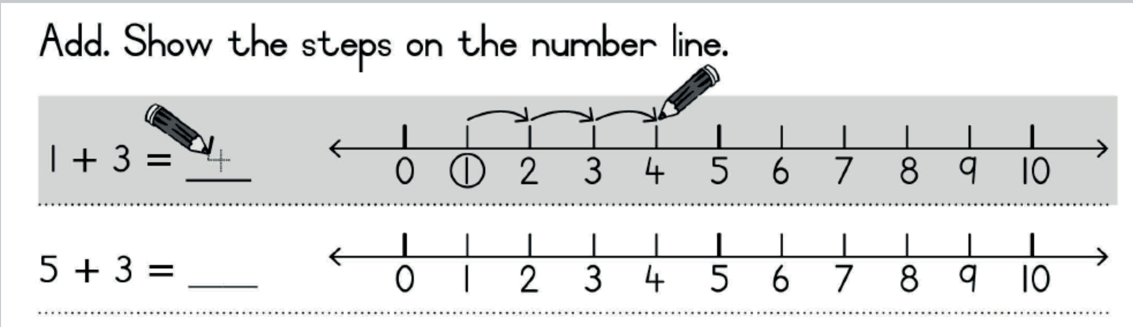
Game



Activity for learners

Adding on a number line

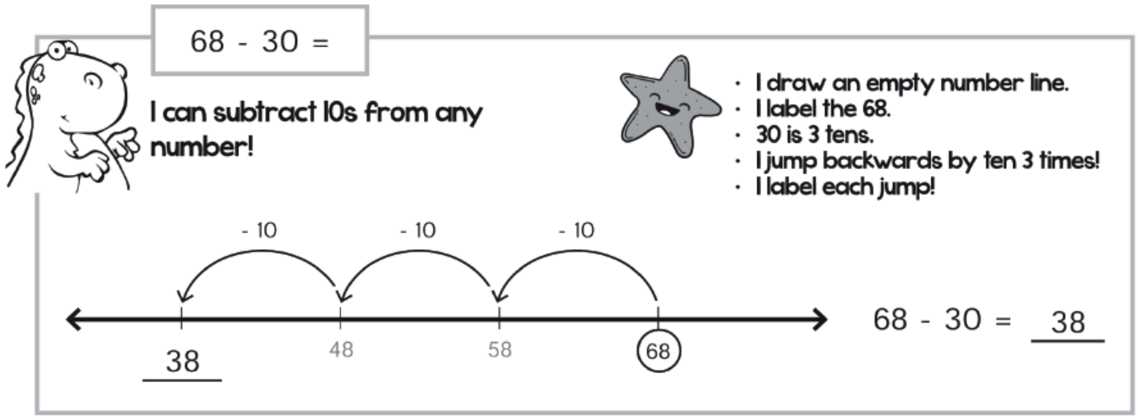
To add, children "put the first number in their head". They circle that number. Then they **hop** in ones to the right. Where they land is the answer.



To subtract, children "put the first number in their head". They circle the first number. Then they **hop** in ones to the left.

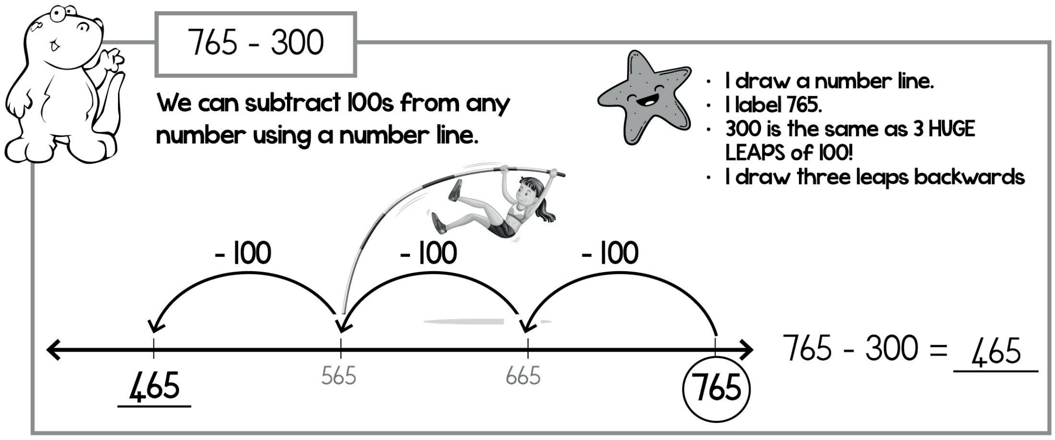
Activity for learners

When children shift from using the small number song, to the medium and large number song they don't hop they **jump**.

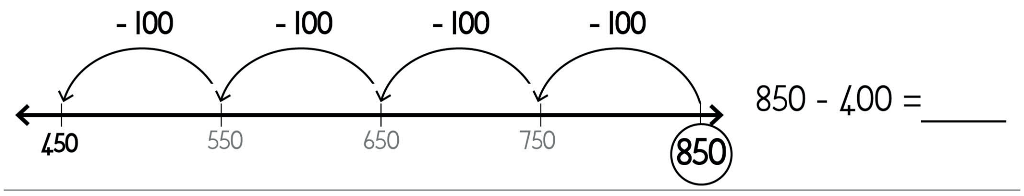


Solve by showing on the number line.

Children use the small number song (hops in 1s), the the medium number song (jumps in 10s) and the large number song (leaps in 100s).



Solve by showing on the number line.



Tasks: Conceptual understanding

Task 3.1.1 Ordinal number names

1. In English we write the word " two" for the cardinal number name for a set of 2 objects. Write the ordinal number name for position number 2. ____________________

2. What is a standard unit for measuring time that is smaller than 1 minute? ____________________.
Notice Q1 and Q2: This is an example where the same English word has two different meanings

3. Write the ordinal number name for position number 3. ____________________

4. When we divide something into 3 equal parts, each part is called 1 __________________.
Notice Q3 and Q4: This is an example where the same English word has two different meanings

5. Write the ordinal number name for position number 9. ____________________

6. Which whole number comes after 6? ____________________

7. Which whole number comes before 6? ____________________

8. Translate question 6 into another South African language.

9. Translate question 7 into another South African language.

10. Complete this table.

Attribute	**Standard unit of measurement**	**Measurement instrument**
Length		Metre stick / Ruler
	gram (g)	
Volume/Capacity		
	Month	
	Second	
		Thermometre

Cinga

In English we can recognise the **ordinal number** names bigger than three, by the suffix "th". In English, there are exceptions:

- The ordinal number name for one is "first" (not oneth),
- The ordinal number name for two is "second" (not twoth)
- The ordinal number name for three is "third" (not threeth).

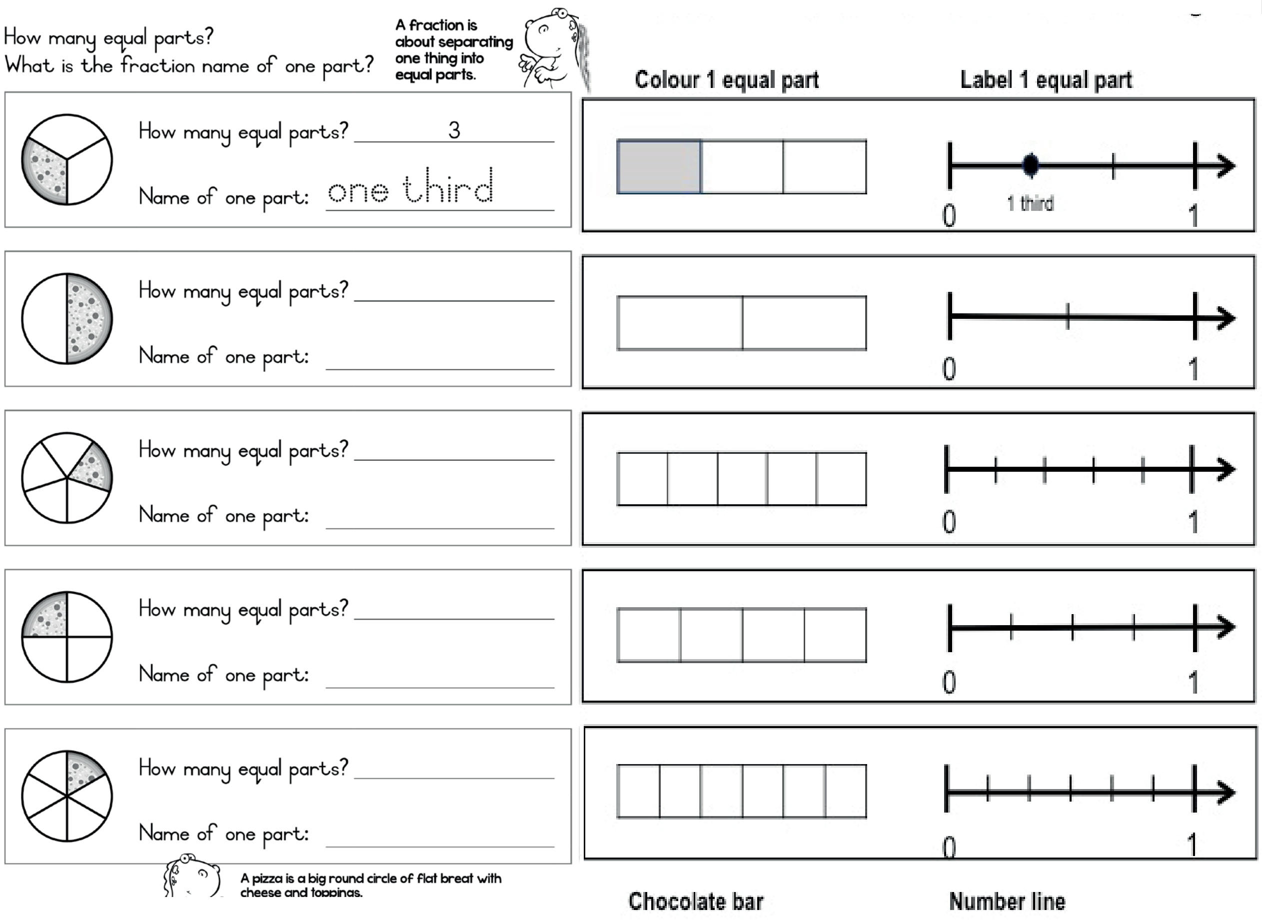
In English, we also use **ordinal number** names to name the fraction parts:

- When we divide something equally into five parts we get fifths: 1 fifth, 2 fifths, 3 fifths, 4 fifths
- When we divide something equally into ten parts we get tenths: 1 tenth, 2 tenths, 3 tenths, etc.

