MATHS4
Primary teachers

Multiplying and Dividing

2026

MATHS 4 PRIMARY TEACHERS

Name: ______________________________

Surname: ______________________________

Contact: ______________________________

University: ______________________________

This **multiplying and dividing** module aligns to the following PrimTEd standards

GPM General pedagogical standards for mathematic teaching	GPM1	Plan effective learning experiences
	GPM 2	Take learner knowledge into account
	GPM 3	Engage learners productively in mathematics
	GPM 4	Teach a balanced mathematics curriculum
MAT Mathematical acting and thinking	MAT 1	Playful engagement to search for and develop mathematical insight
	MAT 2	Represent and use mathematics
	MAT 3	Reason mathematically
	MAT 4	Reflect for action
NA 4 Multiplicative relations with natural numbers	NA 4.1	Demonstrate and use the relation between multiplication and division
	NA 4.2	Demonstrate knowledge of equivalent relations, in general and equality in particular, in relation to multiplication and division problems
	NA 4.3	Demonstrate the necessity for the exclusion of division by zero
	NA 4.4	Demonstrate knowledge of number theory, in particular odd and even numbers, factors, prime factorisation, the divisibility rules and links to multiplication and division
	NA 4.6	Describe and use multiplication and division problem types and the range of useful associated representations
NA 5 Rational numbers and operations on rational numbers	NA 5.1	Demonstrate how to teach learners to order, add, subtract, multiply and divide rational numbers
	NA 5.3	Identify and use part-whole relationships when working with fractions, decimals, percentages, ratios and rates
	NA 5.4	Describe and use a range of representations, calculation strategies and tasks that are used to solve problems involving rational numbers
NA 7 Early algebra	NA 7	Early algebra
NA 8 Overarching mathematical ideas	NA 8	Overarching mathematical ideas for school arithmetic

MULTIPLYING AND DIVIDING

This *Maths 4 Primary Teachers* course for Bachelor of Education students is undertaken as design-based research initiative. It was first piloted in 2025.

2026 first intervention cycle

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In this course you use jumptrak for:

1. The PRE-TEST and POST-TEST;
2. Two minute TANGO practice;
3. PEER MARKER for tasks; and
4. Weekly QUIZZES.

Your lecturer will give you a username and password.

https://tinyurl.com/yc7yzcdv

Find additional resources for this module here:

M4PT Multiplying and dividing

STUDENT FOLDER.

Acknowledgements

An initiative such as this takes a team effort.

University	Lead teachers and course designers	
Cape Peninsula University of Technology	Rosemary Brien Sharon McAuliffe	Byron Abrahams
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University of Limpopo	Manare Setati	

It is framed as a design-based research project.

Extensive use is made of the learner activity books designed by Dr Kim Porteus, Nobuntu Mazeka and Qhama Ngcobo and colleagues, developed and used in the *Magic Classroom Collective* by the Nelson Mandela Institute (NMI). https://www.mandelainstitute.org.za/

Several multiplication and division components draw on the *Primary Mastery professional development materials* of the National Centre for Excellence in the Teaching of Mathematics (NCETM). https://www.ncetm.org.uk

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UNIT 2: LEARNING THE TIMES TABLES

1. **Commutative property.** Multiplication can be done in any order
2. **Division is the inverse of multiplication.** For every multiplication fact, there is a family of 4 division, and 4 multiplication facts which are all equivalent
3. **Distributive property.** You can break up a number using addition or subtraction and multiply each part.
4. We keep track of multiplication facts using various representations

</td></tr>
<tr><td>2.1 Multiplicative structures with 2s, 5s and 10s</td><td>The ×2 table are the doubling facts.
Halving is the inverse of doubling.
The products/ multiples in the ×10 table end in 0.
The products/ multiples in the ×5 table end in 0 or 5.</td><td>Tens (× and ÷)
Twos (× and ÷)
Fives (× and ÷)
Factor × factor = product.</td><td>Multiplication table of 100 facts
Families of related number sentences</td><td>93</td></tr>
<tr><td>2.2 Times tables with 2s, 4s and 8s</td><td>The 2, 4 and 8-times tables are closely related to each other.
Products in the 2, 4 and 8 times tables share the same factors.
You can factorise a number to simplify the multiplication. Factorise means split into smaller factors.</td><td>Twos (× and ÷)
Fours (× and ÷)
Eights (× and ÷)</td><td></td><td>135</td></tr>
<tr><td>2.3 Times tables with 3s, 6s and 9s</td><td>The 3, 6 and 9-times tables are closely related to each other.
Products in the 3, 6 and 9 times tables share the same factors.
You can factorise a number to simplify the multiplication 3, 6 or 9</td><td>Threes (× and ÷)
Sixes (× and ÷)</td><td></td><td>172</td></tr>
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UMTHAMO	KEY IDEAS	MENTAL FLUENCIES	REPRESENTATIONS	

UNIT 3: Multiplication and division word problems

1. Multiplicative thinking it about **multiplication and division.** Teachers should **link division to multiplication.**
2. **Multiplication is commutative.** The order does not matter. 5 × 2 = 2 × 5
3. **Division is not commutative.** The order does matter. 10 ÷ 2 ≠ 2 ÷ 10.
4. There are important **representations** for multiplicative thinking
   a. Whole – equal part diagrams
   b. Arrays
   c. T-tables
   d. Clue boards

UMTHAMO	KEY IDEAS	MENTAL FLUENCIES	REPRESENTATIONS	
3.1 Word problems and using arrays, t-tables and clue-boards	Multiplicative thinking it about **multiplication and division.** Teachers should **link division to multiplication.** For division you should include problems: that result in **left overs or remainders** that result in a **fraction**; and about 'finding a fraction' (of a discrete object, or a measurement, or of a group of discrete objects)	Threes (× and ÷) Sixes (× and ÷) Nines (× and ÷)	Arrays, t-tables, clue-boards	**201**
3.2 Multiplying and dividing big numbers	**Distributive property**: You can break up a number and multiply each part. You can break up a number and divide each part.	Factorise (find factor pairs)	Areas of rectangles Division by chunking Long division	**232**

mistakes in
MATH
are ok!

MATH
helps me
become a
THINKER.

The
PROCESS
is more
important
than the
ANSWER!

my
MISTAKES
can help my
PEERS!

MATH
can be
FUN!

I have a
POSITIVE
math
esteem!

I can be a
RISK-TAKER
and try new
STRATEGIES
in math!

I CAN
DO
MATH!

I CAN
do difficult
things - even
MATH!

UNIT 1: BIG IDEAS FOR MULTIPLICATIVE THINKING

Key ideas

1. Multiplicative thinking it about multiplication and division.
   Teachers should link division to multiplication.
2. Multiplicative problems should be on both pathways:
   a. The counting pathway using discrete objects
   b. The measurement pathway using continuous extents/measures
3. How to succeed in this course.

Two pathways into number

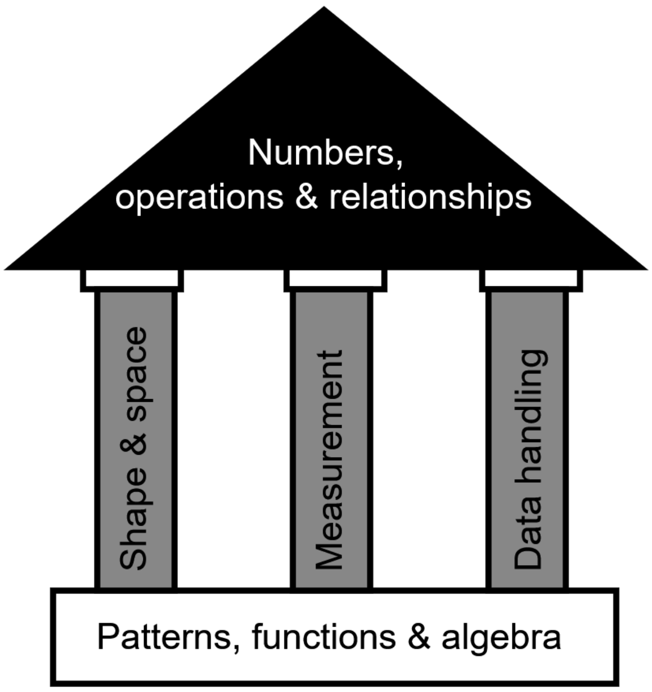
Mathematics involves thinking about and working with numbers, shape and space. Working with numbers is about quantifying things: comparing, ordering, sequencing "things". The "things" we quantify can either be:

- **Discrete objects** or **discrete events** that can be separated from each other which can be counted using a counting unit; or
- **Continuous quantities or extents**, that can be measured by focusing on a particular attribute and using standard or non-standard units of measurement.

A big idea in mathematics is finding and describing **patterns, structures or functions**.
The South African Curriculum and Assessment Policy (CAPS) identifies five content areas for primary school mathematics.

South African curriculum for primary school

Diagram



Numbers operations and relationships is the focus in primary school. It is given the most time
- Types of **numbers** (whole, natural, rational then integers),
- Operations (add, subtract, multiply and divide) and
- Relationships such as equal or unequal or bigger than or less than

There are 3 contexts in which to use and apply numbers, operations and relationships:
- Shape and space (2D, 3D, positions, directions, transformations)
- Measurement (length, time, mass, capacity, volume, temperature)
- Data handling (collect, organise, analyse, present, interpret)

Patterns, functions and algebra is the foundation. It is the way of learning about numbers, operations and relationships in the three different contexts.

There are two main routes to developing children's number sense:

- **A counting pathway** where the starting point is discrete quantities (discrete objects or discrete events); and
- **A measurement pathway** where the starting point is continuous quantities that can be measured using a particular attribute such as length or volume.

In *Maths4Primary teachers* we consider both pathways, and an underlying approach of structuring numbers to identify, describe and generalise the patterns of 2s, 5s and 10s.

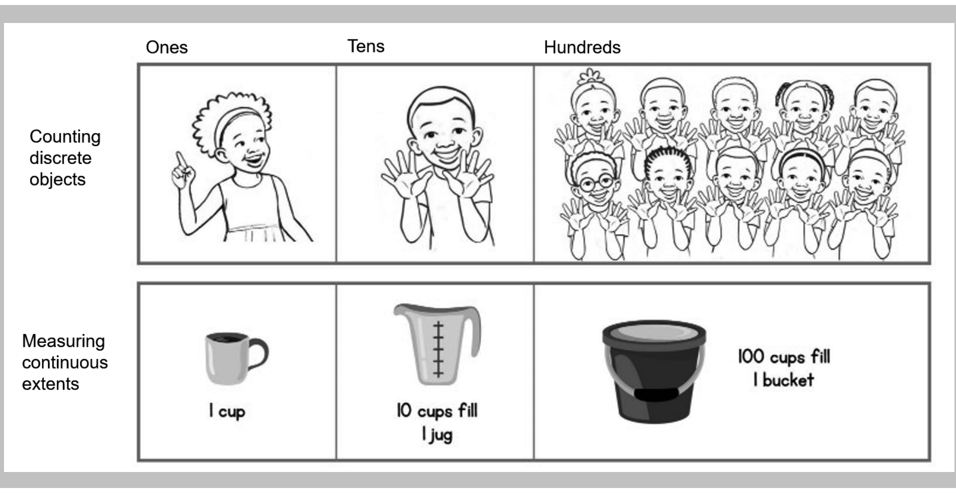
Discrete and continuous quantities

It is your job as a teacher to make sure that you use **both pathways into number**. When you choose an example, ask yourself:

- Is this example or problem about **discrete objects or events**?
- Is this example or problem about **continuous quantities that can be measured**?
- Does this example or problem have **no context**?

You should try and balance your examples of problems.

Diagram



Make sure you use contexts, especially in the **Foundation Phase** – both counting and measuring pathways.

In Intermediate Phase you should constantly bring in all the measurement contexts: **length, mass, capacity, volume, time, money and temperature**. Make sure you start to generalise and use 'context free' or 'bare calculation; examples using × and ÷ signs, when the concepts are secure.

How to succeed in this course

Weekly rhythm for learning

To be successful in this university course, you will need to work every week. We want you to pass and to pass well. We give you lots of opportunities to

- learn how to teach maths and
- improve your own maths.

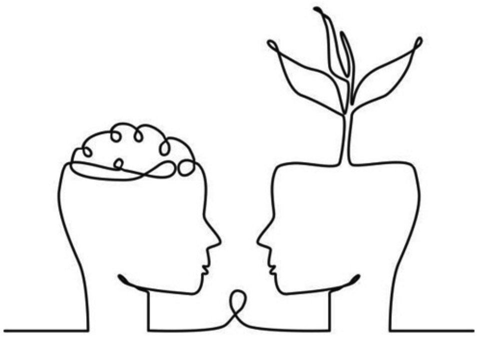
If your own maths is shaky and if you hate maths you wont be able to teach it well. Now is your chance to work hard to develop a better relationship with maths, be better at doing maths (maths should make sense); and learn to teach primary maths well.

Each week is an umthamo (bite sized chunk) of work. Each umthamo consists of lectures, tutorials, portfolio tasks which you and your peers mark, fluency practice and a quiz. If you work each week – in this rhythm for each week – you will succeed.

You will get feedback on your work through marking and reflecting on tasks, and from your quizzes. Use this feedback to improve.

Build a Growth Mindset

Research studies of learning and the brain, from the fields of education and neuroscience, have been brought together in the last decade to produce findings that are critically important for schools.



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https://www.youcubed.org/resource/growth-mindset/

See **Jo Boaler's** work at Stanford University. She draws on the work of **Carol Dweck** and applies it to learning school mathematics.
The research findings include:

- **Ability and intelligence grow** with effort and practice.
- When students believe that **everybody's ability can grow**, their achievement improves significantly.
- When teachers believe that **everybody's ability can grow**, and they give all students opportunities to achieve at high levels, students achieve at high levels.

JUMPTRAK

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1. The PRE-TEST and POST-TEST;
2. Two minute TANGO practice;
3. PEER MARKER for tasks; and
4. Weekly QUIZZES.

Your lecturer will give you a username and password.

Some classroom norms

Your should agree on the classroom norms you would like to uphold for your class at university – and for your classrooms where you teach in schools. You may want to elect a class representative. Start a whatsapp group to help each other.

Mathematics classrooms are places where students believe:

- Everyone can do well in maths.
- Mathematics problems can be solved with many different insights and methods.
- Mistakes are valuable, they encourage brain growth and learning.
- Mathematics will help them in their lives, not because they will see the same types of problems in the real world but because they are learning to think quantitatively and abstractly and developing in inquiry relationship with math.

UMTHAMO 1.1

Making sense of the times tables

Most South African adults probably learnt the times-tables by rote when they were in primary school. Knowing the times table was a ticket to enter the class. We put our hands on our heads and chanted as a chorus. We were picked on and had to stand up and say it. **How did learning the time tables make you feel?**

Some ama-Y2k's might have not had these humiliating experiences.
Did you learn your times tables? If so, how?

VERSION A	VERSION B
	5 Times table
1 × 5 = 5	5 x 1 = 5
2 × 5 = 10	5 x 2 = 10
3 × 5 = 15	5 x 3 = 15
4 × 5 = 20	5 x 4 = 20
5 × 5 = 25	5 x 5 = 25
6 × 5 = 30	5 x 6 = 30
7 × 5 = 35	5 x 7 = 35
8 × 5 = 40	5 x 8 = 40
9 × 5 = 45	5 x 9 = 45
10 × 5 = 50	5 x 10 = 50
11 × 5 = 55	5 x 11 = 55
12 × 5 = 60	5 x 12 = 60

You probably learnt the times tables using the word **"times".**
Which version of the times table did you learn?

- **VERSION A**: "1 times 5 is 5… 2 times 5 is 10" or "1 five is 5, 2 fives are 10", or maybe you learnt
- **VERSION B:** "5 times 1 is 5…, 5 times 2 is 10"? or "5 ones are 5, 5 twos and 10, etc

Did you know what you were saying?

Key Ideas

After this umthamo you should know that

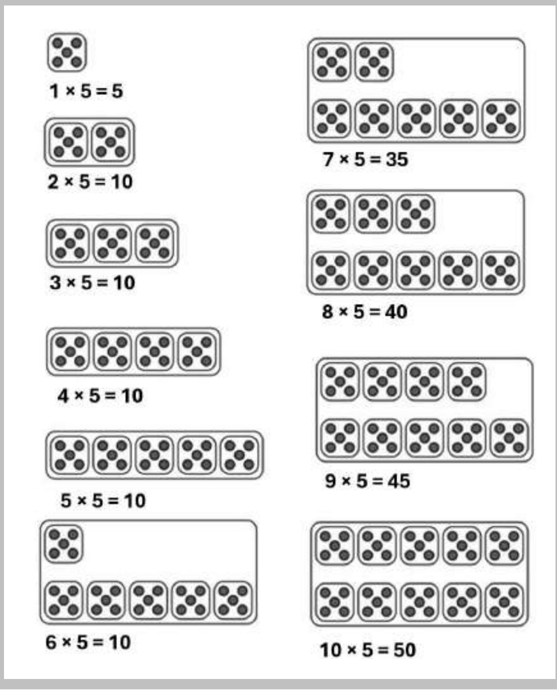
1. Times has two meanings: **"Repeated … times" or "Groups of …"**
2. Early in **Foundation Phase** we don't teach "times". We let children work with multiplication and division situations. These can be picture problems, talk problems or word problems.
3. The times tables always **keep track of 2 different units**. We can keep track of the 2 counts using fingers, dot pictures, number lines, function machines horizontal tables, t-tables or clue boards.