In English, there are exceptions:

- when we divide into 2 equal parts, we get halves (not twoths)
- when we divide into 3 equal parts, we get thirds (not threeths)
- when we divide into 4 equal parts, we get quarters (not fourths)

How are **ordinal numbers** named in other South African African languages?
In isiXhosa, what does the prefix okwe- mean?
In isiXhosa, fractional parts are not named using ordinal number names.


How many equal parts?
What is the fraction name of one part?

A fraction is about separating one thing into equal parts.

Colour 1 equal part | Label 1 equal part

How many equal parts? 3
Name of one part: one third
1 third

How many equal parts?
Name of one part:

How many equal parts?
Name of one part:

How many equal parts?
Name of one part:

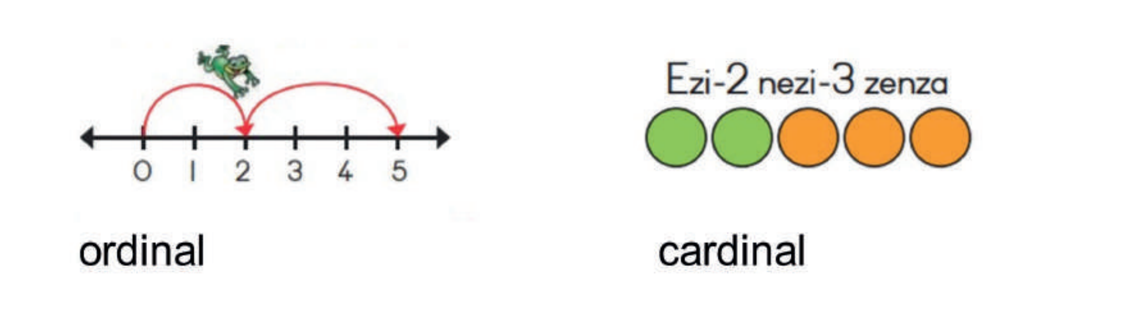
How many equal parts?
Name of one part:

A pizza is a big round circle of flat bread with cheese and toppings.

Chocolate bar | Number line

Task 3.1.2: Cardinal (number pictures) and ordinal (number line) representations

This example shows an ordinal and cardinal way to represent 2 + 3 = 5.



Source: DBE Rainbow book

1. Circle the correct word or phrase.

In the ordinal example:

a. zero is: the **place to start** OR the **empty set.**

b. the numbers are: **poles** or **fences**?

c. The numbers represent: **discrete objects** or **equal jumps along on a line**

2. Circle the correct word or phrase.

In the cardinal example:

a. zero is: the **place to start** OR the **empty set.**

b. the numbers are: **poles** or **fences**?

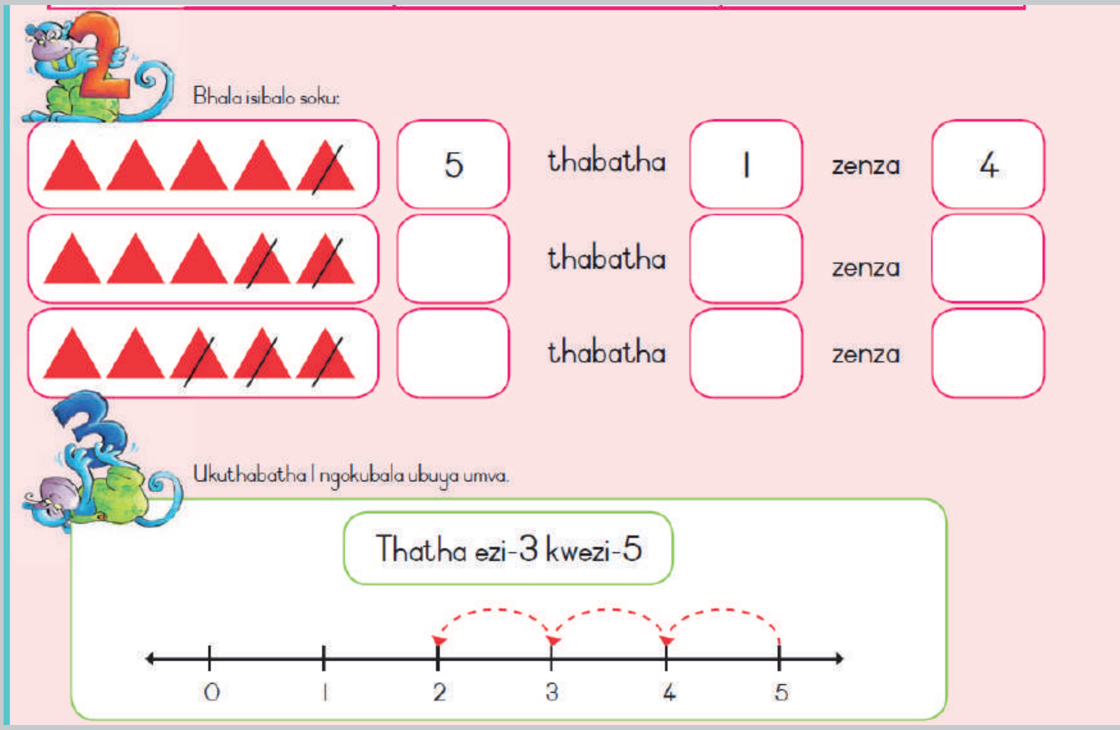
c. the numbers represent: **discrete objects** or **equal jumps along on a line**

3. Draw a number line (ordinal example) for 20 + 30 = 50

4. Draw a number picture for 20 + 30 = 50.

Activity for learners

This is an example of from a grade 1 learner workbook. It shows subtraction as take-away (tabatha)



Source: DBE Rainbow book

Circle the correct word:

5. The triangles an example of <u>cardinal / ordinal</u> representation of number.

6. Is the number line an example of <u>cardinal / ordinal</u> representation of number.

7. When a teacher shows subtractions as discrete objects being removed (or taken away), this is a <u>cardinal / ordinal</u> representation of number.

8. When a teacher shows subtraction as jumping backwards (to the left) on a number line, this is a <u>cardinal / ordinal</u> representation of number.

9. Draw a number picture to show 50 - 30 = 20

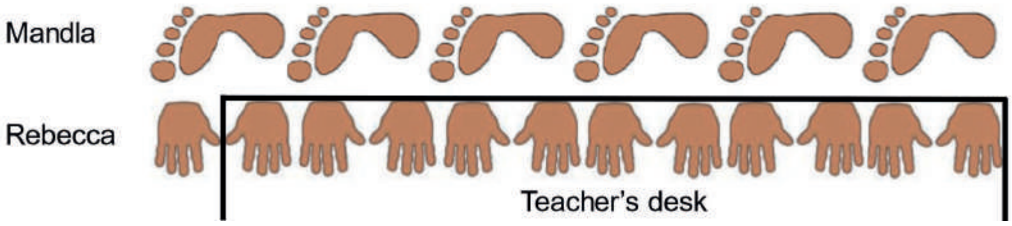
10. Draw a number line to show 50 - 30 = 20.

Task 3.1.3 Measuring using hands and feet

Grade 3s are measuring lengths.

Boys use their feet as a unit.
Girls use their hands as a unit.

Mandla and Rebecca measure the length of teacher's desk

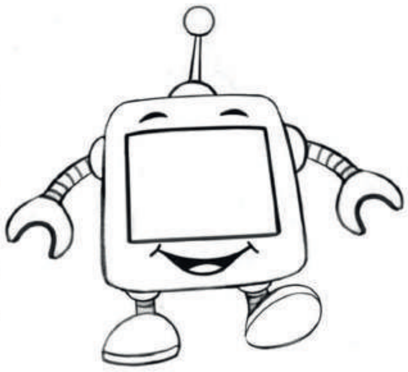


1. Mandla measures the teacher's desk to be ______________ feet long

2. Rebecca measures the teacher's desk to be __________________ hands long.

3. Complete the function, to show the relationship between Mandla and Rebecca's units

INPUT	FUNCTION	OUTPUT
Mandla's feet		Rebecca's hands
1	→	2
2	→	_____
3	→	_____
5	→	_____
10	→	_____
100	→	_____



PrimTEd NA7

Overaching mathematical ideas
Understand and apply knowledge of relations and functions

Cinga

In algebra we don't write × **for multiplication**.
We use brackets, or if it is variables and number, then we use no sign.

- 5 times 6 is written 5(6)
- 5 times f is written 5f.
- m times n is written mn

4. Complete the different ways of writing the relationship between the feet and hands:

Half the number of hands = the number of ___________

Number of ___________ × 2 = number of ___________

Let the number of hands be h. Let the number of feet be f. Then,

h = _________

5. What about the inverse function. If we know the number of Rebecca's hands, how do we work out the number of Mandla's feet? Write a number sentence to show the relationship between Mandla's feet and Rebecca's hands.

No. of feet = ______________________

f = ______________________

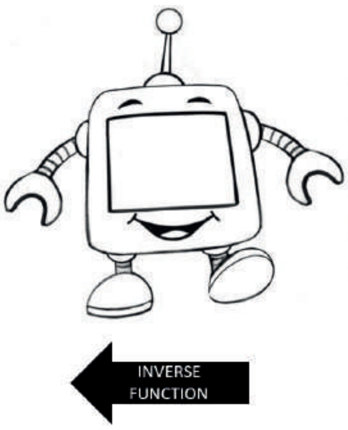
Cinga

In algebra we don't write ÷ **for division**, we use a fraction
We use fraction notation.

- 6 divided by 2 written $\frac{6}{2}$
- Half my number is 6. What's my number?
- Half my "my number" is written: $\frac{1}{2}$ my number
- f divided by 2 is written: $\frac{f}{2}$
- m divided n is written: $\frac{m}{n}$

6. Complete the function machine. Show how to get from Rebecca;s hands to Mandla's feet.

OUTPUT Mandla's feet	INPUT Rebecca's hands
_____	2
_____	4
_____	6
_____	10
_____	100
_____	1000



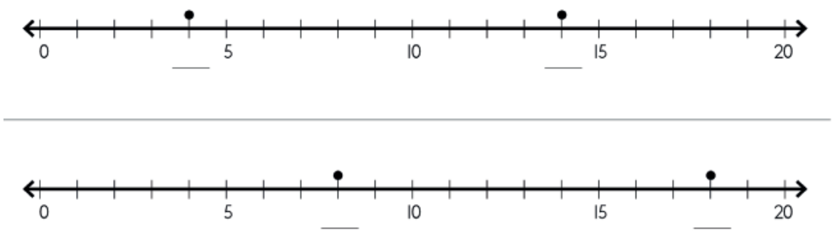
Tasks: Procedural fluencies and structuring

Task 3.1.4: Estimate 0-100 on number lines

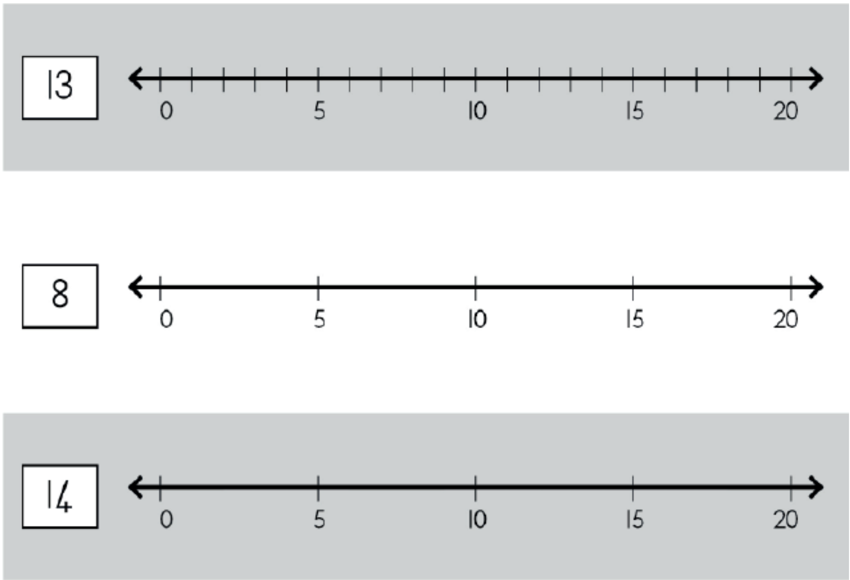
Structuring representations

Structured number lines.

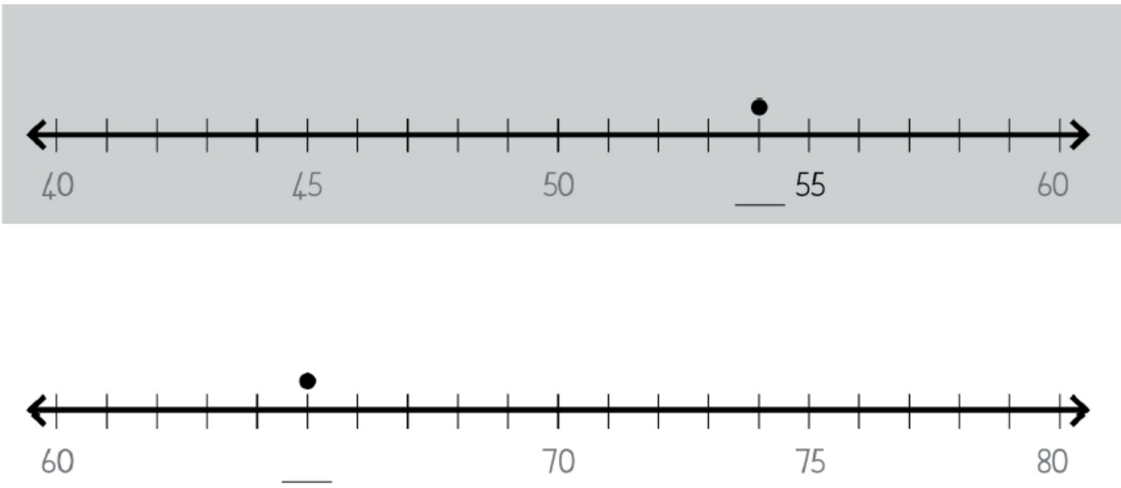
1. Write the numbers below each dot



2. Draw a dot and write the number below the line.



3. Write the number below the dot

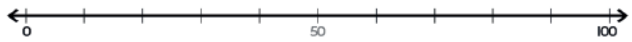


PrimTEd GM2

Knowledge of measurement
Estimate

4. Draw a dot and write the number on the line.

20



70

40

5. Look carefully at the start and end of each number line. What is the size of the interval?. Mark and label half way, for each number line.

Now, draw a dot for 57 and write 57. Look carefully at each line.

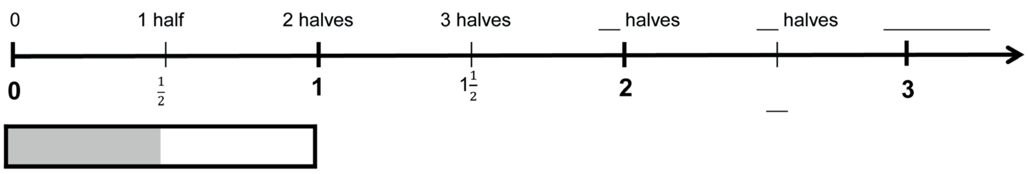
50 60

0 100

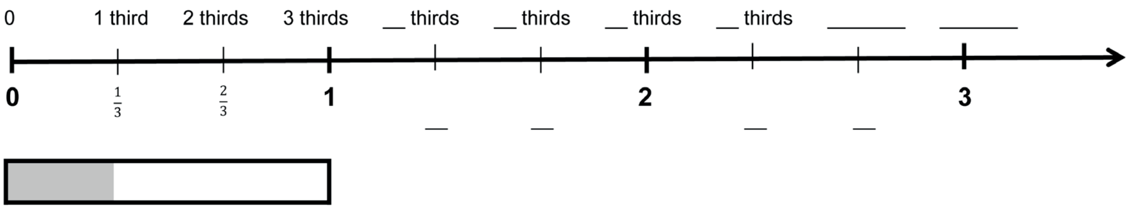
40 60

30 60

6. How many equal parts in 1? ________. Name of one part:__________



7. How many equal parts in 1? ________. Name of one part:__________



Fluencies

Multiply and divide by 2, by 10; by 100
To multiply a big number, you break it up and multiply each part
To divide a big number, you break it up and divide each part

Task 3.1.5: Two minute tango

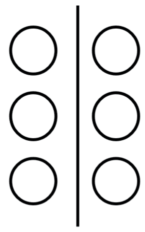
1	5 repeated 10 times = _____	11	72 ; 71 ; 70 ; ____; ...
2	50 × 10 = ____	12	372 ; 362 ; ____; 342 ; ...
3	500 multiplied by _____ = 500	12	If I decrease 500 by 1, I get ____
4	Double 18 = ______	14	1892 + 100 = ____
5	Double _____ = 18	15	(7×10) + (2×10) = ____×10
6	800 shared equally between 8 = _____	16	(60×2) + (____×2) = 63×2
7	80 ÷ ____ = 8	17	____ + 2 = 20
8	_____ ÷ 10 = 80	18	____ + 20 = 200
9	Half of 36 = _____	19	41 + ____ = 50
10	Half of 360 = _____	20	____ - 9 = 500

Multiplication involves choosing a unit and repeating it, to find how many there are.