Two versions of the times table

We now return to the two versions of the times tables. What do they mean? Which one is correct?

Two ways of thinking about ×

VERSION A: 5 times table as groups of 5



Version A makes sense if you think of the first number as **how many groups of 5**:

- 1 group of 5 is 5,
- 2 groups of 5 is 10,
- 3 groups of 5 is 15, ...

The unit is 1 group of 5. You repeat that unit to get the times tables

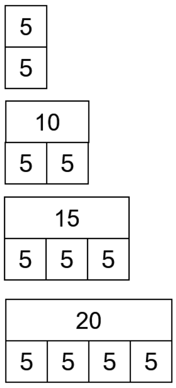
Version A makes sense if you think of your **unit as five**, and you repeat that unit:

- 1 five is 5,
- 2 fives are 10,
- 3 fives are 15, etc...

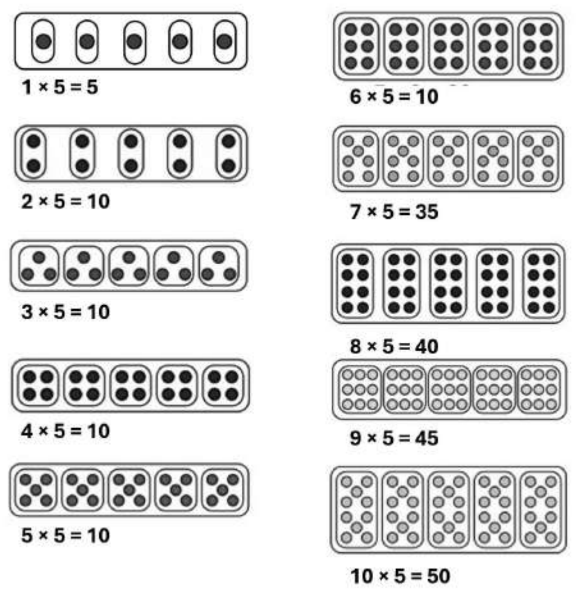
Notice that there are two counts involved in this times-table. **The group of 5 stays constant.** A group of 5 appears in every case. What changes is the number of groups.

Groups	Dots
1	5
2	10
3	15
4	20
5	25

1 group of 5 is 5 dots
2 groups of 5 is 10 dots
3 groups of 5 is 15 dots
4 groups of 5 is 20 dots



VERSION B: 5 times table as repeat 5 times



Version B makes sense if you think of the first number as the unit, and **repeat that unit 5 times**

- 1 repeated 5 times is 5,
- 2 repeated 5 times is 10,
- 3 repeated 5 times 15, ...

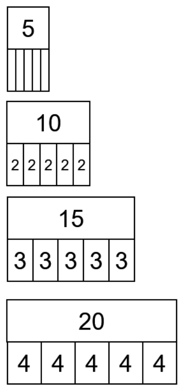
The number of repeats is always 5. You use different units, and repeat them 5 times get the 5 times table.

This is the phinda-phinda or pheta-pheta way of thinking.
Version B makes sense if you think of the first number as the unit, and then **repeat that unit 5 times**:

- 1 repeated 5 times is 5, (5 ones are 5)
- 2 repeated 5 times is 10, (5 twos are 10)
- 3 repeated 5 times is 15...(5 threes are 15)

Notice that there are two counts involved in this times-table. **The number of repeats is always 5**. The number of repeats stays constant. What changes is the unit that is being repeated.

Unit	Dots	
1	5	1 repeated 5 times
2	10	2 repeated 5 times
3	15	3 repeated 5 times
4	20	4 repeated 5 times
5	25	...



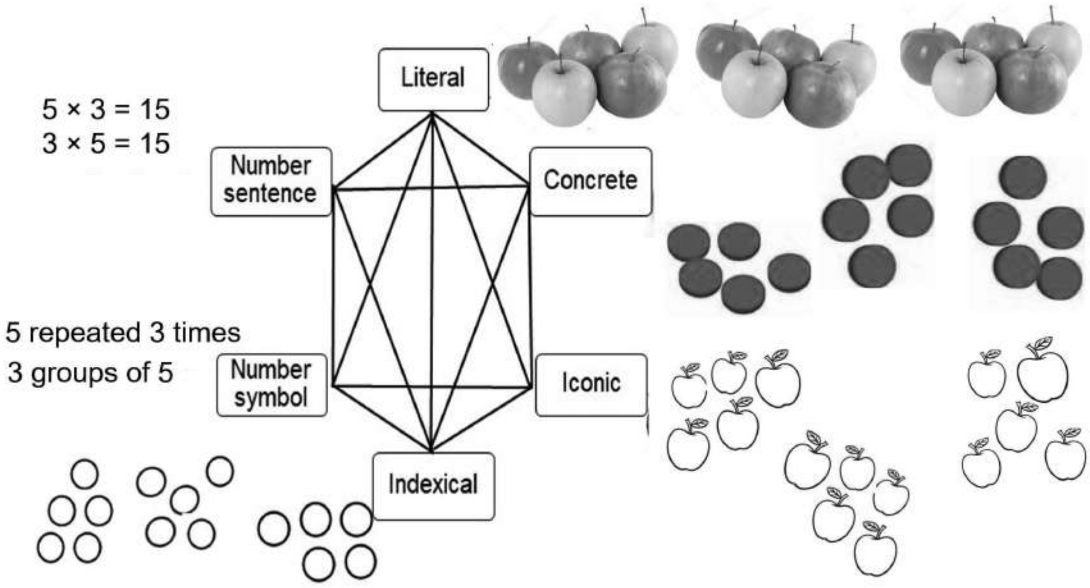
Both versions are correct. Which one makes most sense to you? It is important to understand both versions. Both are helpful ways to think about "times" or multiplication.

PrimTEd Std: NA 4.2
Demonstrate knowledge of equivalent relations, in general and equality in particular, in relation to multiplication and division.

Moving between concrete pictorial and abstract

We now know there are far better ways for children to learn their times tables. **Children can learn their tables in a way that makes sense**. Learning your timetables takes time and practice, and lots of repetition.

For each times table: first children work concretely, then pictorially then abstractly.



They move between working on real problems, learning the counting song, keeping track while they sing the song, and ultimately the known facts of the times table. There is an ongoing dance between imagining the concrete, working in the pictorial and thinking symbolically.

Concrete	**Literal objects** are real physical objects in their surroundings: sticks, stones, forks, bowls, leaves, hats, shoes **Concrete objects** are physical objects which represent the literal objects such as bottle tops, or unifix cubes. They are also called manipulatives. They are concrete as they can be touched, joined and arranged.
Pictorial	**Iconic pictures:** Learners also use drawings or picture of collections of objects. They can draw a hand, or a few stones or a pair of shoes. **Indexical picture:** Overtime, they start to draw with less detail. Instead of drawing 5 apples, they just draw 5 circles to show how many there are.
Abstract	**Number symbols:** Only in Grade R and 1 do they start to recognise and learn to write number symbols. **Number sentences (symbolic syntactical):** A number sentence describes the relationship between collections of discrete objects or between measurement units. Learners start to write number sentences using symbols like + and – and = in Grade 1. The × and ÷ symbols are used by Grade 3.

Building knowledge of each times-table shifts from concrete to pictorial and abstract. And **children dance between the abstract pictorial and concrete** as they build their growing network of ideas about multiplying and dividing.

To free up working memory, we want Grade 3 and Grade 4 children to know:

- as many times-tables as **rapid recall**, as possible;
- **how to work out any times-table, if they don't know it**, by making connections with what they do know.

Different times tables are learnt at different grades.

2s	**2s, 10s, and 5s**	**×2, ×10, ×5**	**×2, ×4, ×8**	**×3, ×6, ×9**	**×7, ×11, ×12**
Skip count forwards in 2s to 20. Start at any even number.	Skip count forwards in 2s,5s and 10s. Start at any multiple of each of these numbers.	Recall multiplication and division facts for 2, 5 and 10 up to 20, 50 and 100 respectively	Recall multiplication and division facts for 2, 5 and 10 up to 20, 50 and 100 respectively	Recall multiplication and division facts up to 30, 60 and 90 respectively	Recall multiplication and division facts up to 70, 110 and 120 respectively
Even and odd numbers, **Double and halve** Doubles are the evens. End in 2,4,6 or 8	**Count in 2s:** Even counting song **Count in 10s:** medium song **Count in 5s.** Time in minutes	**Multiples of 2** (evens). **Multiples of 10** (end in 0). **Multiples of 5** (end in 0 or 5).	**Multiples of 2** (evens/doubles). **Multiples of 4** (even, double-doubles). **Multiples of 8** (even, double, double double).	**Multiples of 3,** the digit sum is 3,6 or 9 **Multiple of 6** **(**even digit sum is 3, 6 or 9) **Multiples of 9** **(**digit sum is 9)	Multiples of 7 (use commutative property) Explore **multiples of 11** **Multiples of 12** link to 3 and 4 times tables

Steps for learning any times table

There is a key sequence from exploring picture and word problems, to ultimately having the times tables as known facts for rapid recall.

So, for example, this is a **learning trajectory for the 5 times-table**:

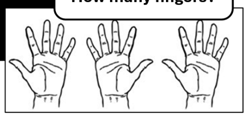
1. Start with **picture problems**, talk problems and word problems.
   Give a real multiplicative context.
   a. Arrange objects (like counters or stones) into little piles or groups of 5.
   b. Circle groups of 5 objects in pictures of many objects.
   c. Arrange cubes into towers of 5.
   d. Share equally into 5 groups.
2. Learn the 5s **song**: Count forwards in 5s. Start at 5. These are the multiples of 5.
3. Use the 5s song to **solve real problems.** These can be word problems or picture problems.
4. **Model multiplying and dividing** by 5 with the **manipulatives**.
   The game of the week is towers of 5. Each tower has 5 cubes.

1. Keep track of the counts, when singing the 5s song.
   Keep track of both units and involved in the times-table.
2. Learn the 5 times table as rapid recall number facts.
3. Explore divisibility rules and factors (including prime factorization)

STEP 1

Picture problems and number talks

How many hands?
How many fingers?

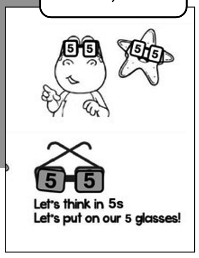


STEP 2

Learn the number song,
Skip or group counting

.

5, 10, 15, 20, 25, 30, ...

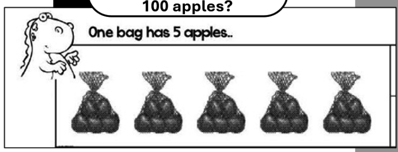


STEP 3

Use the song to solve problems

How many bags?
How many apples?

What if, 5 bags, 6 bags, 7 bags,..
100 bags?
100 apples?



STEP 4

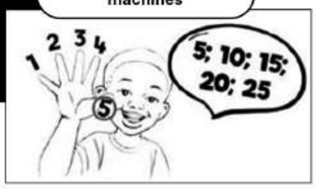
Model multiplication and division with manipulatives

Make towers of 5.
4 towers, how many blocks?
20 blocks, how many towers?
22 blocks how many towers?

STEP 5

Keep track of the counts

Fingers, horizontal tables,
number lines,
t-tables,
clue boards, function machines



STEP 6

Know the times table

The times tables connect to each other

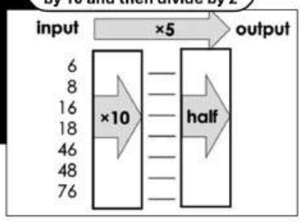
10 has 2 and 5 as factors
4 and 8 have 2 as factors

STEP 7

Factorise
Divisibility rules

I know 15 is a multiple of 5, because it ends in 5

I factorise 10 into 2 × 5
To multiply by 5 I multiply by 10 and then divide by 2



STEP 1:

Start with picture problems. Give a real multiplicative context

A **picture problem** is when children are given a picture and they need to reason about it.
A **talk problem** is when a teacher tells a story problem. The children cant read, but the teacher talks about a problem, and they children work on it.

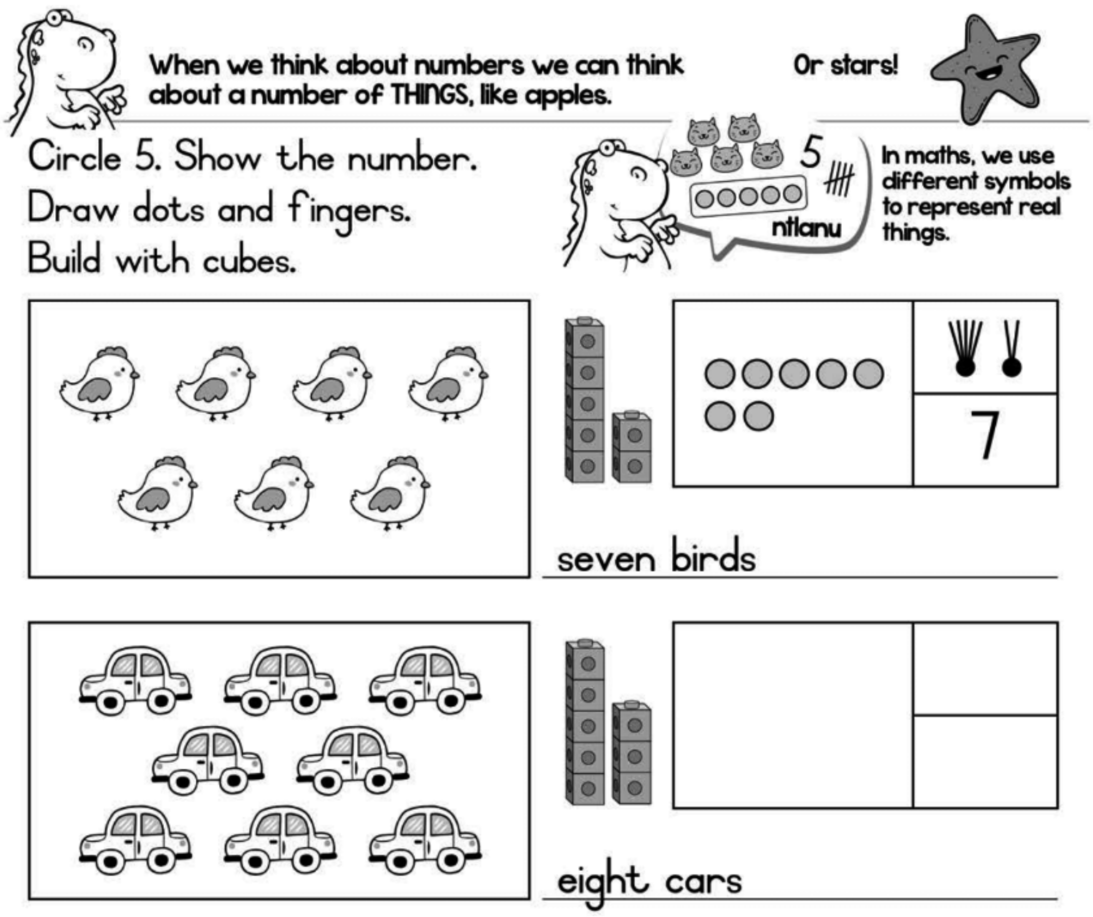
Perhaps they have to:

- join two collections (add) together
- remove, take away or compare two collections (subtract)
- repeat to make equal groups (multiply)
- separate or circle equal groups (divide)

A mentor teacher, who had taught grades 1-3 for years said:

> *"In foundation phase we don't teach multiplication and division...*
> *We let children solve multiplication or division situations or problems.*
> *We don't use the* **×** *sign or the* **÷** *sign. They reason about situations."*

CIRCLE 5, DRAW and WRITE



Find 5 differences.

Picture problem

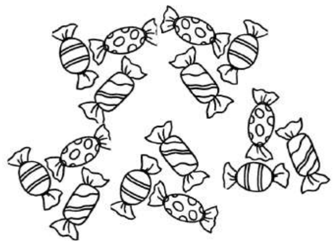
How many sweets?

We want to pack them into groups of 5.

Draw around 5 sweets to make a group.

Draw around another 5 sweets.

How many groups can we make?



some sweets

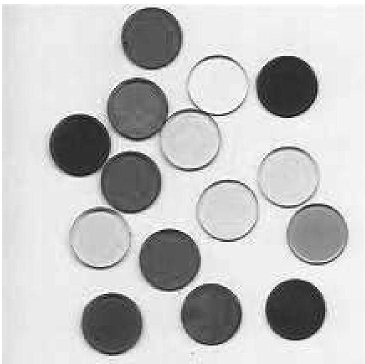
Word/talk problem

Fetch some counters

Now everyone has some counters

Arrange them into groups, with 5 in each group.

- How many counters altogether?
- How many groups of 5?
- How many left over?



Some counters

Word/talk problem

Here is some string.

- Fold it into 5 equal parts
- Can you cut it into five equal pieces?

Some string

Word/talk problem

There are 12 suckers or lollipops
Share them equally between 5 children.

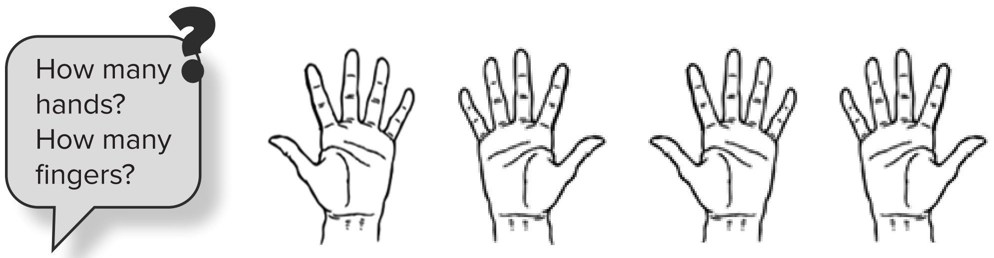
- How many suckers does each child get?
- How many are left over?

Some suckers

Make sure you include:

1. both **discrete and continuous** contexts
2. problems where there are multiples of 5 (**no remainders**) and problems which are not multiples of 5 (**with remainders** or left overs)
3. problems involving **sharing**, and problems involving **packing or grouping.**

Introduce more structured situations



Ask **what if** questions..

- **What if** ...there were 3 hands?. Or 5 hands?
- **What if** ...there were 10 fingers, how many hands?

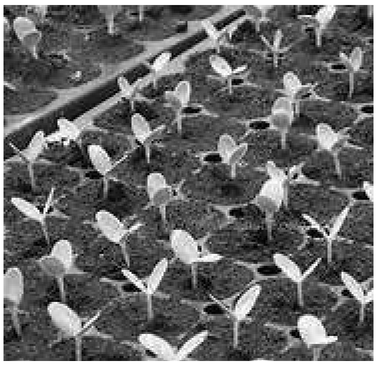
How many cupcakes? How much money?

Ask **what if** questions..

- **What if** ... there were 2 cupcakes, or 5 cupcakes?
- **What if** ... I had R10. How many could I buy?

Use situations that need rows and columns (arrays). Planting or gardening is a good context.

Zibalo is planting carrot seedlings. She plants 5 seedlings in each row. She has 15 seedlings. Use counters or a drawing to show me Zibalo's rows of carrots.

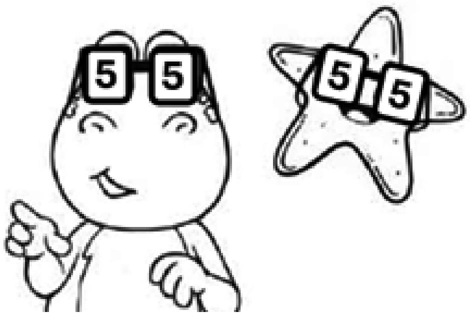


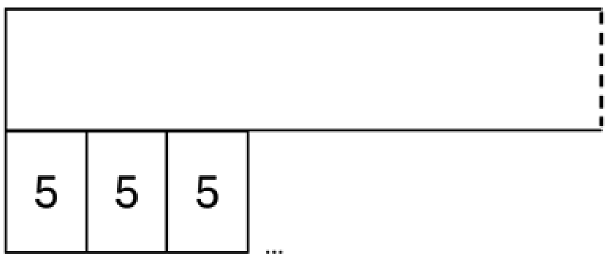
Some carrot seedlings

STEP 2:

Learn the 5s song. Practice counting in 5s, starting at 5

This involves thinking about numbers as made of equal groups of 5.





5 Is the unit that repeats. How many 5s make up the whole?

The whole-equal part diagram, has equal parts.
With our 5 glasses on, each part is always 5.

Practice counting forwards in 5s, starting at 5. This is **counting the multiples of 5**.
It is the **fives song**. Using a **100 chart** display can help at first.

- Teacher points, and learners say. Learners follow the teacher
- Then learners say, and teacher points. Teacher follows the learners

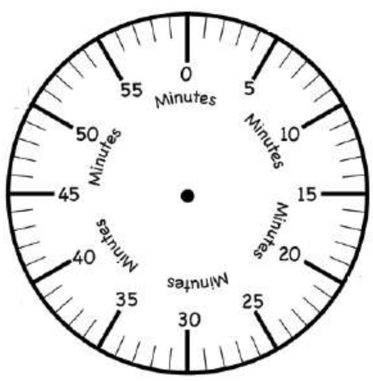
Number Chart: 1 - 100

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

The 100 chart is a visual scaffold to help learner learn to the fives song. It should be taken away over time, so they know the song without looking at the numbers.

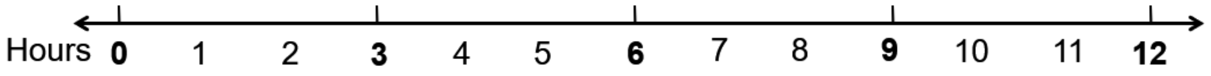
Another visual scaffold is an analogue clock, which has 2 number lines.
They are both curved into a circle.

The first number line is for the **short hand.**
It measures hours, in ones.

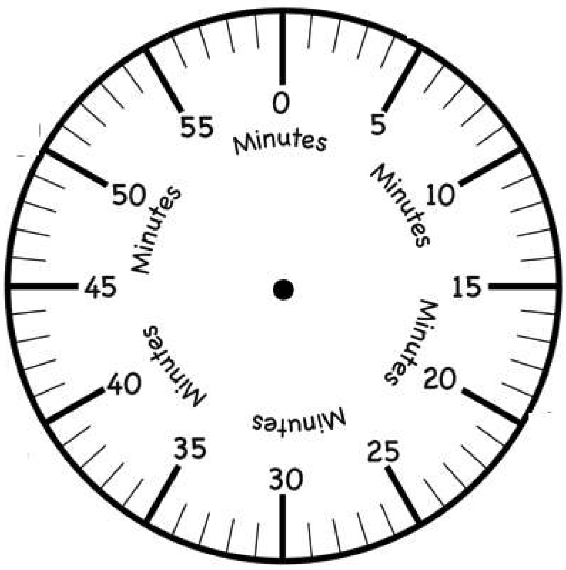


The second number line is for the **long hand.**
It measures minutes, in fives.
It is not usually shown on the clock.
We have to imagine it.

The first number line is for the **short hand.** It measures hours, in ones.



The second number line is for the **long hand.** It measures minutes, in fives.
The analogue clock can also be a useful visual scaffold for learning the fives song.



You can use a clock face as a visual scaffold for the **fives song**.

- You count **the minutes**, starting at 5 minutes.
- Teacher points, learners say. Learners follow the teacher.
- Then learners say, and teacher points. Teacher follows the learners.

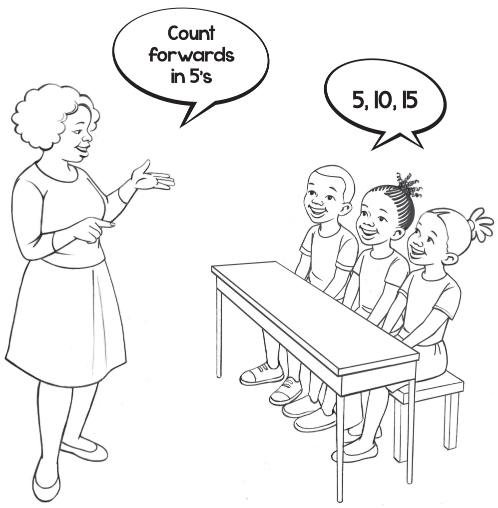
Counting forward in fives, starting at 5 should be a regular part of your mental maths lesson starter,

When you sing the **fives song**, you are counting the **multiples of 5.**

COUNTING:
Fowards in 5's

Count forwards in 5's, starting from 5.
We are counting the multiples of 5's.

Count from 5 to 60.



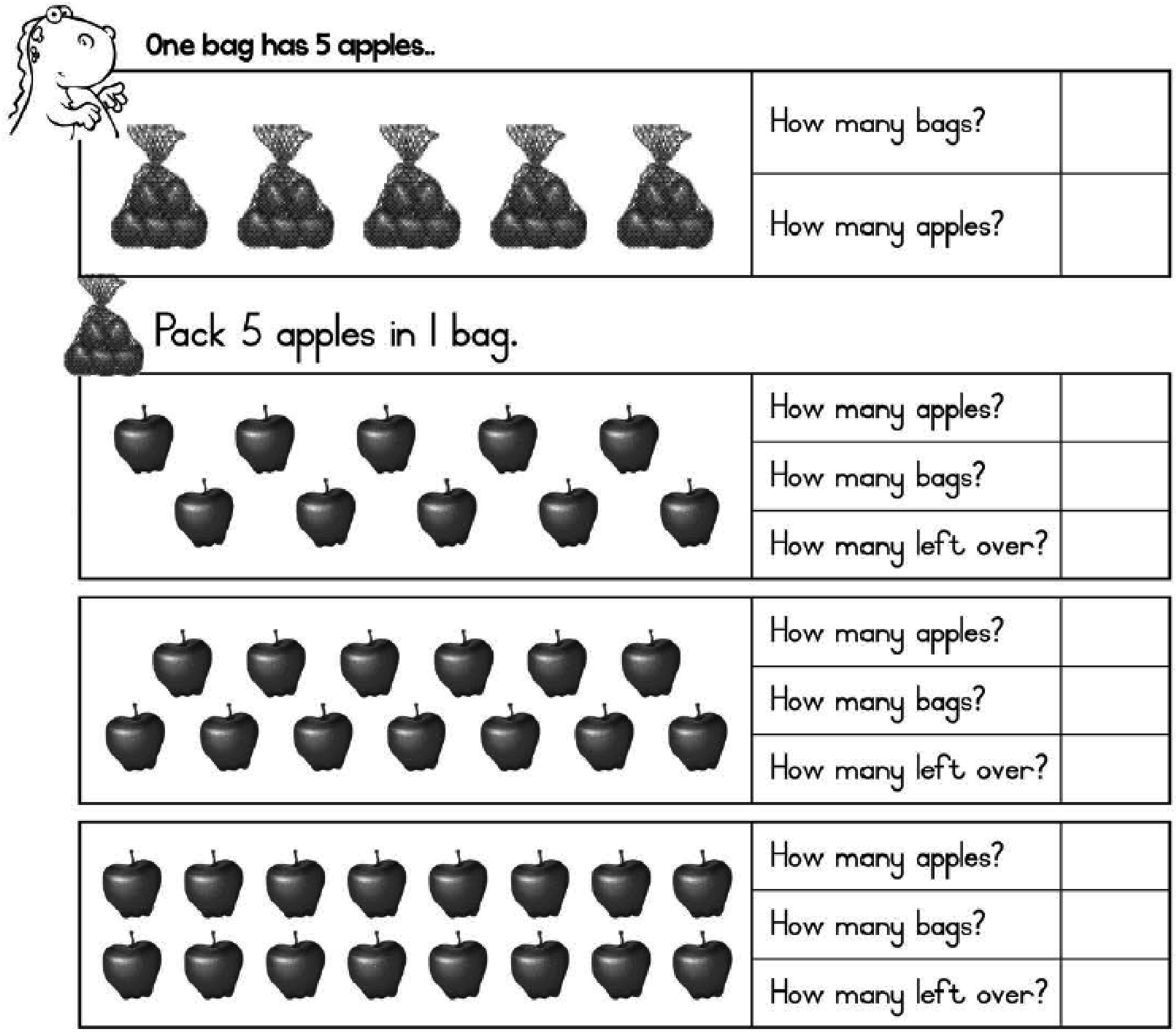
STEP 3:

Use the counting in fives song to solve real problems

First you can work with problems where a **collection of ones** (such as apples), must be **structured into equal groups of fives** (5 apples in each bag).

Now when children work on real problems (picture problems, talk/word problems) they can use the **fives song**. They should count in fives. They should not count in ones.

You create one unit of 5, which is then repeated. You circle the fives. **This is in pictures.**

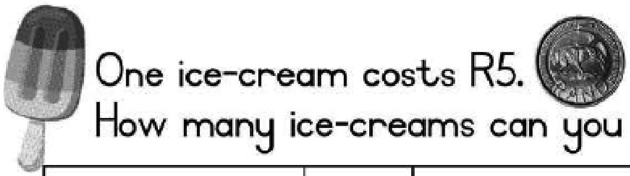


In this example you have a "**composite unit**". There are apples (which are ones). Then there bags of apples (which are 5 ones). The **composite** unit is the bag – 1 bag has 5 apples, 2 bags has 10 apples. Each group of 5 apples, is thought of together as being "1 bag". Counting equal groups is counting composit units. We always do this when multiplying and dividing. We keep track of 2 units:

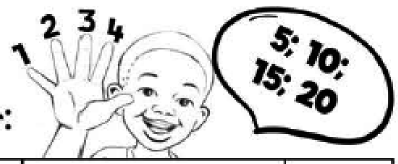
- We keep track of the **apples**
- We keep track of the **bags** (of 5 apples each time).

In this example the idea of **left overs**, is also brought into focus. You have numbers that are not multiples of 5. When the number is not a multiple of 5, there is a left over (the left overs for fives can be 1, 2, 3 or 4).

Then you can work with a symbolic 5. So five is the unit which must be repeated.
We don't need to count each 1, to make 5, we just think of the groups of 5.



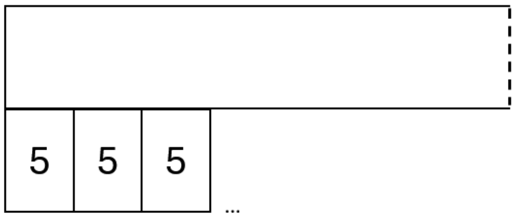
One ice-cream costs R5.
How many ice-creams can you buy for:



R 15		R 20		R 30	
R 25		R 10		R 50	

STEP 4:

Model multiplying and dividing by 5 with manipulatives



We put on our 5 glasses.

The **whole-equal part diagram**, has several parts.
We put on our 5 glasses, to see how many 5s we need to make any whole number.

GAME OF THE WEEK:

Multiply by 5

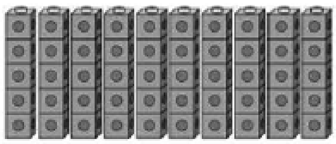
- Work in pairs.
- Prepare by building 10 towers of 5 blocks:

- Your teacher calls a number: eg, 4
  Take 4 towers
- How many towers? How many cubes?
- Count forwards in 5s
  Say a number sentence: 5 repeated 4 times is 10

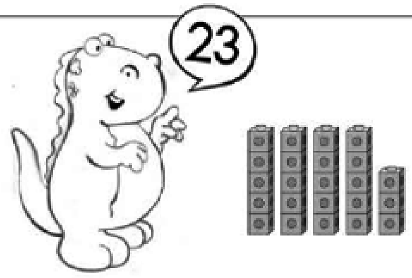
Children can use cubes to make towers of 5 cubes.
Each tower is always 5 cubes. We see all numbers as made up of units of five.
How many towers of 5 make up the whole?

GAME OF THE WEEK: Divide by 5

- Work in pairs.
- Prepare by building 10 towers of 5 blocks:



- Your teacher calls a number.
- Show the number with towers of 5.
- How many left over?



Say aloud: There are 4 fives in 23, and three left over.

This is a helpful point to reinforce the idea of **remainders or left overs**.

- The **multiples of 5** (the counting song of fives) have **no left overs.**
- When you break a multiple of 5 into groups of five, there are no left overs.

Lets talk maths

Let's Talk Math in English!

NgesiXhosa sithi:	In English we say:
Xa siphindaphinda, siphinda amaqela alinganayo. Xa siphindaphinda oo-5, siphinda amaqela oo-5. Sibala sisiya phambili ngoo-5. Sisebenzisa iminwe yethu ukugcina umzila. Inani esilibiza ekugqibeleni yimpendulo. 5 phindaphinda ka-4 ngama-20.	When we multiply, we repeat equal groups. When we multiply 5s, we repeat groups of 5. We count forward in 5s. We use our fingers to keep track. The last number we say is the answer. 5 phinda ka 4 nge 20.
Xa sisahlula, sihlukanisa amaqela alinganayo. Xa sisahlula ngo-5 sibala ngoo-5. Sibala sisiya phambili ngoo-5. Sisebenzisa iminwe yethu ukugcina umzila. Iminwe yethu isinika impendulo. 20 yahlula ka-5 ngu-4.	When we divide, we separate equal groups. When we divide by 5 we count in 5s. We count forward in 5s. We use our fingers to keep track. Our fingers give us the answer. 20 hlula ka 5 nge 4.