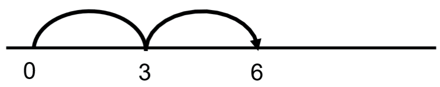
- For double 3, the unit is 3. It is repeated 2 times. The result is 6.
- For triple 2, the unit is 2. It is repeated 3 times. The result is 6.

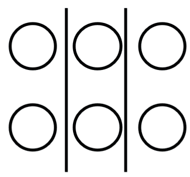
Representations

Multiplication as **repeating equal groups** in a **number picture**.
Multiplication as **repeating equal jumps** on a **number line**

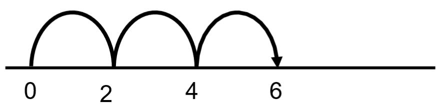


Double 3
3 repeated 2 times
3 **× 2** = 6
Jump 3. Repeat 2 times





Triple 2
2 repeated 3 times
2 **× 3** = 6
Jump 2. Repeat 3 times



SUMMARY

There are 2 pathways to learn about number.

Complete the table.

Quantities	Discrete quantities that can be counted	Continuous quantities that can be measured
Pathway into number sense		
Unit		
Metaphor		
Representations		
Number meaning		

Which **concepts** will you be tested on in Quiz 3.1?

What **mental fluencies** should you be practicing for Quiz 3.1?

UMTHAMO 3.2

ORDINALITY

Key Ideas

1. Any number always occurs in a sequence of other numbers.
2. Continuous quantities are measured using marks (poles) to show equal units of length (fences) along a line.
3. The ordinal meaning of number describes a position along a line or a position in time.
4. Children move along a learning trajectory from structured number tracks to empty number lines.

Any number always occurs in a sequence of other numbers.

Any number always occurs in a **sequence of other numbers**. Children learn the counting sequence as an ordered list of words. They know the sequence “one-two-three-four-five”. They start to learn that:

- three comes **before** four; and that
- three comes **after** two.

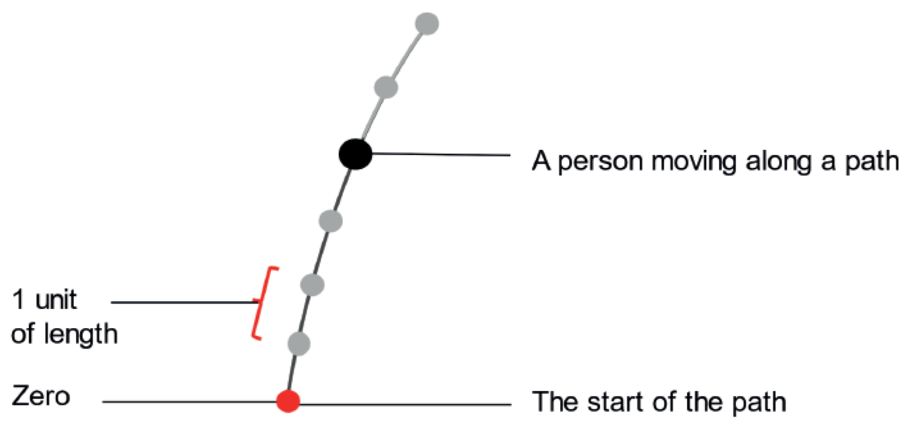
They are able to **order and compare** objects by particular attributes:

- Tall, taller, tallest.
- Slow, slower slowest.
- Heavy, heavier, heaviest.

These are all **continuous measurement** contexts.
the measurement can be: mass, length, capacity, volume, time, or temperature.

Children make lists and use the ordinal numbers to plan actions: First, second, third.
This is the **ordinal meaning** of number. A number describes a sequence: 1st, 2nd, 3rd,...

Diagram: A number line showing 4 steps along a path



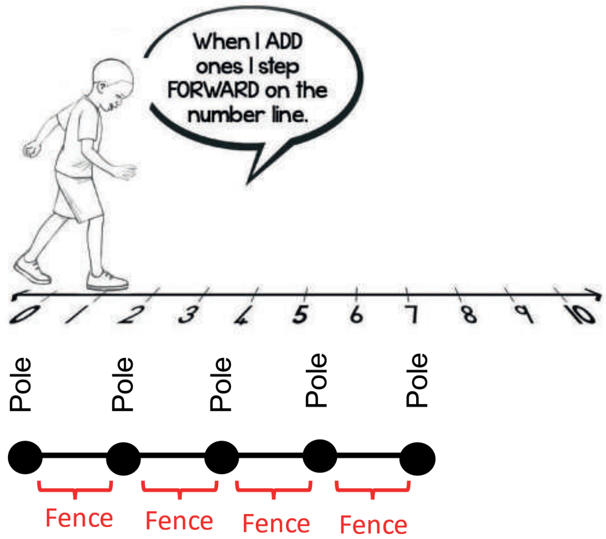
This is the **ordinal use of number**: the 1st step, 2nd step, 3rd step as we move along a path.

Continuous quantities are measured using marks (poles) to show equal units of length (fences) along a line.

The number line is a sequence of numbers placed in order along a horizontal or vertical line. Numbers are positioned or placed relative to other numbers. Calculations are done by **hopping (in ones)** or **jumping (in bigger units**). Here **movement** - jumping forwards (to the right) or jumping backwards (to the left) is used for calculations.

When children first start using numbers, they often **confuse the cardinal and ordinal meanings.** This is especially the case when they start using number lines.
If they only know the cardinal meaning of numbers, then using a number line is difficult.
Using the idea of fences and poles can help remove this confusion.

Diagram: Fences and poles



The **poles** are the equally spaced markers on the number line, for each length. The first pole is 0. The second pole is 1.
The **fence** is the 'interval' between the markers on a number line. It is a measure of length. Each section of fence is the same length.

We measure continuous quantities by introducing a measuring unit and repeating that unit. With formal measurements we use an **ordinal scale** (which is a structured number line). **The spatial** idea of numbers being in an order, and arranged from smallest to biggest or numbers as moving along a line is a very important way of thinking about and using numbers.

Let's talk maths

Write down and discuss the isiXhosa words and phrases used to describe
an interval (fence) and
a marking (pole)?

What are the various ways to describe zero? (in English and in isiXhosa). Do any of the words or phrases denote an empty set (absense) or a starting point?

The ordinal meaning of number describes a position along a line, or a position in time.

You have learnt that people think about and use numbers to quantify collections of discrete objects and use the idea of a container, to group those objects. This is the **cardinal meaning** of number.

But people also think about and use numbers as being in an order or as a sequence. This is the **ordinal meaning** of number.

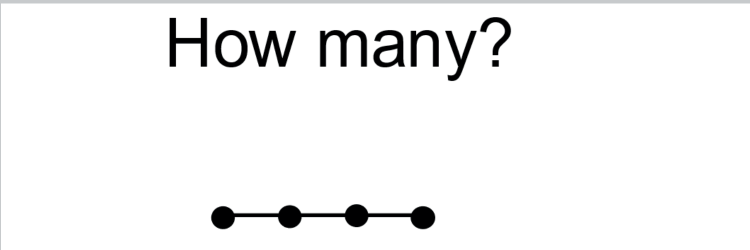
The sequence of numbers can be arranged in space. There is a **start**. They are ordered from left to right, or from bottom to top. This is using the **bodily position sense** (to know left versus right and bottom versus top). It is using the **sight sense** and **visual sketchpad** to imagine **a line or path.**

The sequence of numbers can said as a string of words, or a number song, forwards or backwards. The words are said over time, and so a smaller number is said before a larger number. The metaphor of stepping along a path creates the dynamic idea of movement over time. Children can image moving their body through space and walking along a path.

SOME IDEAS FOR YOUR CLASSROOM

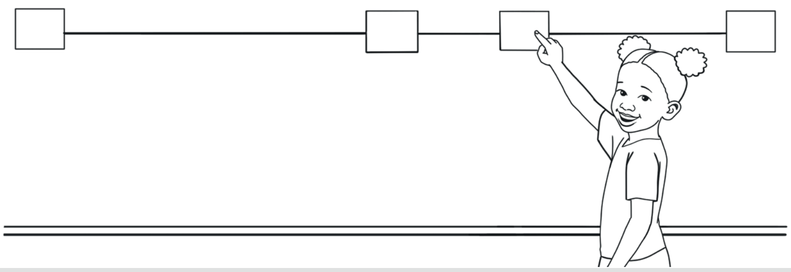
Activity for learners

Fences and poles



A teacher draws four dots that are equally spaced along a line. She asks: how many are there? Alan says 3, and Vusi says 4. Discuss.

Is Alan counting fences or poles?
Is Vusi counting fences or poles?



Children should practice placing numbers in order on a number line . 0 on the left. 10 on the right.

PrimTEd GM2

Knowledge of measurement
Recognise the attribute - identify the unit - iterate the unit

Teachers should make children aware that **the number line starts at 0** (at the pole). When we count on a number line we count the **fences**.

We label the fences at the pole to the right of the fence.

The first fence needs both of its two poles to hold it up. If the pole at position 0 is removed, then the fence will fall down.

This number line is structured in ones. There are 3 important features of all structured number lines:

- They all start at 0.
- Each interval (fence) is the same size. In this case each fence is 'one' long)
- Horizontal number lines have the smallest number on the left, and biggest on the right.
- Vertical number lines have the smallest number at the bottom, and biggest number at the top.

Children move along a learning trajectory from structured number tracks to empty number lines.

From number tracks to structured number lines to empty number lines
Length is key representation for children. It uses the metaphor of a path. The path could be vertical (from top to bottom) or horizontal (from left to right).

At first children use a structured number track for the whole numbers 0 to 10. Then then use a structured number line from 0 to 10. Over time children get used to working with less structured number lines. They should stop counting in ones. They stop hopping in ones, and make big jumps.

<u>Diagram: A learning trajectory for number lines</u>

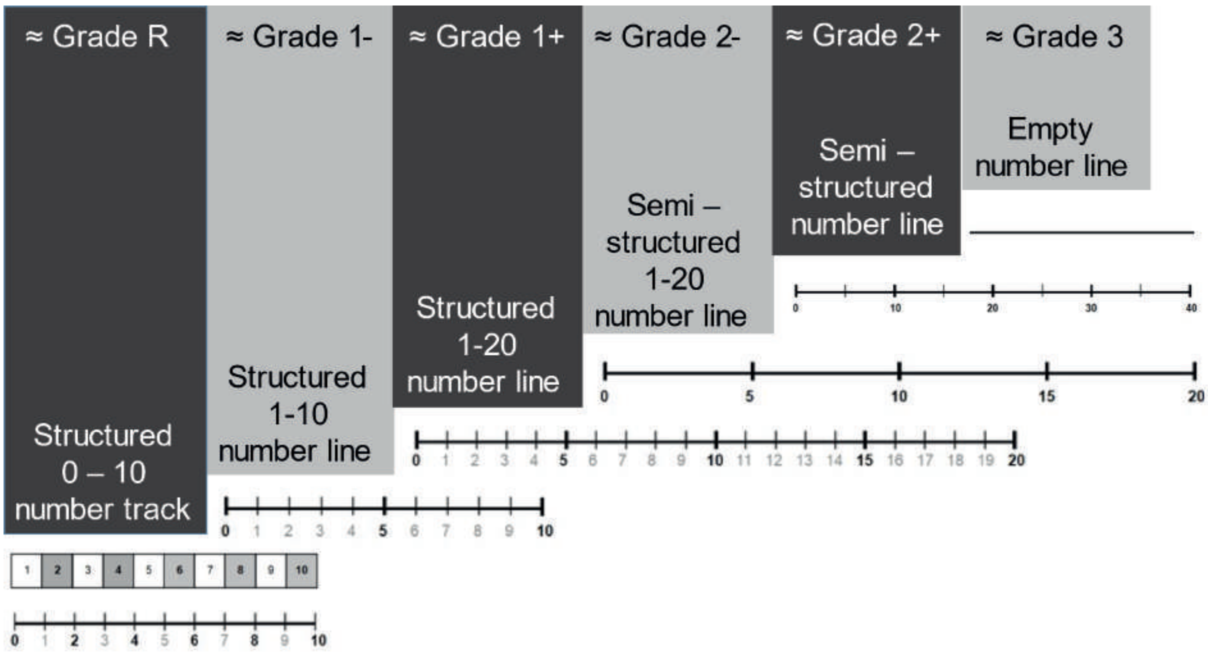
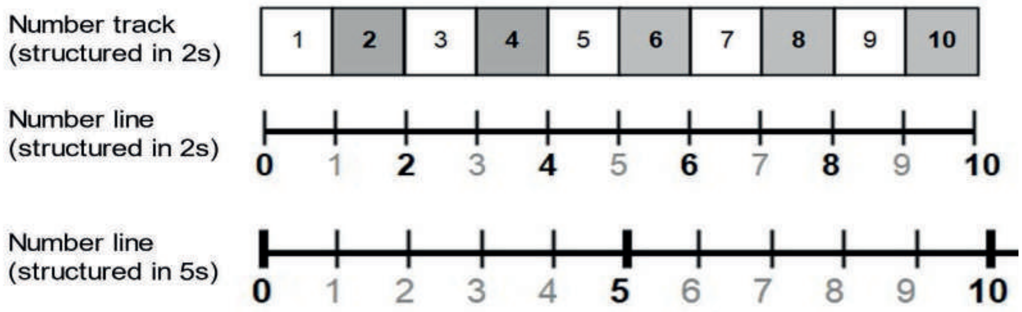


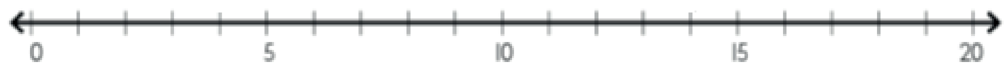
Diagram: Number tracks to number lines



Introduce number lines as **continuous measurement starting at 0**. You are walking or hopping away from the starting point (0.). Each step is an interval (fence). The markings (poles show where your foot lands.

Number lines can be **structured in 1s, 2s and in 5s**. In grades R and 1 children work on number lines that are given to them. By grade 2 they start working with semi-structured number lines.

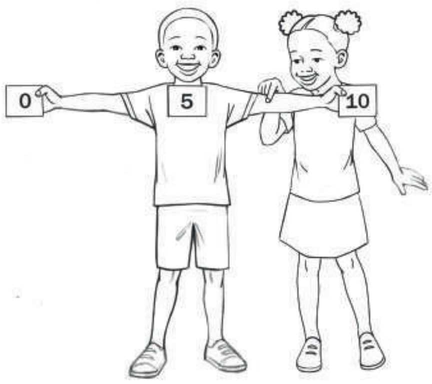
This is a number line structured in 5s. It is semi-structured as not all the poles are labelled.



This is a number line structured in 10s. It is semi-structured as not all the poles are labelled.

The number line is structured in 10s. The measuring unit is 10s, not 1s.

By about grade 3 (but often earlier) children start to draw their own number lines. By this stage, children are using numberlines as tool to track their thinking. Their own number lines do not need to be neat and precise. They do not need to start at 0.



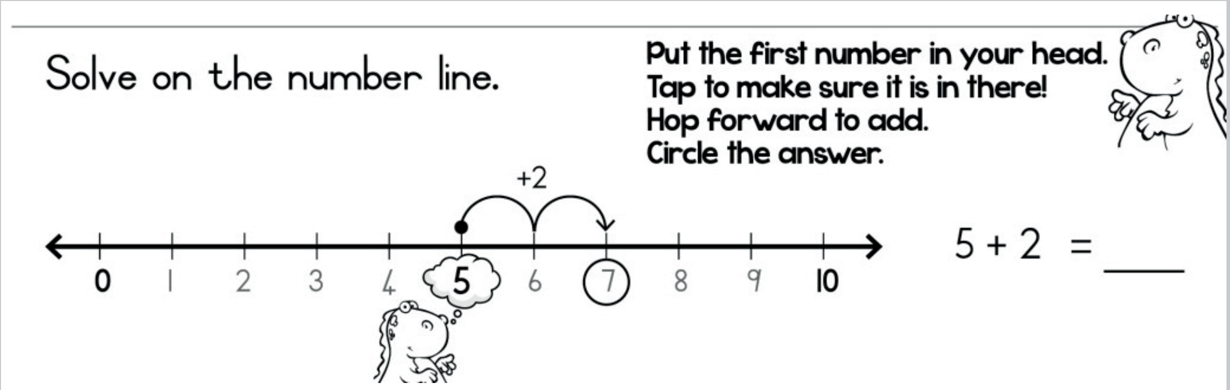
Chidlren can use their bodies to create number lines. 0 is on the left. 10 is on the right. 5 is in the middle.

PrimTEd GM2

Knowledge of measurement
2.6 Integrate knowledge of number with knowledge of measurement

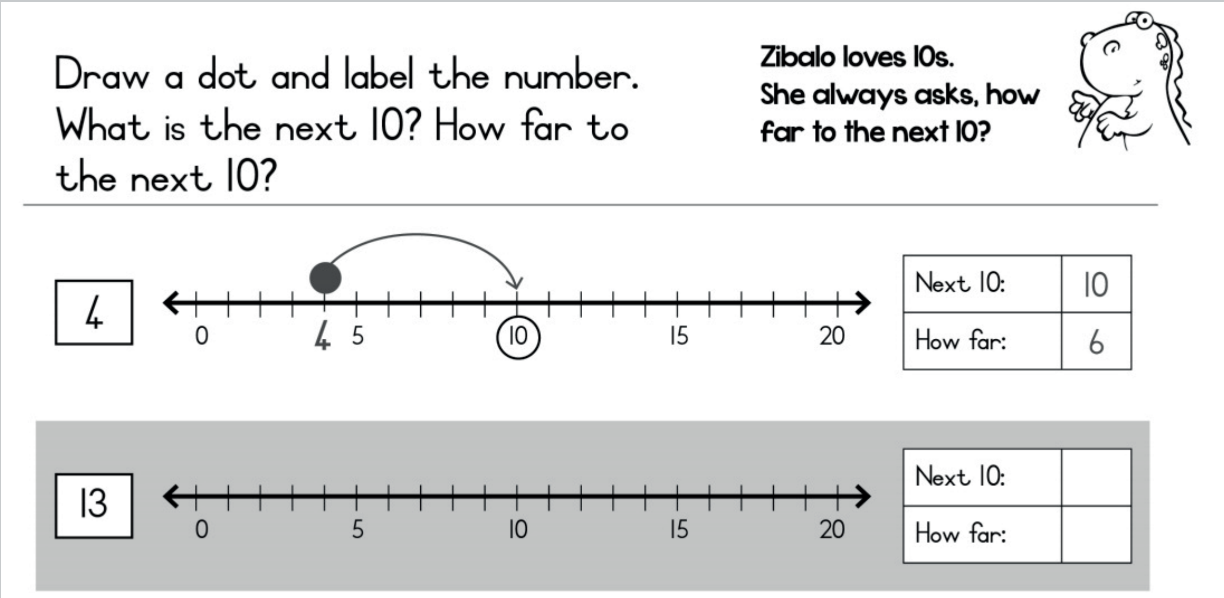
-✂-- Activity for learners

The number line in this activity is structured in 1s.