STEP 5:

Keep track of the counts

You can keep track of the counts on your **fingers, on a number line, in a horizontal table, in a t-table, or using a function machine**.

Keep track on your fingers

When children are secure counting fives starting at 5, they need to start to keep track of their counts on their fingers. When they know the **fives song** (the multiples of 5), they can start to keep track of the **factors** (how many 5s).

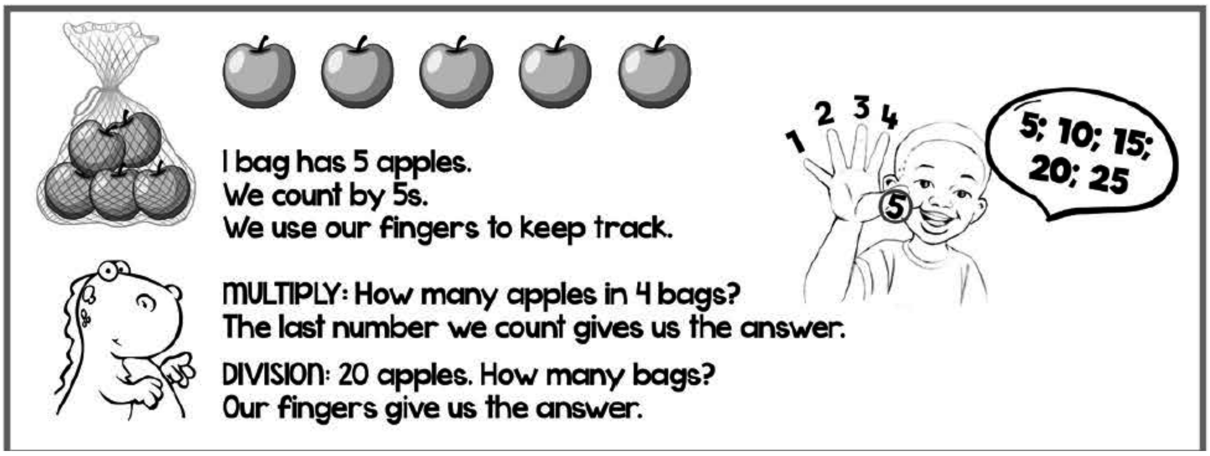
Factors are always in pairs. They are multiplied together to make a **multiple**.

factor × factor = multiple.

1 × 5 = 5 and 2 × 5 = 10 and 3 × 5 = 15.

The multiples of 5 are: 5, 10, 15, 20, ...
1 and 5 are factors of 5. 2 and 5 are factors of 10. 3 and 5 are factors of 15 etc.
For the multiplication song there are always 2 counts (2 different units):

- There are the apples (the fives song or the **multiples**)
- There are the bags (the counts or the **factors** that make a pair with 5)



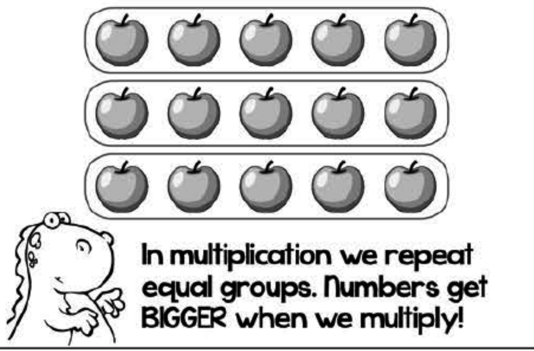
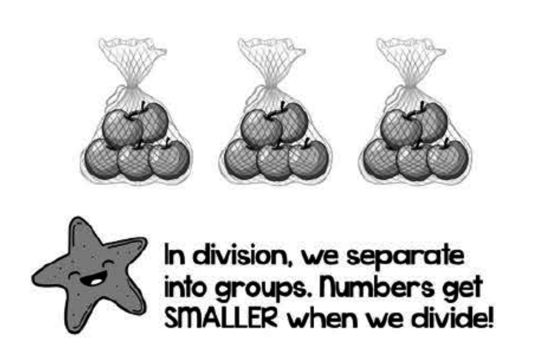
When we multiply we ask:

- How many all together?
- How many apples?
- We find the answer in the multiples (what we say)...

When we divide we focus on:

- How many equal groups?
- How many bags?
- We find the answer by keeping track of our counts (what is on our fingers)...

PrimTED std: NA 4.1
Demonstrate and use the relationship between multiplication and division.

Multiplication	Division
	
5 apples x 3 bags $5 \times 3 = 15$ apples	15 apples. How many bags? $15 \div 5 = 3$ bags

The situation of apples and bags is a **discrete** context. It is using collections or sets. This is the **counting pathway**.

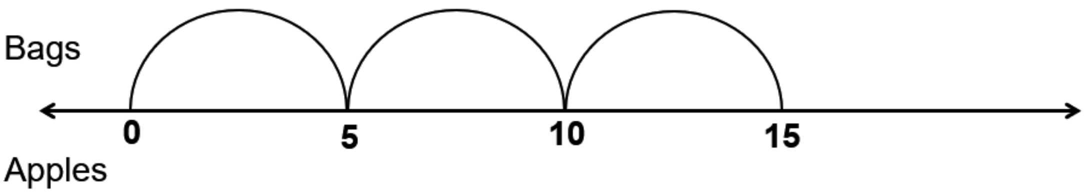
Keep track on a number line

How can we think about multiplication on the measurement pathway?

With multiplying and dividing there are always two units, and we keep track of both units.
We can think about equal groups of 5, as bags of apples. Each bag has 5 apples.
We can represent this situation on a number line.

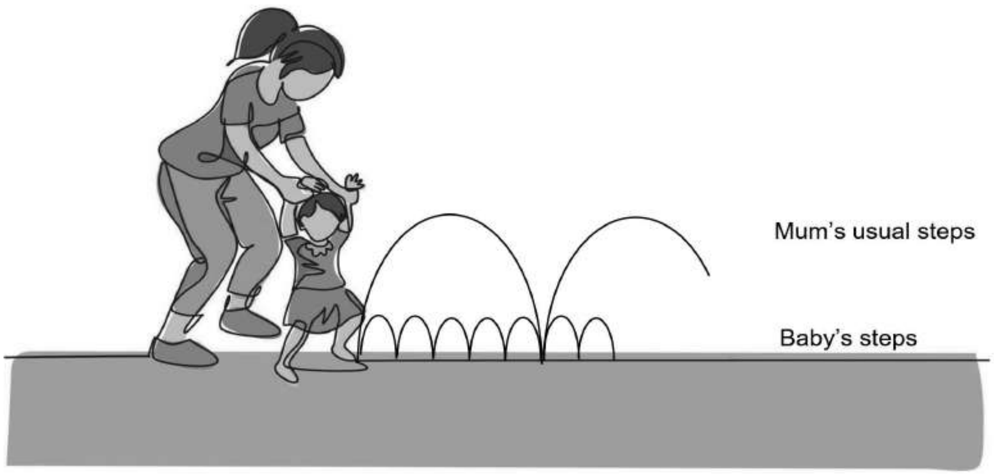
Working on a number line, we jump in 5s. Each jump is a unit of five.

- When you **multiply** by 5, you **jump forwards** in fives. Where you **land** is the answer.
- When you **divide by 5**, you **jump forwards** in fives **to reach a target**.
  The **number of jumps** is the answer.

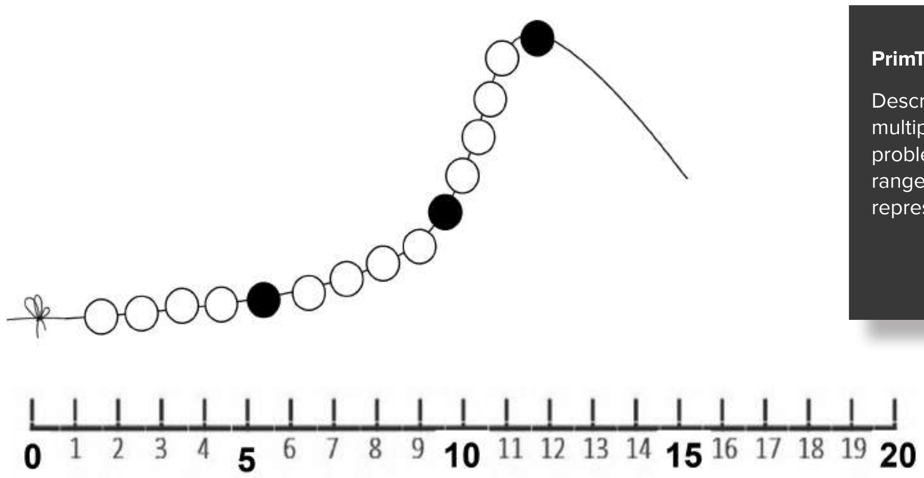


When the number line is introduced it may be easier to use a real **measurement** context such as:

- **Steps**: Mama takes big steps. Baby takes small steps. For each step mom makes, baby makes 5 steps.



THREADING BEADS



PrimTEd Std: NA 4.6

Describe and use multiplication ad division problem types and the range of useful associated representations.

We start with a knot (0 on the number line).

- Then we string 4 white beads, and mark the end of a group of 5 beads with 1 black bead,
- Then repeat: 4 white beads, then mark the end of the group of 5 beads, with 1 black bead.

Each time we thread a group of 5 beads. We mark the end of the group with a black bead.

- How many black beads? ... 3
- How many beads all together? ... 15

The bead threading gives a mix between the counting and measurement pathways. The beads are discrete. When we thread them onto a string they become a line, which has a length. Beads on a string are a helpful way to join the two pathways together.

Keep track using a horizontal table

With multiplying and dividing there are always two units, and we keep track of both units. It is helpful to have a systematic way to keep track of the two different units.

Count by 5s:

Counting by 5s gives us the MULTIPLES of 5. we multiply and divide 5s we count by 5s.

	1	2	3		5					10
	5	10	15							

Compare the vertical T table and the pattern counting with 5s. What is the same? What is different?

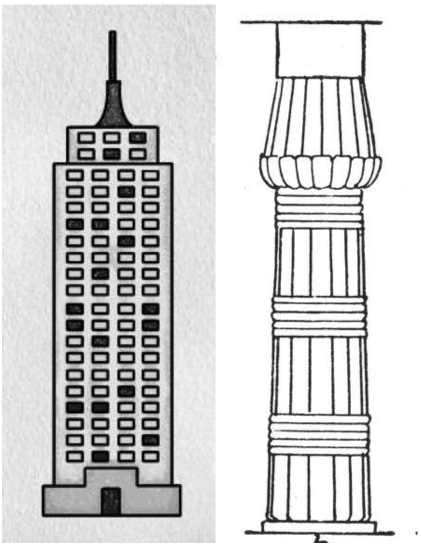
This is a **horizontal table.** It goes across, like the **horizon**.

We read from left to right. A **horizontal table** has the labels on the left. It goes across from left to right. **Rows go across from left to right**. The are repeated downwards. Our eyes go from left to right.

Keep track using a T-table

We can also keep track of the 2 units for multiplication and division using a **vertical table**. We call this a **T-table.** It looks like a big T. It goes up and down. It is vertical. It has columns.

Bags	Apples
1	5



Vertical, is like a **ver**y tall building.

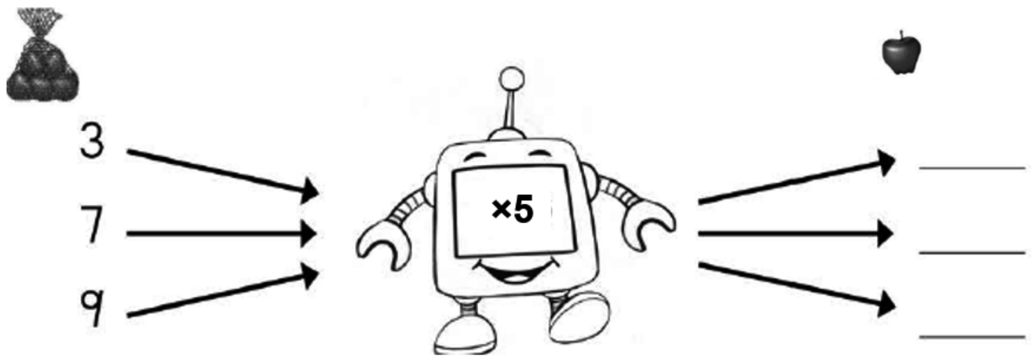
Or an Egyptian **column**.

Columns go from top to bottom. The labels are at the top. We read from top to bottom. Our eyes go up and down. A t-table can keep track of the two units in the multiplication table.

Keep track with a function machine

We can also use a **function** machine to keep track of the two units. A function machine makes us think about what we do to the input, to get the output. For each **input**, there is only one possible **output**.

First, Funky is programmed to change **the number of bags** to **the number of apples.** The function is "repeated 5 times". Here, Funky works from left to right. The inputs are bags. The outputs are apples.



A function machine children keep track of **what if** questions.

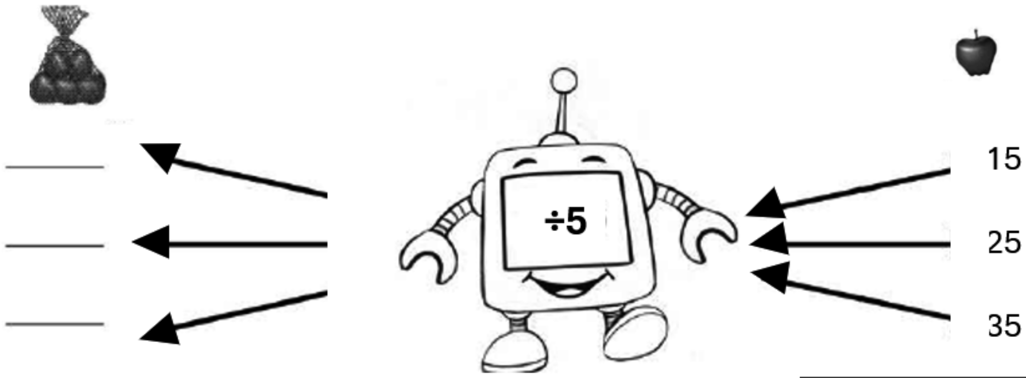
- **What if ...** there are 3 bags?
- Or 7 bags?
- Or 9 bags?

The inputs are bags. The outputs are apples. These are two different units, which we keep track of. The function which changes number of bags to number of apples is: multiply by 5.

No of bags × *5 = Number of apples.*

A function machine is also helpful for working with the **inverse**.
What if Funky was programmed to do the reverse or opposite of ×5?

Now Funky is programmed to change **the number of apples** to **the number of bags.** The function is "make groups of 5" or "How may fives?".



Now funky works from right to left. The inputs are apples. The outputs are bags.

- **What if ...** there were 30 apples? Or 31 apples? Or 17 apples?

The general number sentence (formula which states the relationship) is:

No of apples ÷ 5 = Number of bags.

PrimTEd Std: NA 7 Early algebra.

Function machines are an important part of 'patterns, functions and algebra; in CAPs. They are an important overarching mathematical idea (See NA 8).

STEP 6:

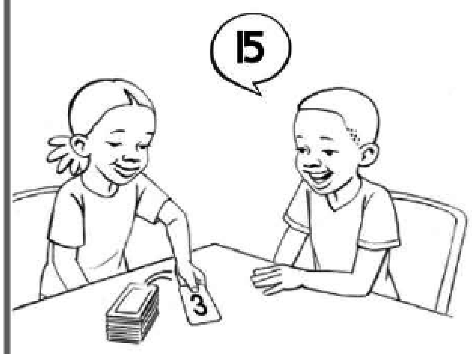
Learn the 5 times table

It is your job as teacher to provide **lots of opportunities to learn and practice the time tables**. You could use 1,2,3 Veza times 5, or Dice times 5, or cards times 5. Fizz Pop times 5 etc.

123 VESA

Multiply by 5

- Play in pairs.
- Put hands behind back.
- Say together, "1, 2, 3! Vesa!" (Show)
- One child shows some fingers using ONE hand.
- Samp shows some fingers using one hand
  Bean multiplies by 5
  First they might count in 5s.
- Soon they should just say the answer
- Do again! Faster!