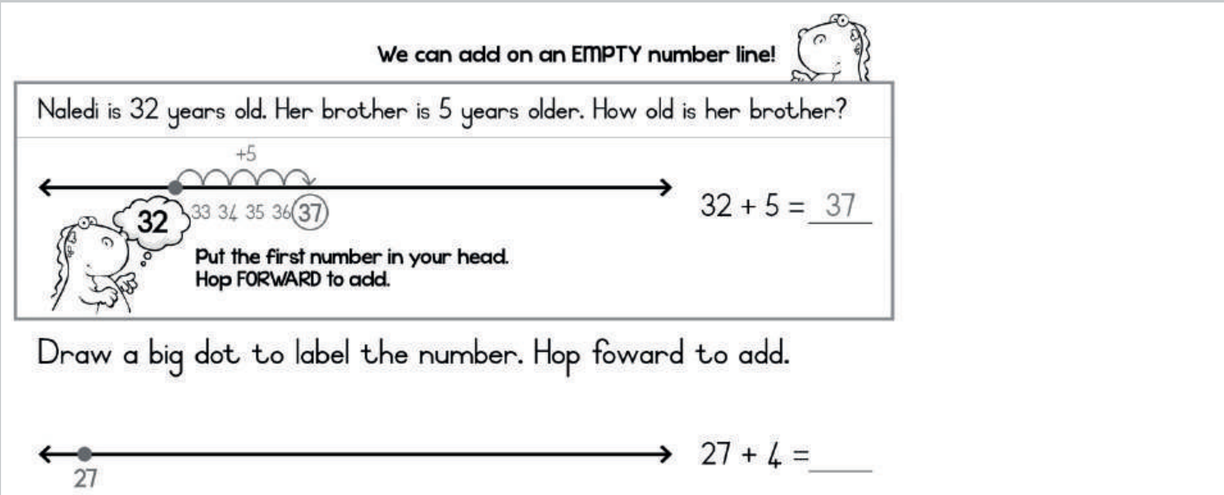
-✂-- Activity for learners

The number line in this activity is semi-structured in 5s.



-✂-- Activity for learners

The number line in this activity is empty.



Tasks: Conceptual understanding

Task 3.2.1 Investigate bottles and litres (use a number line)

1. Count in 2s, on the multiples of 2.

Count by 2s from 2 to 100.
Colour each block.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Count forwards by 2s.

2	4	6				

22	24	26				

Count backwards by 2s.

14	12	10				

20	18	16				

2. How many bottles, how many litres of coke? (Specialise for a particular example)

How many bottles?	
How many litres?	

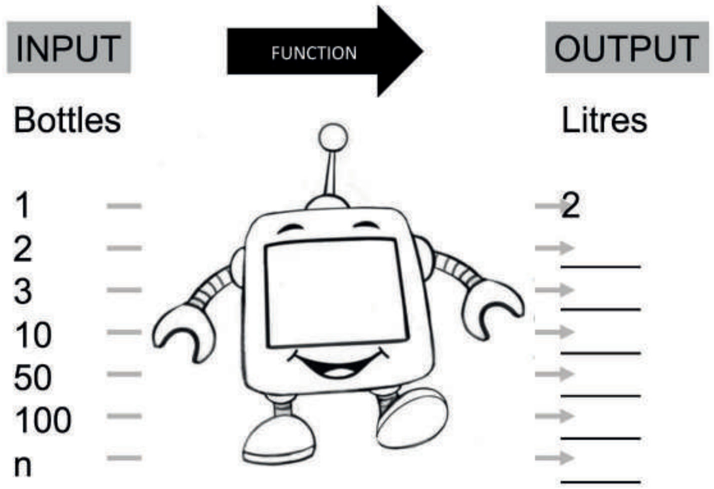
3. Complete the number line with equal jumps of 2 each time (Organise on a number line.)

4. What is the relationship between bottles and litres? (Generalise)

If I know how many bottles I must _______ to get the number of litres.'

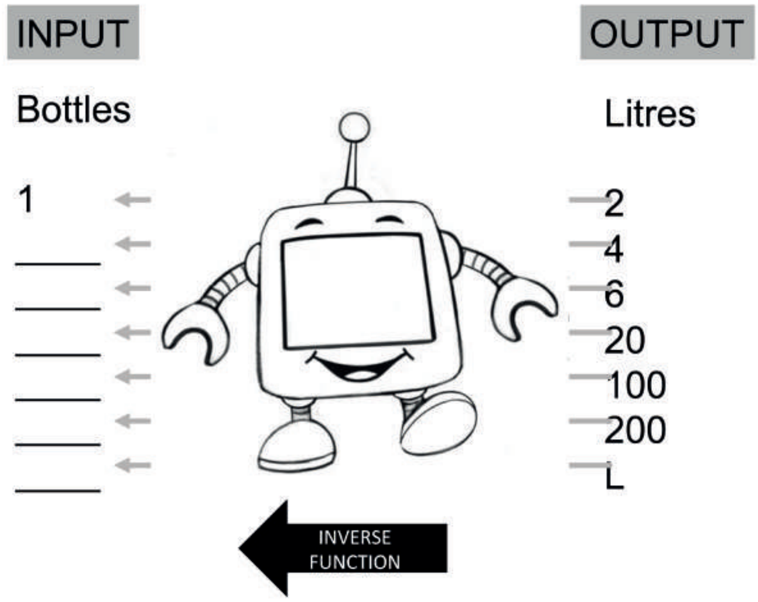
If I know how many litres, I must ________ to get the number of bottles.

5. Organise using a function machine or a flow diagram



6. The number sentence or rule, that relates the number of **litres** to the number of **bottles** is:

7. Complete the inverse function



8. The number sentence or rule, that relates the number of bottles to the number of litres is:

No. of bottles =

Let the no. of litres be L, and the no. of bottles be n.

n =

Cinga

In algebra we don't write ÷ **for division**, we use a fraction

We use fraction notation.

6 divided by 2 is written $\frac{6}{2}$

Half my number is 6. What's my number?

Half my number is written ½ my number.

f divided by 2 is written: $\frac{f}{2}$

m divided n is written: $\frac{m}{n}$

9. Label the number line from 0 to 10. **Start at 1. Count on in twos**.
Complete the number line with equal jumps of 2 each time.

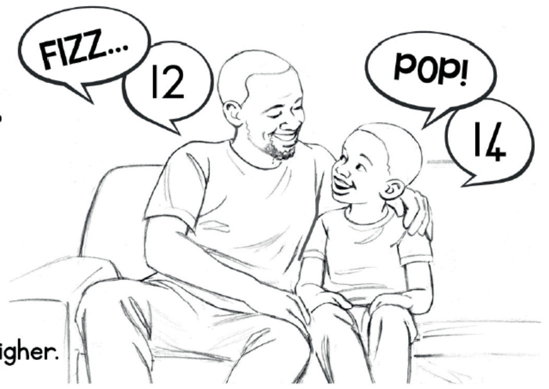
10. The function for this fizz-pop game is "2 more" or "add two".

Game 3: FIZZ POP

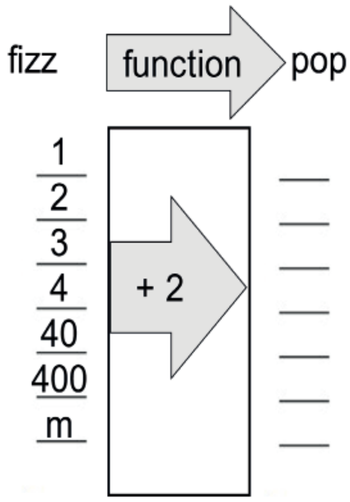
2 More and 2 Less

- You say, "Fizz." Your child says, "Pop."
- You say, "12"
- Your child adds 2 more.
- Your childs says "14."
- Keep going!
- Now try 2 less.

Start with numbers 0 to 20, then go higher.



Complete the function machine for "2 more". Write down the rule.