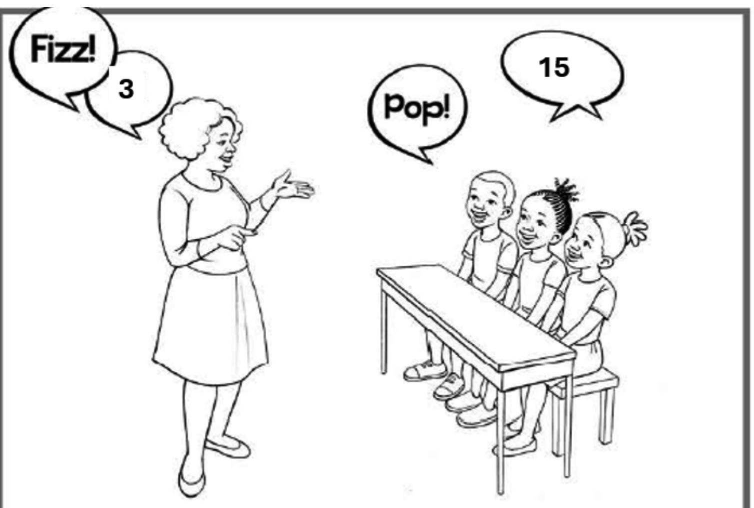
MULTIPLY 5s!

- Play in pairs, with one pile of cards.
- Mix up cards 1 to 10.
- First learner flips a card.
- The second learner multiplies by 5.
- Do it again! Faster!

FIZZ POP:

Multiply by 5

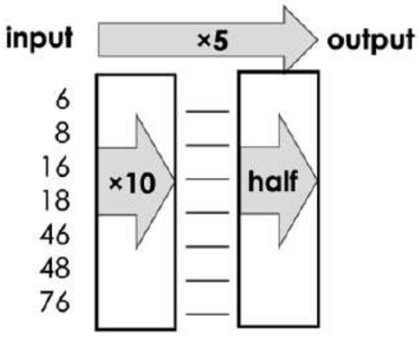
- Use numbers from 0 to 20.

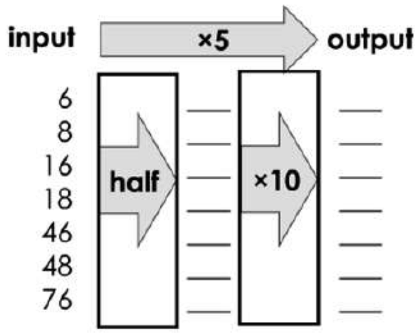


Even with lots of practice, some children may still need help. You can help them make connections to what they do know. T**hese are helpful connections to guide children to learn the 5 times table:**

- **Use multiply by 10 and halve**. So you use what you know about ×10 and ×2, to work out ×5 in two steps, or
- **Use a clue board**; where they also need to know how ×10 and ×2. Over time, remove the clue board

Multiply by 10 and then halve is a quick two-step function, to work out ×5.





You can also halve and then × 10, but that is usually a bit harder for children. Halving odd numbers is a little more tricky.

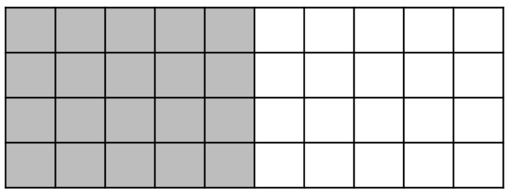
PrimTEd Std. NA 4.1 Demonstrate knowledge of equivalent relations.

The two-step function x10 and then halve, is equivalent to the one-step function x5.

Example

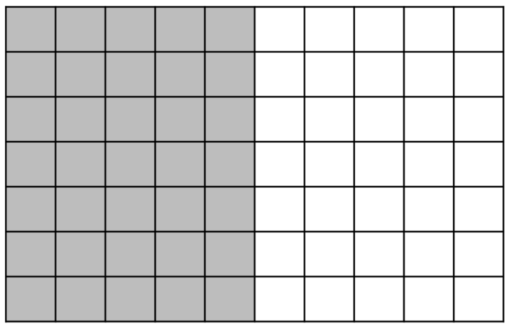
4 × 5 can be worked out by group counting.
But 4 × 5 is probably quickest if you multiply by 10 and then halve.
I know that 4 × 10 = 40.
I know that half of 40 is 20.
Therefore, 4 × 5 = 20.

Each row in the 100 chart has 10



4 **× 10** = 40
But ×5 is half of ×10.
Therefore
4 **× 5** = 20

Each row in the 100 chart has 10



7 **× 10** = 70
But ×5 is half of ×10.
Therefore
7 **× 5** = 35

You can see if a number can be divided by 5, because it ends with 0 or with 5.

A **clue board** is another way to work out any times-table.
All you need to know is the ×10 and ×2 times tables.

Clue boards are very helpful to **scaffold learning** the times tables for Grade 4s (and even 5s and 6s).

Scaffolds are used to build a building structure. When the building is finished, then the scaffold is removed. **Learning scaffolds** are used to build a knowledge structure.
The structure of multiplicative thinking is a whole and equal parts.

Learning scaffolds should fade away over time. The scaffold should no longer be needed, when children know their times table. Clue boards are a condensed t-table. They only use the numbers 1, 2, 4, 8 and 10 on the left had side of the table.

To build a clue board you only need to know the following number facts:

- Doubles and halves
- ×10

And those are easy! They should have been learnt in Grades 1 and 2.
Remember when teaching multiplication tables in grades 1-3:

> *"In foundation phase we don't teach multiplication and division...*
> *We let children solve multiplication or division situations or problems.*
> *We don't use the* **×** *sign or the* **÷** *sign. They reason about situations."*

So it is **always easier to bring in a context**. We will not just think about ×5 and build a clue-board. We think of a structure that always comes in groups of 5. We use that structure.

 1 hand has 5 fingers	 1 bag has 5 apples	 Every 5 minutes the minute hand moves to the next number on the clock

When we have a real context we get the vocabulary to label the two counts.
We can think of **hands** and **fingers**, rather than **number of groups** and **how many in the groups.**

Example

This is a ×5 clue board

×5

Hands	Fingers
1	5
2	10
4	20
8	40
10	50

You create a ×5 clue board like this.

Draw the T for the table.

Write ×5 at the top.

It is helpful to think of a context where there are groups of fives. Eg hands and fingers.

PrimTEd Std. 4.5

Describe and use multiplication and division problem types and the use of useful associated representations. A clue board is a useful representation. It features in the Intermediate Phase CAPS.

For the left hand side:
Start with 1
Double 1 to get 2.
Double 2 to get 4.
Double 4 to get 8
Those are the doubling facts.
Stop and think.
The other easy number fact is x10
So write 10.

For the right had side:
This is the 5 times table.
1 repeated 5 times is 5.
Double 5 to get 10.
Double 10 to get 20.
Double 20 to get 40
Stop and think.
The other easy number fact is ×10
So 5 ×10 = 40.

You can use the ×5 clue board to work out all the other ×5 number facts.

3 x 5 =
5 x 5 =
6 x 5 =

7 x 5 =
9 x 5 =
100 x 5 =

A **clue board** is a technology, a way of doing something, which helps to work out all time-tables. Instead of using a calculator to help you multiply, you can use a clue board. It was invented in the Netherlands at the Freudenthal Institute. Dutch children use it to help both multiplication and division calculations.

(See Anghileri et al, 2002, https://link.springer.com/article/10.1023/A:1016273328213)

SCAN ME

So **if you are stuck, you can quickly make a clue-board** to help you. A clue board is also a helpful **scaffold** when learning the times tables. You can use a clue board to help learn the times tables. The clue board should **fade away** as you get more practice. When you know the 5 times table, by rote, then you can remove the clue board.

Here the × 5 clue board is used to work out:
3 × 5, 5 × 5, 6 × 5, 7 × 5 and 9 × 5
(the × 5 timetable facts that are missing from the clue board).
It is also used to work out bigger numbers, such as 40 × 5.

×5

	Hands	Fingers	
3	1	5	**15**
	2	10	
	4	20	
	8	40	
	10	50	

×5

	Hands	Fingers	
	1	5	
5	2	10	**25**
	4	20	
	8	40	
	10	50	

×5

	Hands	Fingers	
	1	5	
6	2	10	**30**
	4	20	
	8	40	
	10	50	

×5

	Hands	Fingers	
	1	5	
7	2	10	**35**
	4	20	
	8	40	
	10	50	

×5

	Hands	Fingers	
	1	5	
9	2	10	**45**
	4	20	
	8	40	
	10	50	

×5

Hands	Fingers
1	5
2	10
40	**200**
8	40
10	50

TASK

Use your clueboard to work out all the x7 facts for 1-10.	Make a clueboard for x7. The left hand side is DAYS. The right hand side is WEEKS.

Tasks 1.1

Hippo investigation

Hippo SENIOR takes BIG bites of **5 parts** each time.
Hippo JUNIOR takes SMALL bites of **2 parts** each time.
Investigate…

SENIOR **JUNIOR**

Bites	Parts

Tasks 1.1.2

Clue board for investigating multiplicative situations

1. **Make a clue board for ×3**

PrimTEd Std. MAT 1

Playful engagement to search for, and develop, mathematical insight

Use the clue board to answer these questions

a. 7 × 3 =

b. 3 × 3 =

c. 12 × 3 =

d. 6 × 3 =

e. 15 ÷ 3 =

f. 24 ÷ 3 =

g. 120 ÷ 3 =

h. 60 ÷ 3 =

2. Eggs are packed with 6 eggs in each box.

Make a clue board for ×6.
Then complete the other table to explore the relationships between number of eggs and the number of boxes.

a.

boxes	eggs
1	6

b.

boxes	eggs
5	
20	
	90
12	
18	
	120

c. Draw a function machine. The input is number of boxes. The output is number of eggs. Specialise: choose 5 different inputs and write the outputs. Now write the function.

d. Generalise. Write the number sentence or formula to show how to get from the number of boxes to find the number of eggs.

e. Draw a function machine. The input is number of eggs. The output is number of boxes. **Specialise**: Choose 5 different inputs and write the outputs. Now write the function.

Tasks 1.1.3

Two minute tango 2s and 10s

If you completed the **Maths 4 Primary Teachers** *Emergent Number Sense* module, you had an opportunity to practice basic mental fluencies. You should practice Tango 1, 4 and 6.

Tango 1
Ones, tens and hundreds

Tango 4
Doubling and halving

Tango 6
Multiplying and dividing by 2 and 10

SCAN ME

JUMPTRAK

In this course you use jumptrak for:

Two minute TANGO practice;

Use jumptrak online.

Or go to your app store and download: Jumptrak.

You don't need a clueboard for ×2.
You should know how to double.

I know a number is divisible by 2 when...

The factor pairs of 2 are ....

Therefore 2 is prime/composite.

2 written as product of primes is _______.

Each child has 2 eyes.

Number theory is the area of mathematics that focuses on **whole numbers** and the relationships between them.

In primary mathematics, it includes ideas such as **odd and even numbers, multiples and factors, and divisibility.**

Understanding these relationships helps teachers support learners to see **patterns, structure, and meaning in numbers**, rather than only learning procedures, laying a foundation for multiplication, fractions, and later algebra.

SUMMARY 1.1 MAKING SENSE OF THE TIMES TABLES

There are two ways to understand 3 × 5:

1. Draw a picture problem for **3 repeated 5 times**.

2. Draw a picture problem for **3 groups of 5**.

3. Multiplicative thinking always involved **keeping track of two units**.
   Think of a situation where the structure is always a whole and 10 equal parts.

4. Draw a **whole equal part** diagram for the ×10 situation.

5. Show all the different ways that you can keep track of the two units, in your ×10 situation.

PrimTEd std. MAT 1.4

Reflect for action. Writing a summary gives times to pause. Think about what you have learnt so far. Ask questions ...Get clarity. Collaborate.

UMTHAMO 1.2

ADDITIVE VERSUS MULTIPLICATIVE THINKING

Key ideas

1. **Additive thinking** concerns the relationship between a **whole and unequal parts**.
   - Additive situations involve **joining, separating or comparing** collections of discrete objects or continuous measures
   - Additive thinking involves **addition or subtraction**
   - We can link subtraction to addition.
   - 7 – 5 = ... is equivalent to 5 + ... = 7
2. **Multiplicative thinking** concerns the relationship between a **whole and equal parts**
   - Multiplicative situations involve **defining a unit and repeating that unit**.
   - Multiplicative thinking involves **multiplication or division**.
   - We can link division to multiplication.
   - 20 ÷ 5 = ... is the same as 5 × ... = 20
3. **Doubling and halving problems** can be word problems or picture problems. They should include:
   - Problems where there are 2 equal groups. So **a unit is repeated 2 times.** And
   - Problems where children count in 2s. **Pairs or 2s are repeated**.

Additive thinking vs multiplicative thinking

We use ***additive thinking*** when we compare two numbers and think about the difference between them.

- When we compare 2 and 8 using *additive thinking* we would consider 8 – 2 = 6, so their difference is 6, or 2 is 6 less than 10.

This is in contrast to ***multiplicative thinking*** when we think about how many of one number, fits into, or can be repeated to make, another number.

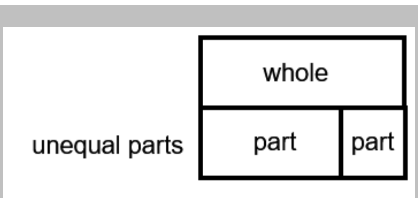
- When we compare 2 and 8 using *multiplicative thinking* we would consider 8 ÷ 2 = 4, so there are 4 twos in 8, or 8 is 4 times more than 2.

These are two very different ways of thinking.

Additive thinking uses **adding** and **subtracting**.

Additive thinking

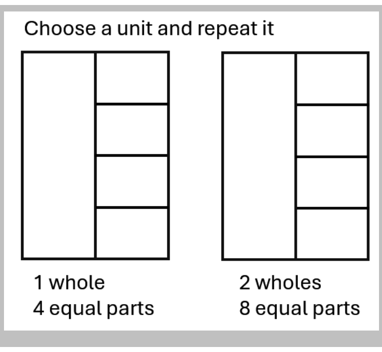
Diagram



Additive thinking concerns the relationship between a whole and **unequal parts.**
Additive thinking is about **whole-part-part** relationships. The two parts do not need to be equal. There can be a small part and a big part. You **combine or join** the two parts to get the whole. We can **compare** a whole to one part.

Multiplicative thinking

Diagram

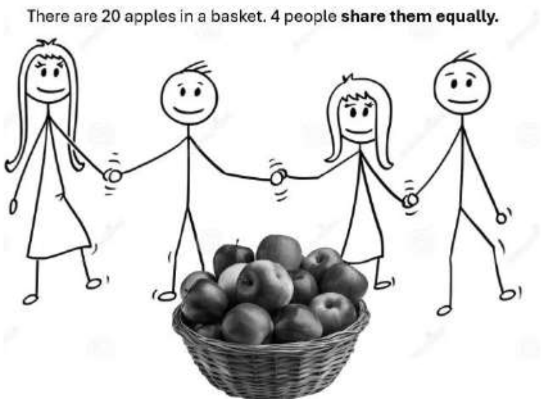


Multiplicative thinking concerns the relationship between a whole and equal parts.
Multiplicative thinking is **whole-equal-part** relationships.
We have an equal part which we repeat to make a whole.
We **repeat** the smaller part to get the whole.

Multiplicative thinking uses **multiplying** and **dividing**.

Here we have a box of 4 apples.
The box is repeated 5 times.
4 repeated 5 times is 20 apples.
4 phinda ka 5 = 20.

Division is the inverse of multiplication



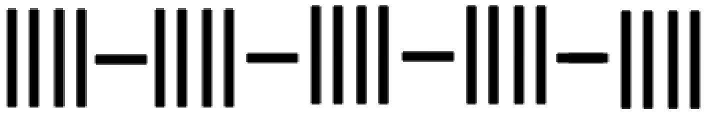
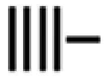
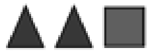
We can think about division as starting with a big number, and **sharing** that big number **equally** amongst a group of people.

We can think about division as starting with a big number, and **packing** that big number of things into equal groups. How many boxes do you need?

Geometric patterns in **Foundation Phase** are usually multiplicative. There is a group or unit, and that unit is repeated again and again.

Activity for learners

Copy and extend the pattern.

 Circle the unit that repeats.	The unit that repeats is: 
Extend the pattern.  Circle the unit that repeats.	The unit that repeats is: 

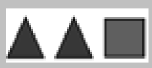
TASK

To make a pattern we repeat the unit. We explore how many equal parts there are for different numbers of wholes.

1. For this pattern  each unit that repeats (the whole) has 5 lines.

If you repeat the whole 10 times, how many lines will you draw? ______

If you repeat the whole 10 times, how many horizontal lines will you draw? ______

2. For this pattern  each unit that repeats has 3 shapes.

If you repeat the unit 5 times, how many shapes will you draw? ______

If you repeat the unit 5 times, how many triangles will you draw? ______

3. Make your own repeating patterns. Circle the unit that repeats.

Consider these two contrasting examples:

Example

Example A
You are watching cars come out of a parking lot. You notice that every car has 4 tyres.
You are not sure if there is a spare tyre in the boot, so you focus on the tyres you can see.