PrimTEd MAT3

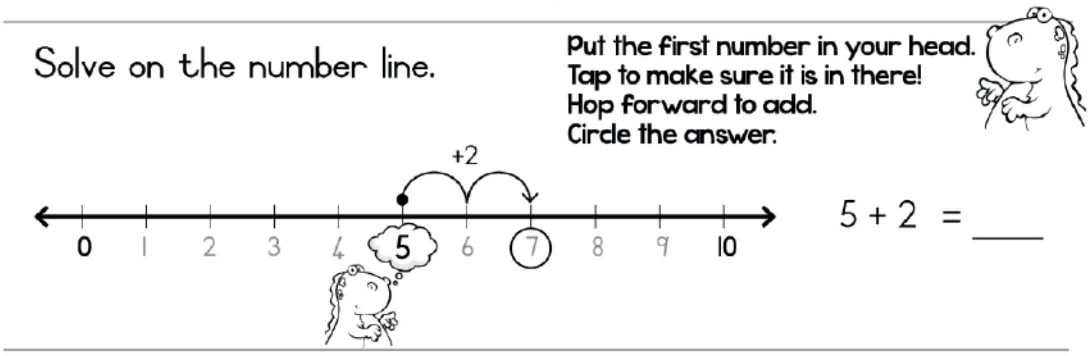
Reflect for action
Is the rule
"Add 2 more each time" the same or different to the function "Add 2".

What about the function
"Multiply by 2"?
Critique - attend to precision - regulate

Cinga

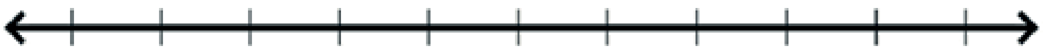
A function is special kind of relationship in mathematics:
For each input there is only one output.
An example of a function is **add two**: input + 2 = output.
Another example of a function is **double**: input × 2 = output.

11. This example shows that '2 more than 5 is 7'.



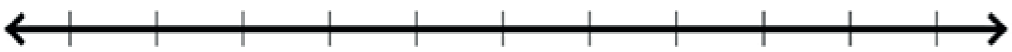
a. Draw a number line to represent 6 + 2 = 8.

b. The fences or intervals are 10s. the first pole (on the left) is 50. Draw a number line to represent 60 + 20 = 80.

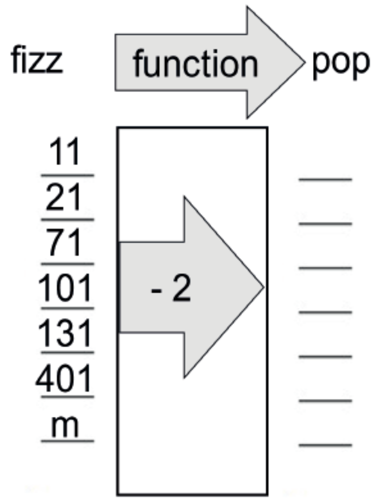


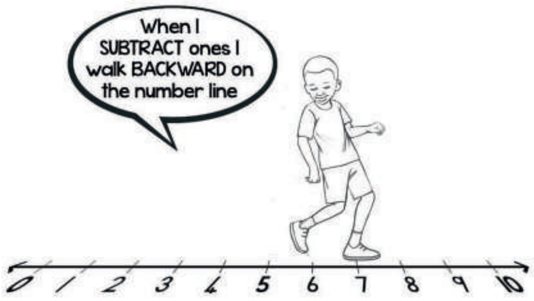
c. The fences or intervals are 100s. the first pole is 0.
Draw a number line to represent: 2 hundred more than 6 hundred = 8 hundred.

d. Each fence or interval is 1 half. Start at 0. Label each pole.



12. Complete a function machine for a fizz pop rule of '2 less':





13. The counting unit is 'ones' (small number song).

a. Draw a number line to represent 7 – 2 = 5. Start at 7. Circle the 7. Jump back 2.

b. The measuring unit is 'tens' (medium number song).
Draw a number line to represent: 70 – 20 = 50. The number line does not start at 0.
The last pole on the right is 100.

c. Draw a number line to represent 2 hundred less than 7 hundred = 5 hundred.
The last pole on the right is 800.

The counting unit is _______________.

d. Draw a number line to represent 7 million – 2 million = 5 million. The number line does not need to start at 0.

The counting unit is _______________.

Task 3.2.2 Counting on, and back, in tens (use a number line)

1. The medium number song is counting on in tens, starting at any number. Start at 0. Count on in tens.

Count by 10s from 10 to 100.
Colour each block.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Count forwards by 10s.

10	20					

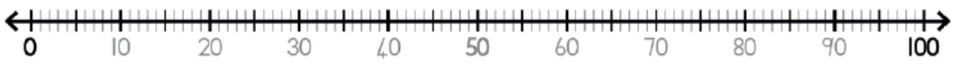
40	50					

Count backwards by 10s.

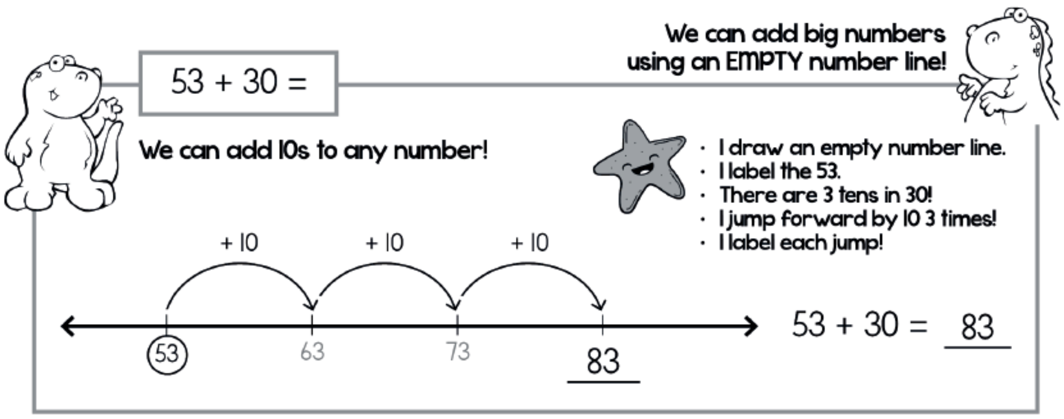
100	90					

70	60					

2. Start at 9. Jump forward in tens. Draw it on the number line.



We can use the medium number sequence for adding.



Notice that you start on the left, and jump forward in 10s to the right.

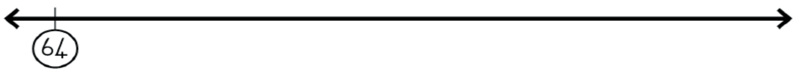
3. Solve by showing on a number line.

20 + 40 =

50 + 30 =

70 + 30 =

4. Solve by showing on a number line



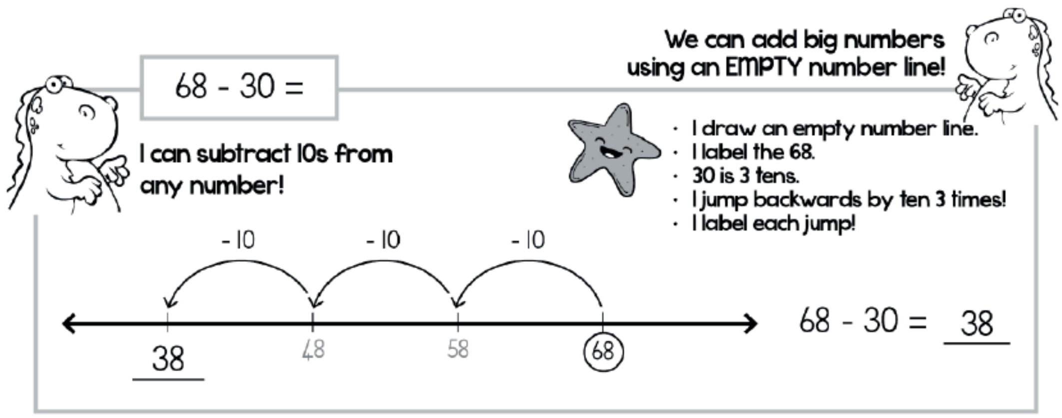
64 + 20 = ____

73 + 20 = ____

57 + 30 = ____

5. Make a worksheet with 3 of your own examples like the ones in question 4. Draw an empty number line. Write the calculation next to each number line.

We can use the medium number sequence for subtracting.



Notice that you start on the right, and jump back in 10s to the left.

6. Solve by showing on a number line

47

47 - 20 = ____

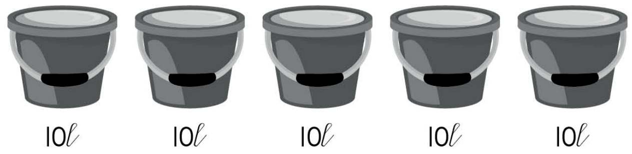
78 - 30 = ____

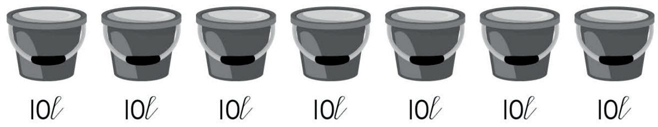
63 - 30 = ____

7. Make a worksheet with 3 of your own examples like the ones in question 6. Draw an empty number line. Write the calculation next to each number line.

Task 3.2.3 Explore buckets and litres

1. First, **specialise.** Complete the examples of specific cases:

	How many buckets	
	How many litres?	

	How many buckets	
	How many litres?	