- The first car comes past, you see 4 tyres.
- The second car comes past also 4 tyres. So that is 8 tyres now.
- The third car comes past also 4 types, so its 12 tyres now.

You stand for half an hour and 10 cars have come past.
How many tyres have gone past?
Can you get an exact answer or must you estimate, between a maximum and minimum?

Example

Example B

You are watching cars come out of a parking lot. You notice that the first car has 4 people in it. You are not sure if there are people in the boot, so you focus on the people you can see in the car.

- The first car comes past, you see 4 people.
- The second car comes past also 4 people. So that 8 people now.
- The third car comes past with 3 people, so its 11 people now.

You stand for half an hour and 10 cars have come past.
How many people have gone past?
Can you get an exact answer or must you estimate, between a maximum and minimum?

What is the same and what is different about these two examples?

What is the same?

- Both examples are about watching cars come out of packing lot.
- Both examples are about counting things related to each car.
- In both examples the first and the second car each have 4 'things'.
- In both examples you see 10 cars go past.
- Both examples can be solved using repeated addition.

What is different?

Example A	Example B
• In example A you count tyres. • There are always 4 each time. The ratio of cars:tyres is always 1:4. • This structure is fixed and cannot change. It just repeats.	• In example B you count people. • There is always at least 1 person (the driver), • The third car has 3 people, not 4. It breaks the pattern. • But there can be 2, 3, 4, 5 or (at a squash 6 or 7) people in a car. • The structure is not fixed. The number of people in each car can change. • It might repeat, but sometimes it does not.

1. Which example presents an additive situation? _______________
2. Which example presents a multiplicative situation? ________________
3. Write a word problem that has an **additive situation**.

4. Write a word problem that has a **multiplicative situation**.

Multiplicative thinking gives rise to fractions

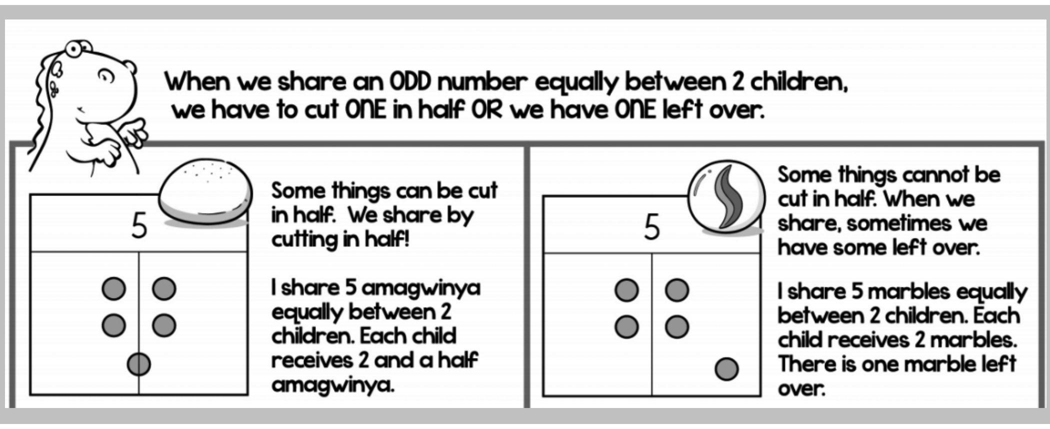
PrimTEd Stds. NA 3 Rational numbers and operations on rational numbers.
Rational numbers extend whole numbers, to include fractions, decimals and percentages. When we halve odd numbers (divide by 2) children are introduced to fractions or remainders

Right from Grade 1 children start working with **fractions**.
They double and halve.
They start working with **unit fractions**: 1 part of a whole.
When do you teach young children about remainders? When do you teach them about fractions?

Right, from Grade 1 if you are halving an odd number you get a remainder or you get a fraction. This is always done in a picture or word problem context

- You get a **fraction** if what you are dividing **CAN** be cut into equal pieced (like amagwinya or fat cakes)
- You get a **remainder** if what you are dividing **CANNOT** be cut into equal pieced (like marbles)

Example



When introducing the concept of a fraction there three important concepts for **unit fractions** in the early grades:

1. **A whole** can be divided into any number of **equal parts**.
2. **One equal** part of a whole is called a **unit fraction**. **Each unit fraction has a name**. We write 1 third. The fraction name "third" tells us how many equal parts in the whole.
3. A fraction (one equal part of a whole) is always **a fraction of something**:
   a. a fraction of one apple (**one discrete object**)
   b. a fraction of a paper strip (**a length or measurement unit**)
   c. a fraction of a group of people (**a group of discrete objects**).

Note that writing fractions using fraction notation, with a numerator on top of a denominator, is only introduced in Grade 4. **In Foundation Phase children write the fraction name (denominator) as a word**.

- So as a Foundation phase teacher you write 1 half, you do not write ½.
- You say "1 half", you do not say "1 over 2".

This is helpful in Intermediate Phase too. So, although learners start using the fraction notation (numerator over denominator), it is important that they first firmly understand that:

- The **denominator is the unit.** It is the **fraction name** (it names the size of the fractional part).
- The **numerator is how many parts** (how many times you repeat the fraction name).

Going too quickly into fractional notation causes a lot of problems as children read the numerator and the denominator as whole numbers, and don't grasp that the top number is a whole number (how many parts), the bottom number is a unit - it is the fraction name.

There are 3 ways to think about a fraction as **a fraction of something**:

1. A fraction of 1 discrete object

2. A fraction of a measurement

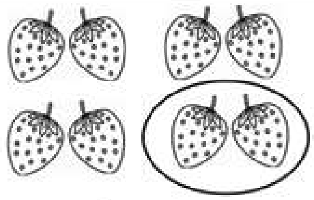
3. A fraction of a group of discrete objects

A fraction of a group of discrete objects

Children can work out a fraction of a group by sharing equally.
They need to make equal groups.

- To **find a half** they share equally between 2.
- 'Find half' of something is the same as divide that something by 2.
- To **find a third** they share equally between 3.
- Find a third of something is the same as divide that something by 3.
- To **find a quarter** they share equally between 4.
- Find a quarter of something is the same as divide that something by 4.

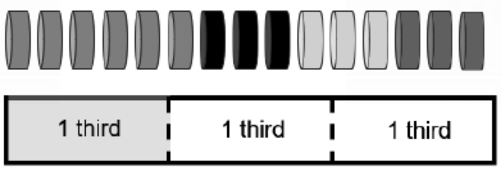
Diagram

Model	Say	Write	Notation
 1 quarter	*'The strawberries have been divided…'*	Write the division bar.	$\frac{1}{4}$
	'…into 4 equal parts…'	Write '4' as the denominator.	
	'…and 1 of the parts is circled.'	Write '1' as the numerator.	

The model of a fraction as part of group of strawberries, uses a discrete context of a group of objects. Another model for a fraction of a group is a packet of sweets.

Diagram

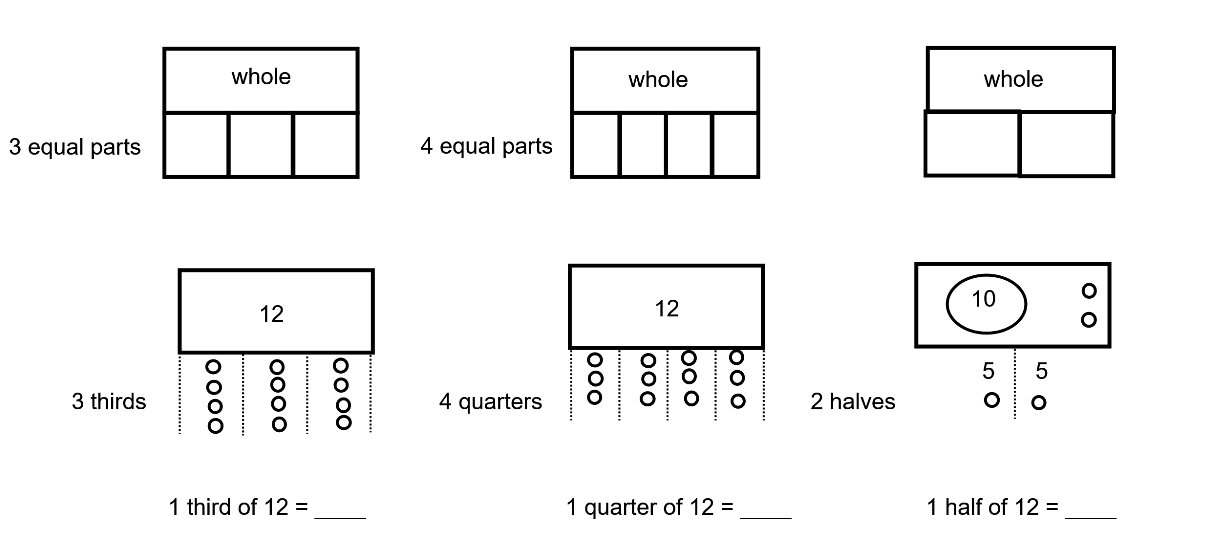
There are 15 wine gums in a packet.



3 children share a bag of 15 sweets equally.
They divide the sweets into 3 equal parts.
Each child gets **1 third** of the sweets.
1 third of 15 = 15 divided by 3

Diagram

Whole equal parts diagram



When I divide 1 group into 3 equal parts, I get 3 thirds.

Therefore, to **find 1 third,** I **divide the group by 3**.

When I divide 1 group into 4 equal parts, I get 4 quarters.

Therefore, to **find 1 quarter,** I **divide the group by 4**.

PrimTEd std. NA 5.3
Demonstrate and use **part-whole relationships** when working with fractions.
When we divide by a whole number, we find a fraction of a group. Divide a group by 3 is the same as finding 1 third of that group.

IDEAS FOR YOUR CLASSROOM

In **Foundation Phase** you should make sure that multiplication and division problems are set in a **real or imaginable context**. Think about **the multiplicative structure** of a whole and equal parts.

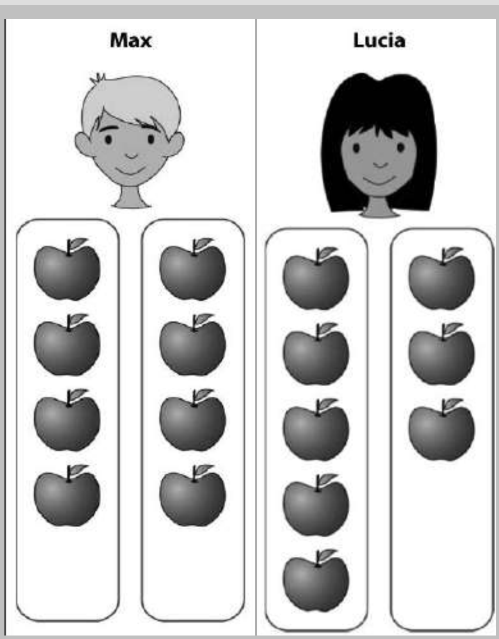
- What are your learners familiar with, that come in pairs or **are structured in 2s**?
  Human eyes, ears, nostrils, legs, shoes, earrings, socks…
- What are your learners familiar with, that come in 10s or **are structured in 10s**?
  A pack of 10 candles, a roll of 10 sweets, 1 person has 10 fingers and 10 toes.
- What are your learners familiar with, that come in 5s or **are structured in 5s?**
  There are 5 fingers on one hand. There are 5 school days in a week, 5 dots on a dice.

Grade R and 1

In Grade R, and 1 children need lots of practice counting in 1s and then counting in 2s (odd and even number songs).

They need to **group collections of objects.** They need to understand **equal** and **unequal groups.**

Activity for learners



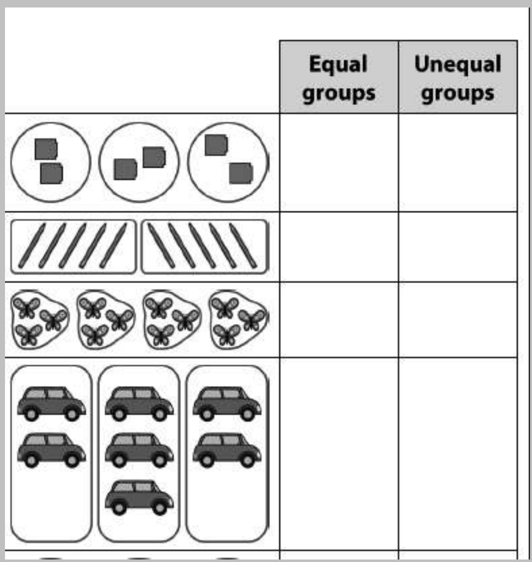
Max groups his apples

Lucia groups her apples.

Max's groups are **equal**. Max has the **same** number of apples in each group.

Lucia's groups are **unequal**. Lucia has a **different** number of apples in each group.

Activity for learners



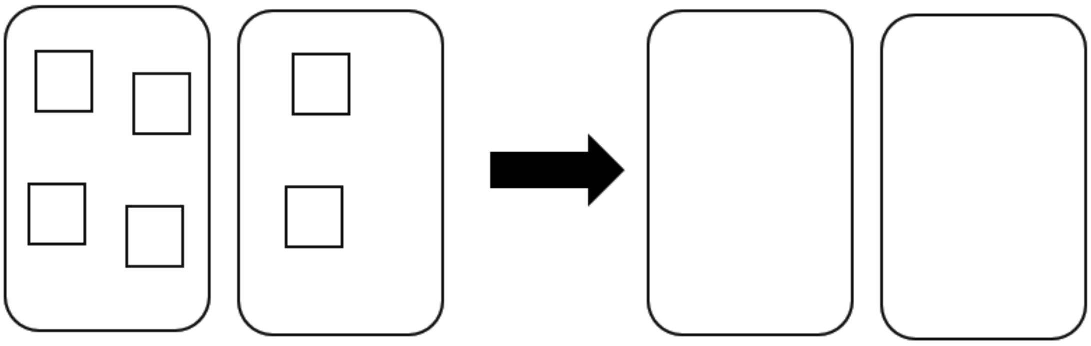
Tick if the groups are equal

Activity for learners

Mandla arranges 12 cubes into equal groups. Finish his drawing for him.

Thami arranges 12 groups into unequal groups. Finish his drawing for him.

Activity for learners



How many? How many?

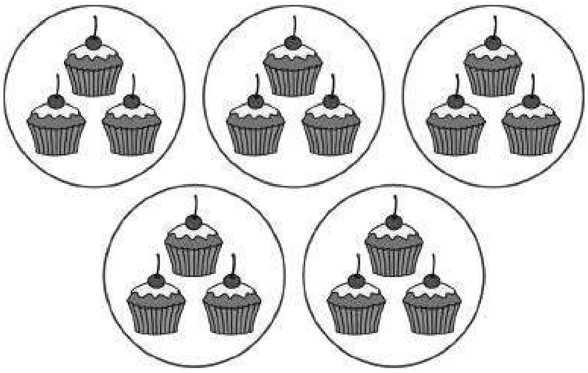
___ squares ___ squares

The 2 groups are **unequal**.

Draw the squares.

Make the 2 groups **equal**.

Activity for learners



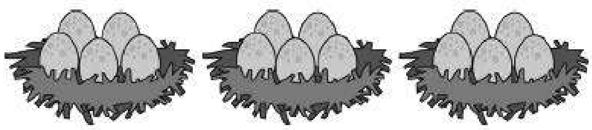
Are the groups **equal** or **unequal**?

How many **plates**?

How may cakes on **each plate**?

How many cakes altogether?

Activity for learners



Complete the sentences:

There are _____ nests.

There are _____ eggs in each nest.

The ____ eggs are repeated _____ times.

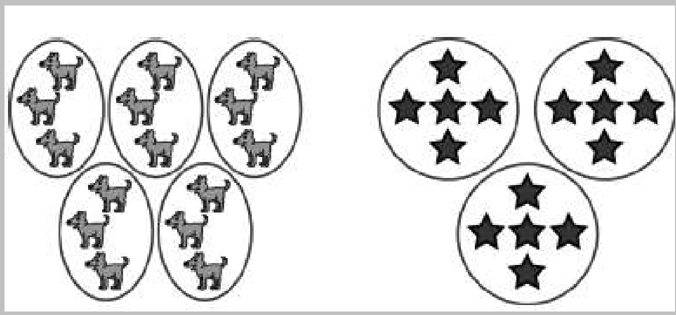
The groups are **equal / unequal**.

There are _____ eggs in each group

There are ____ equal groups.

PrimTEd std. NA 4.5
Describe and use **multiplication and division problem types** and the range of useful associated **representations**. The problems on this page use discrete objects. The concept of equal groups is fundamental to division and multiplication.