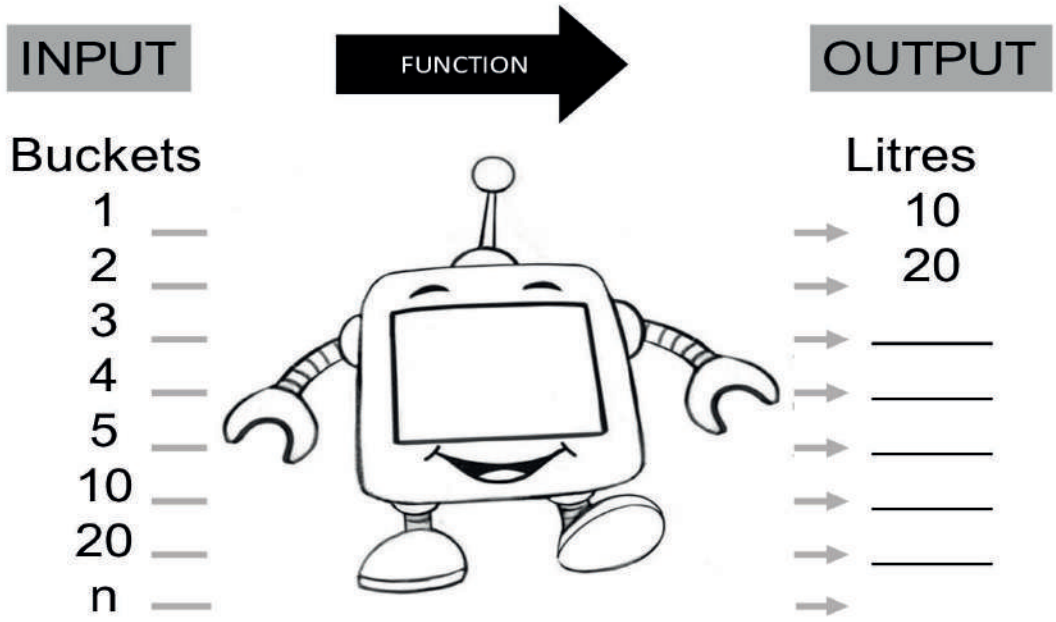
$\boxed{10 \times 4}$

When I multiply by 10, I can count in 10s.
I keep track of how many 10s using my fingers.
We say, "4 groups of 10." We write: 10 x 4 = 40.

Calculate. Use your fingers to keep track!

10 x 3 = ____	10 x 5 = ____	10 x 6 = ____	10 x 2 = ____
10 x 1 = ____	10 x 4 = ____	10 x 8 = ____	10 x 10 = ____

2. Now, **generalise**. If we know the number of buckets, what do we always do to calculate the number of litres?

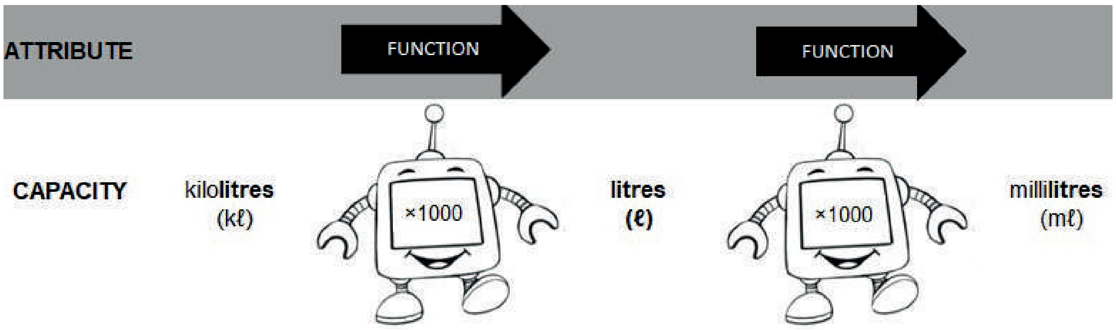


3. Write down an English sentence to explain how to get the number of litres, if you know the number of buckets.

4. Write down a number sentence to explain how to get the number litres (L), if you know the number of buckets (b).

5. The function to get from buckets to litres is ×10. Write down the inverse function, which gets you from litres to buckets.

6. Convert between kilolitres, litres and millilites



a. 1 kilolitre = ________________ litres = ________________ millilitres

b. 2 kilolitres = ________________ litres = ________________ millilitres

c. 5 kilolitres = ________________ litres = ________________ millilitres

d. ________________ kilolitre = 500 litres = ________________ millilitres

e. ________________ kilolitre = 100 litres = ________________ millilitres

Task 2.3.4 Show and explain a calculation (with number pictures and number lines)

As a maths teacher you need to be able to calculate and explain you calculation to primary school children.

Look back at task 1.3 on page 15. You had to show and explain how to do the calculation
92 - 58 = ...
That was near the beginning of the Emergent Number Sense module. You were using your mental maps that you learnt at school. You did not yet know about the counting pathway and the measurement pathway to learn about number.

1. USING NUMBER PICTURES (from the counting pathway),
   show how to calculate 92 - 58 = ...

2. Now write a paragraph to explain what to do to a child. Use the words, metaphors and mental pictures from the COUNTING PATHWAY.

4. USING NUMBER LINES (from the measurement pathway), show how to calculate 92 - 58 = ...

5. Now write a paragraph to explain what to do to a child. Use the words, metaphors and mental pictures from the MEASUREMENT PATHWAY.

6. Create your own subtract calculation that is like 92 - 58 = ...
Show three different ways to calculate.
You can use number pictures, number lines and number symbols)

Task 3.2.5: Two minute tango – bring it all together

1	363 ; 362 ; 361 ; 369 ; ____	11	Double 14 = ____
2	372 ; 362 ; ____; 342 ; ...	12	Double ____ = 140
3	If I decrease 700 by 10, I get ____	12	73 - ____ = 70
4	3 872 - 100 = ____	14	73 + ____ = 80
5	40 + _____ = 100	15	60 ÷ ____ = 6
6	1000 - ____ = 400	16	_____ ÷ 10 = 60
7	____ + 7 = 20	17	600 ÷ ____ = 6
8	____ + 70 = 200	18	____ + 400 = 800
9	377 + ____ = 380	19	(8×10) + (3×10) = ____×10
10	_____ is half way between 1 000 and 900	20	double 30 + double ____ = double 37

You should be fluent with all of the following:

Fluencies
Multiply and divided by 2, by 10; by 100
To multiply a big number, you break it up and multiply each part
To divide a big number, you break it up and divide each part

Fluencies
Make connections with bonds of 10

Fluencies
Doubling and halving

Fluencies
Bonds of 5
Bonds of 10

Fluencies
Use bonds of 10.
Add to visit the next ten.
Subtract to visit the previous ten

Fluencies
Make connections with bonds of 10

Make connections
If you know one number fact (eg. a bond of 10), then use the **small, medium & large number songs**, to work out new facts

Make connections.
If you know one fact (eg a bond of 10), use the medium song to add 10 to both sides, and work out a new fact (eg. a bond of 20).

Representations
Doubling uses a mirror line
Halving is sharing equally between 2.
Doubling and halving are inverse functions

Representations
Number lines to visit the next and previous 10 (or multiple of 10).

Representations
A whole-part-part diagram for a family of number sentences

Representations
Multiplication as **repeating equal groups** in a **number picture**.
Multiplication as **repeating equal jumps** on a **number line**

Representations
Number pictures and number lines for the
- small (1s)
- medium (10s)
- large (100s)

number songs

Representations
Bonds of 10 are "friends in love"
Ten frames are trays of 10 scones.
Structure by 2s - with 2 in each column or Structure by 5s - with 5 in each row.

Representations
If you know one number fact, then use the **small, medium & large numer songs**

SUMMARY

Which **concepts** will you be tested on in Quiz 3.2?
What **mental fluencies** should you be practicing for Quiz 3.2?
What marks did you get for your quizzes?
Which umthamo/imithamo do you still need to work on?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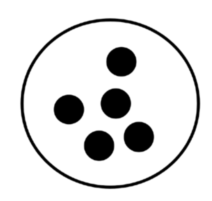
Maths4 Primary teachers is a learning series for Bachelor of Education (B.Ed) students in Foundation Phase and Intermediate Phase specialisations. Each module includes online materials, games, practice tasks, summaries and short quizzes for students and lecturers in initial teacher education settings.

Maths4 Primary teachers is aligned to the **Primary Teacher Education (PrimTEd)** standards which have been agreed as the minimum core content for teachers of mathematics in primary schools in South Africa.

The counting pathway	The measurement pathway
 	 
One way to think about number is using the idea of **elements in a container.** You arrange **discrete objects** into a **set**. You arrange **tomatoes on a plate**. This is the **cardinal** meaning of number. We represent cardinality using **number pictures**.	Another way to think about number is using the idea of **movement along a path**. You measure **equal intervals** along a line. You take equal **steps along a path**. This is the **ordinal** meaning of number. We represent ordinality using a **number line**.

Underlying both ways of thinking about number is a structural approach involving patterns, functions and algebra.

Emergent number sense answers questions such as:

- What are the pathways that children take into developing number sense?
- How do we move children away from always only counting in ones?
- Which representations are helpful to children as they start their journey into school mathematics?
- What games and learner activities can teachers use in maths lessons?

Emergent number sense attunes Intermediate Phase teachers to the origins of mathematics and to the skills and representations that their grade 4, 5 and 6 learners should have already developed. It provides many ideas and teaching tips for remediating gaps in mathematics learning.