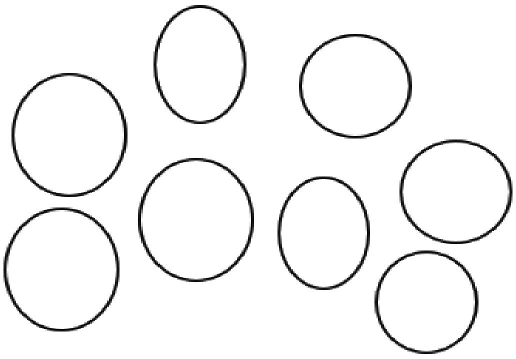
Activity for learners



Which picture shows:
3 repeated 5 times?

Which picture shows
3 groups, with 5 in each group?

Activity for learners



Here are some faces

Choose 3 faces next to each other.

Draw in their eyes

Draw in mouths.

Draw around 1 group of 3 faces.

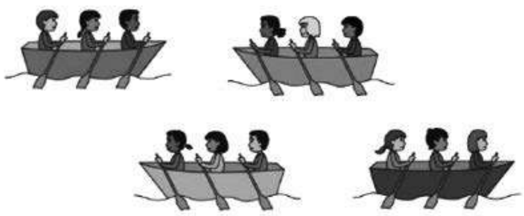
Draw around another group of 3 faces.

Grade 2 and 4

In Grade 2 and 3 children still need support with talking about:

- The unit or **group that repeats**.
- **Equal groups**: How may groups? How many in each group?

Activity for learners



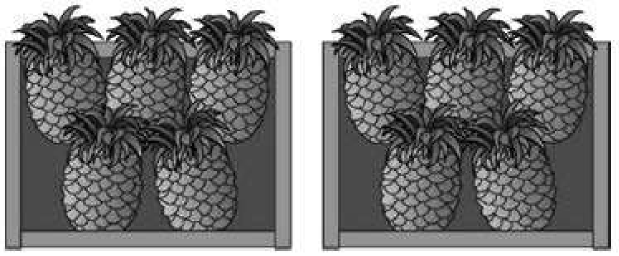
Which sentence is correct?

There are 4 groups, with 3 in each group.

4 is repeated 3 times.

3 is repeated 4 times

Activity for learners



Which sentence is correct?

There are 5 groups, with 2 in each group.

5 is repeated 2 times.

If this is difficult, let children first talk about the pictures using the objects in the pictures.

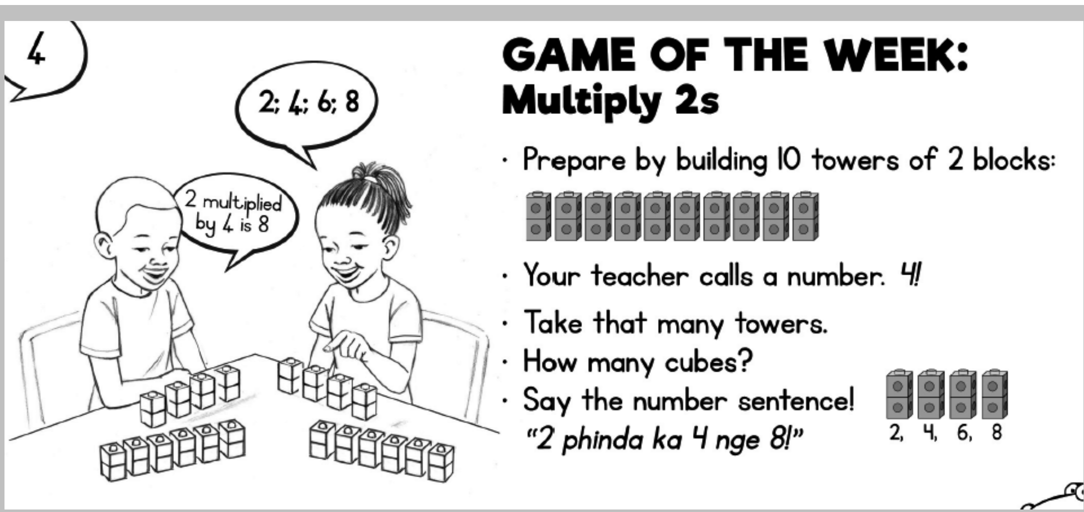
- How many boats? How many people in each boat?
- How many boxes of pineapples? How many pineapples in each box?

They must shift their attention from the unit that repeats (the equal group) to how many there are in each group.

Talk about finding the unit that is repeated. What is the unit that repeats?

In Grade 2 and 3 children need lots of practice counting in 2s, 5s and 10s.

Activity for learners



Give them cubes to count and arrange in equal groups. Give them pictures with a multiplicative structure to count. Let then draw and arrange objects or shapes in equal groups.

Activity for learners

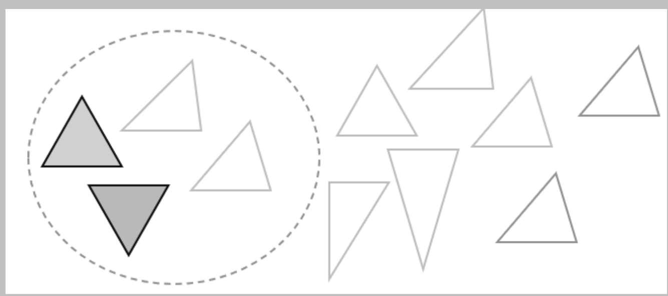
Draw around 4 triangles.

Colour in each triangle in the group of 4.

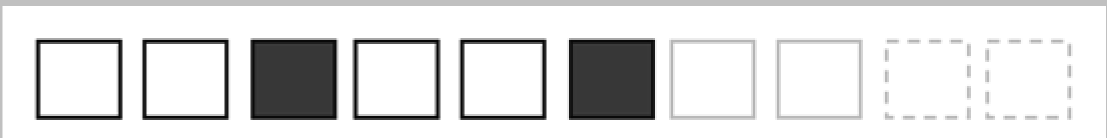
Find another group of 4 triangles.

Draw around to make a group.

Colour in the triangles.



-✂-- Activity for learners

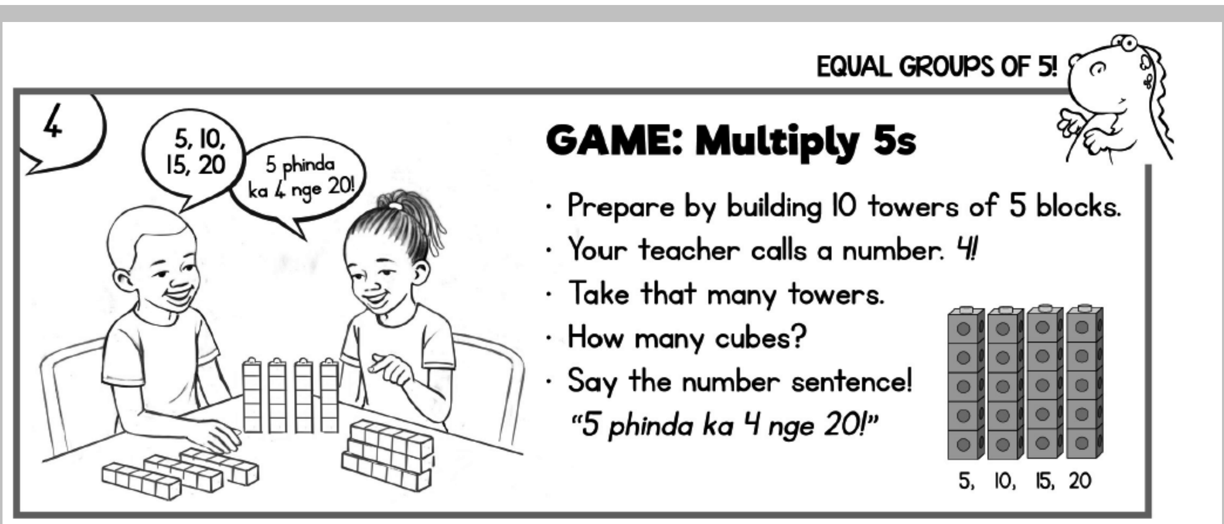


Extend the pattern to the end of the page.

Draw around the unit that **repeats**

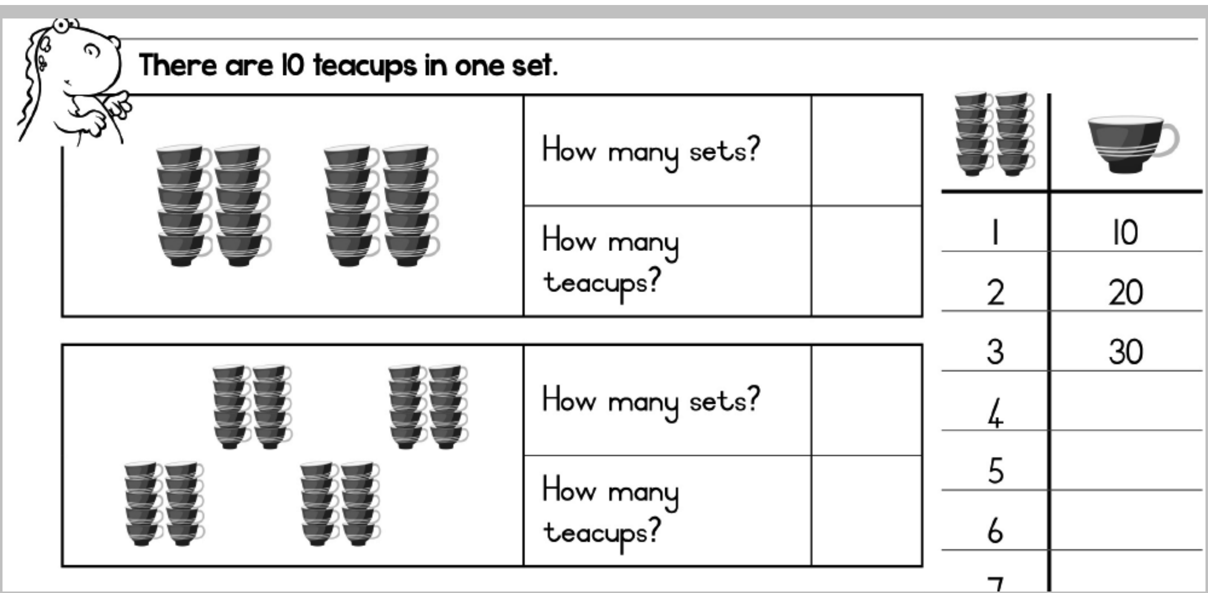
Make sure that children explore multiplication and division using **both the counting pathway** (discrete objects) **and the measurement pathway** (continuous quantities).

-✂-- Activity for learners



Multiplicative thinking using picture problems on the **counting pathway.**

-✂-- Activity for learners



Activity for learners

Multiplicative thinking using picture problems on the **measurement pathway.**

How many buckets	
How many litres?	

How many buckets	
How many litres?	

In **Foundation Phase** children need to practice multiplicative thinking using concrete and picture contexts. They need to work on word problems and picture problems in counting and in measurement contexts.

Activity for learners

A snail moves 10 cm in each hour. Here is the snail after 4 hours.

Write a number sentence for the length that snail has moved using + signs:

____

Write a number sentence for the length that snail has moved using × sign:

____

TASK

1. Pose a problem which uses a number x 10, with discrete objects (the counting pathway).

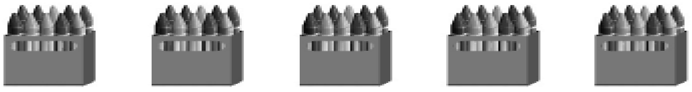
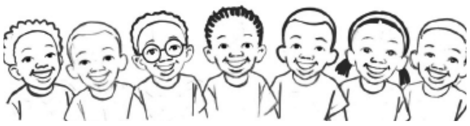
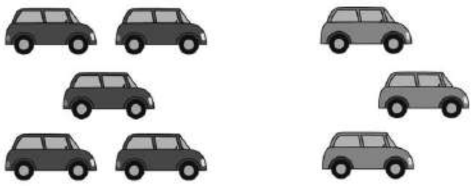
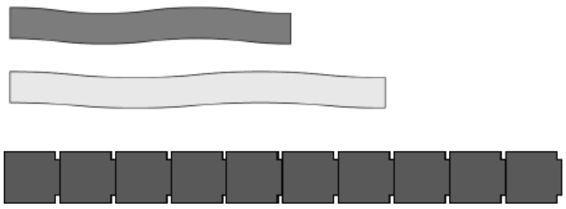
2. Pose a problem which uses a number x 10, with continuous amounts (the measurement pathway).

Tasks 1.2.1

Additive or multiplicative thinking

1. **For each problem**

   - decide whether it is **discrete or continuous** context;
   - decide whether it is an **additive or multiplicative** problem?

	Discrete or continuous	Additive or multiplicative
 10 crayons in each box. How many boxes? How many crayons?		
 How many children? How many eyes?		
 How many red cars? How many blue cars? How many cars altogether?		
 Which ribbon is longer? How much longer?		
 How many shoes? How many pairs?		

2. Create some tasks where children are doubling 6 and they have a picture problem.

The picture contexts can be on the counting or the measuring pathways.

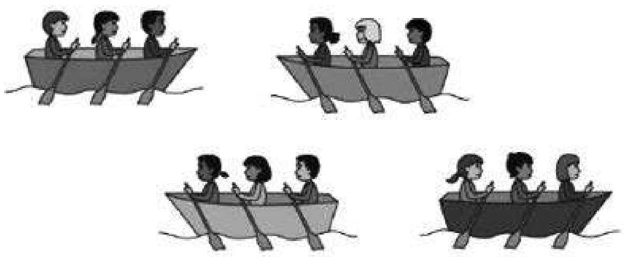
First draw a picture where 6 is repeated 2 times.

a. Write down a question, so learners work out double 6.

Now draw a picture where 2s are repeated 6 times.

b. Write down a question, so learners must count in 2s, six times.

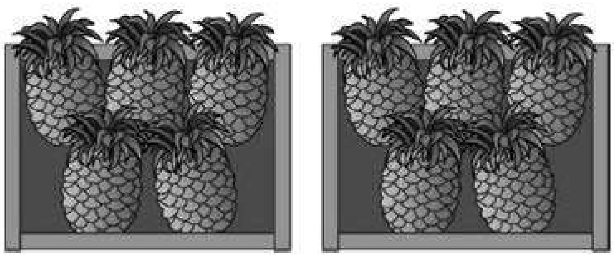
3. Which sentence is correct. Draw a tick next to the correct sentence.
Put a cross next to the incorrect sentence.



Which sentence is correct?

There are 4 groups, with 3 in each group.

4 is repeated 3 times.



Which sentence is correct?

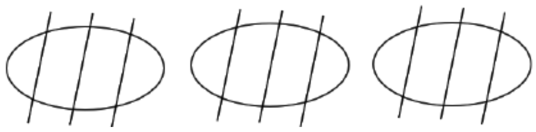
There are 5 groups, with 2 in each group.

5 is repeated 2 times.

Investigation

4. Investigate this pattern

a. Draw the next shape and the next shape.

Ovals	1	2	3	4	5	6
						
Number of straight lines	3	6	9			

b. How may straight lines for 1 oval? ________________________________

c. How many straight lines for 2 ovals? ________________________________

d. How many straight lines for 6 ovals? ________________________________

e. How many straight lines for 60 ovals?________________________________

f. How many straight lines for 600 ovals? ______________________________

g. I have 30 sticks to use as straight lines. How many ovals do I need? ________

h. I have 300 sticks to use as straight lines. How many ovals do I need? _______

i. Write a number sentence which shows how you get the number of straight lines, if you know the number of ovals.

j. Write a number sentence which shows how you get the number of ovals, if you know the number of straight lines.

k. Look at the picture of boats in question 3. How many ovals (in this pattern), is most like the picture of boats?.

PrimTEd Std. MAT 1.1
Playful engagement to search for and develop mathematical insight. Any multiplication or division word problem or picture problem can be extended to an investigation. Just ask What if? questions.

- What if there was another shape?
- What if there was 100 shapes?
- What if there was 4 straight lines?

Investigation

A platform shoe has a 10 cm heel and a 4 cm platform (raised part below the toe).

Actual height is measured with bare feet, and with feet flat.

This shoe makes girls' taller.

The platform shoe is a function. It changes the input (women's usual height) to the output (women's new height with shoe)

5.a **Explore using some different representations (function machine, horizontal table, t-table)**

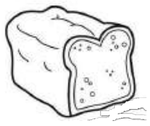
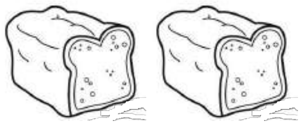
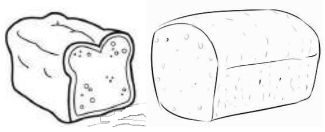
5.b What is the relationship between the **actual height** and **new height**?

5.c Is this an additive function or a multiplicative function?

Tasks 1.2.2

Unit fractions

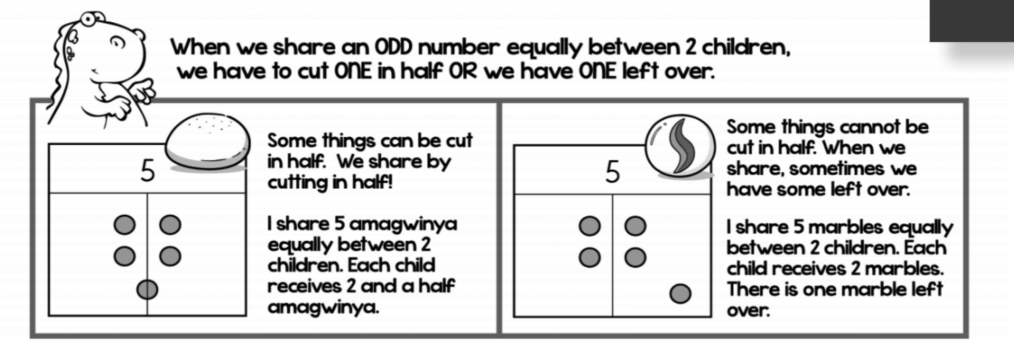
1. How many loaves? Write the fraction. **Use words for the fraction number. Use number symbols for how many there are.** Do not write as a numerator over a denominator.

	1 half
	
	

PrimTed Std. 5.4
Describe and use a range of representations, calculation strategies and tasks that are used to solve problems involving **rational numbers**.

Rational numbers include whole numbers and fractions. 1 half is a rational number. 3 halves is a rational number.

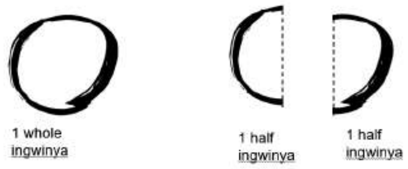
2. Sharing amagwinya versus sharing marbles.



Draw to show how to halve a group or share a group equally between two.

a. Half of 9 amagwinya	b. Half of 9 marbles
c. Half of 7 hotdogs	d. Half of 7 balloons

3. Draw **number pictures for fractions**



Amagwinya	Number picture
2	
2 and 1 half	
3	
3 and 1 half	

4. Sharing fizzers. Fizzers are sweets that you can cut into smaller parts.

a. If 2 people share 1 fizzer equally, then each person gets ________________ of a fizzer.

b. If 4 people share 2 fizzers equally, then each person gets ________________ of a fizzer.

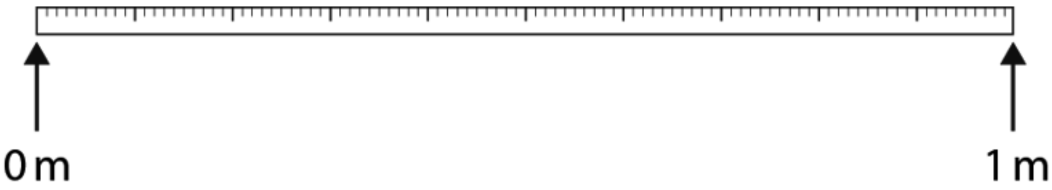
a. I have 3 fizzers. So, I can get ____________________ half fizzers.

b. Share 3 fizzers equally between 2 people.
Each person gets: __________ fizzers.

c. Draw a line in the middle of 3 fizzers.
Half of 3 fizzers is ________________________ fizzers.

5. Fractions of a length

a. Draw an arrow in the middle of 1 metre. Write: 1 half m.

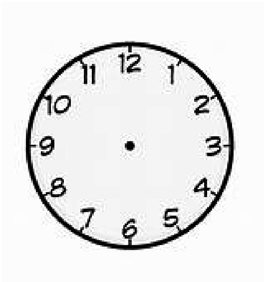


b. 1 metre is the same as 100 centimetres (cm). Therefore, 1 half metre is the same as ______ centimetres (cm).

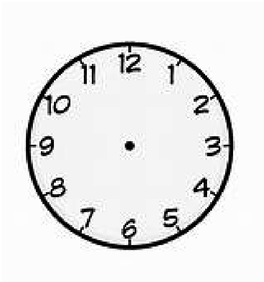
6. Fractions with time

a. I know there are 60 minutes in 1 hour.
Therefore there are _____ mins in 1 half hour.

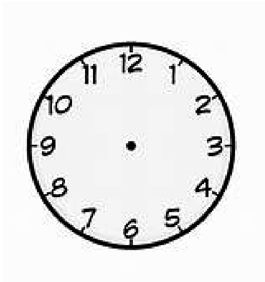
b. We can count in half hours. Start at 12h00 midnight, count on in half hours. **Draw the hands on each analogue clock**. Pay attention to the hour hand as **it should be half way between the hours.** So the hour hand for 01h30 is half way between 1 and 2 o'clock.



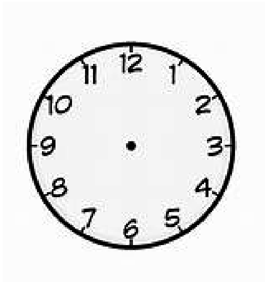
12h00



12h30

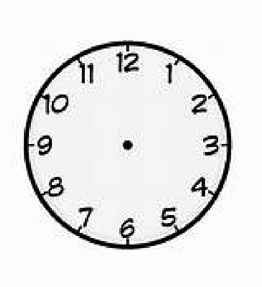
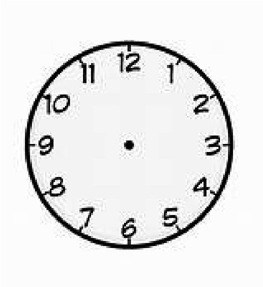
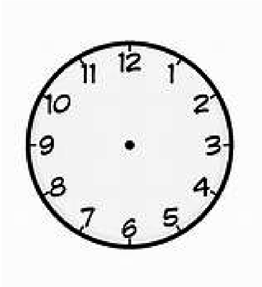
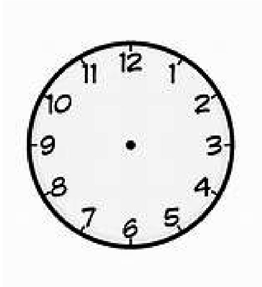


01h00

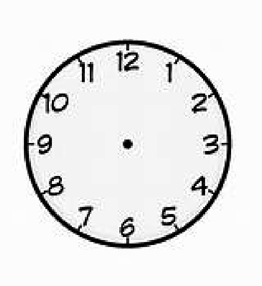
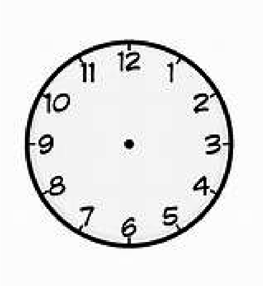
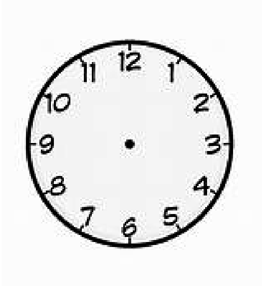
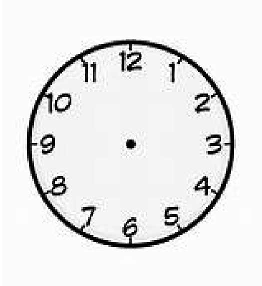


01h30

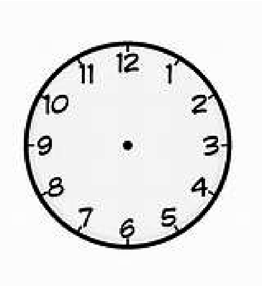
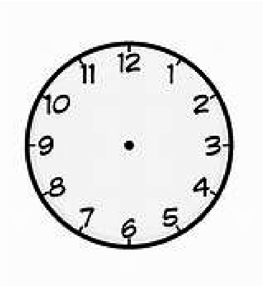
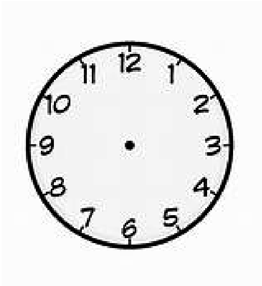
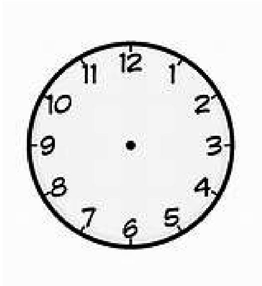
c. Write the digital time below each clock.

12h00 12h30 01h00 01h30

02h00

7. This problem uses both additive and multiplicative thinking. Try this problem for yourself:

When I was 6 years old, my sister was half my age. Now that I am 60 years old, how old is my sister?

- Luisa reasons that my sister is now 30.
- Okuhle reason that she is now 57.

Who is correct, and why?

PrimTEd std MAT 3.
Reason mathematically
As a teacher you should be able to explain your reasoning, conjecture, and convince.

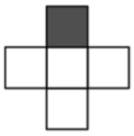
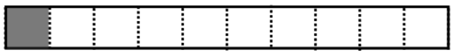
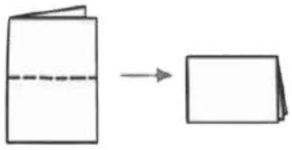
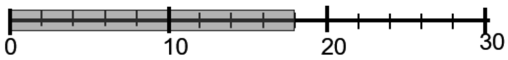
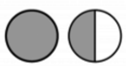
8. Write a word problem for **halving (numbers must be 10 or less)**, where the answer includes 1 half.

9. Write a word problem for **halving (numbers must be 10 or less)**, where the answer includes 1 left over.

PrimTEd std NA 5.4
Describe and use a range of representations, calculation strategies and tasks that are used to solve problems involving **rational numbers**. In this case, dividing an odd number by two results in a remainder of 1, or 1 half (a rational number).

Tasks 1.2.3

Unit fractions

1	1 third means 1 part out of ... equal parts.	11	You fold a strip of paper into 10 equal parts. Each part is 1 ...
2	 1 ... is shaded.	12	 1 ... is shaded.
3	 1 ... is shaded.	13	 1 ... _____ is shaded.
4	 1 ... is shaded.	14	 There are 12 sweets. 1 ... of the sweets = 3.
5	 10 cups fill 1 jug. Therefore, 1 cup is 1 ... of the jug	15	You fold a cloth in half and in half again. Each part is 1 ... 
6	1 hour is 60 minutes. Therefore, 1 quarter of an hour is ... mins.	16	 The amount shown is ...
7	A rope is 6 metres long. 1 third of the rope is ...m	17	1 tenth of 30 is ....
8	 ... halves are shaded.	18	1 tenth of 31 is 3 and 1 ....
9	1 fifth of 10 = ...	19	Half of 7 = 3 and ... half
10	1 tenth of 20 = ...	20	Half of 11 = 5 and 1 ....

A fraction is always a fraction of something.

1. A fraction of 1 discrete object

2. A fraction of a measurement

3. A fraction of a group of discrete objects

SUMMARY

SUMMARY 1.2

1. What is the difference between additive and multiplicative thinking? Give some examples and draw some pictures.

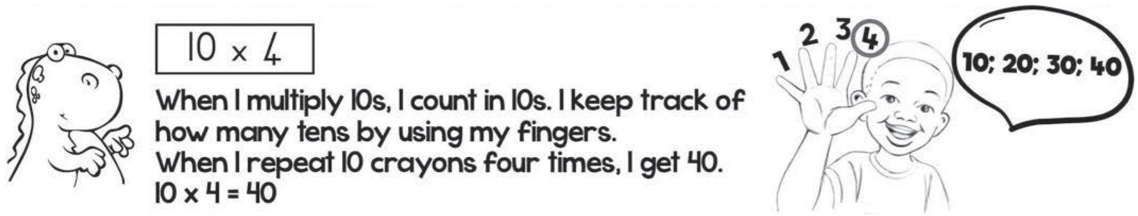
2. A fraction is always a fraction of something. Draw pictures to show
   a. 1 half of 1 discrete object
   b. 1 half of a measurement
   c. 1 half of a group

UMTHAMO 1.3

Sharing and packing/grouping

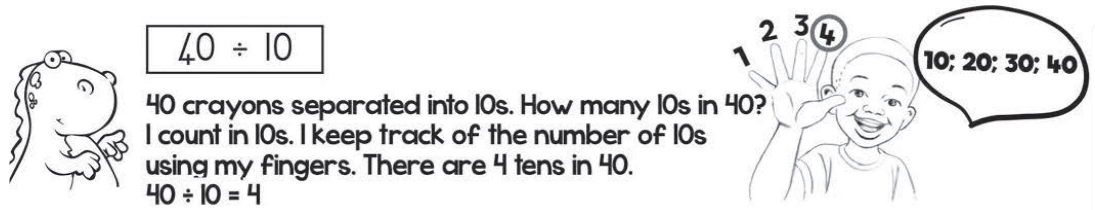
Key ideas

1. The × sign means either **repeat by** or **groups of**.
   a. Foundation Phase children work or word problems or picture problems. They don't use the × or ÷ sign until Grade 3.
   b. In South Africa we start with multiplication as **repeat by** as phindaphinda (isiZulu and isiXhosa) means repeat-repeat.
2. Children should quickly know that with multiplication the order does not matter. $5 \times 3 = 3 \times 5$. **Multiplication is commutative**.
3. Children should know that with division the order does matter. $10 \div 2 \neq 2 \div 10$. **Division is NOT commutative**
4. **The ÷ sign means either sharing or grouping/packing**.
   c. Children work or word problems or picture problems.
   d. Sharing problems are about **sharing equally** between a number of people
   e. Grouping problems are about **packing into equal boxes or bags**.
5. Learning about **10s is easy** and can be done through word problems or picture problems.
   a. Children can **group count to multiply by 10.** They group count forwards in 10s. They keep track of the counts on their fingers.
   b. Children can **group count to divide by 10.** They group count forwards in 10s to reach a target. They keep track of the counts on their fingers.



GROUP COUNTING
Notice that with multiplication and division calculations you always count FORWARDS. You count forwards from the first multiple, keeping track of the counts on your fingers. Eg. Count forwards in 10s, start at 10

STRATEGIC COUNTING
We use strategic counting for addition and subtraction calculations. Here we start at any number. We say what unit we are counting in (1s or 10s or 100s). We count forwards for addition. We count backwards for subtraction. E.g, Count forwards in 10s start at 34.



The meaning of the × sign: 'Repeat by' or 'groups of'

In Foundation Phase we delay the introduction of a symbols × and ÷.

But young children can work on multiplicative tasks. These involve situations of **repeating equal groups** (multiplication) and **sharing** and **grouping/packing** into equal groups (division), in real and imaginable contexts. You can encourage children to skip count (especially counting in 2s, counting in 5s, counting in 10s)

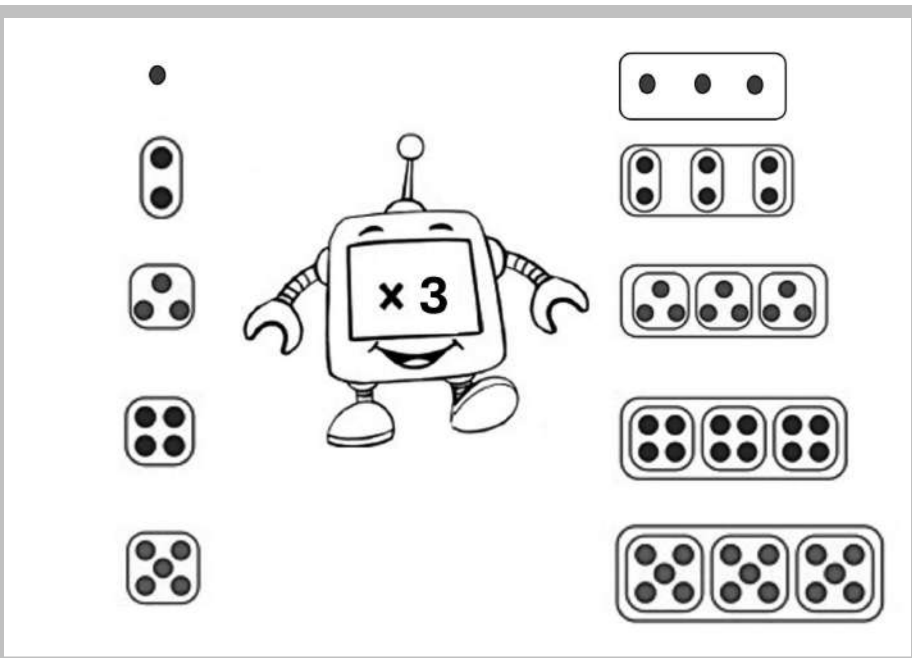
When we do introduce the multiplication sign, this must be done carefully.

It is a matter of convention for × whether the first number is repeated, or the first number tells you how many times to repeat.

Times as 'repeat by'

In some countries × means that **the first number is repeated.**
The second number tells you how many times to repeat.
multiplicand × multiplier = product.
5 × 3 = 15 means 5 **repeated** 3 times gives 15.
Using a functional notion of × 3, an input is repeated 3 times to produce an output.

Diagram



Using this '**repeat by'** convention the times tables for the function **× 3** is written like this:

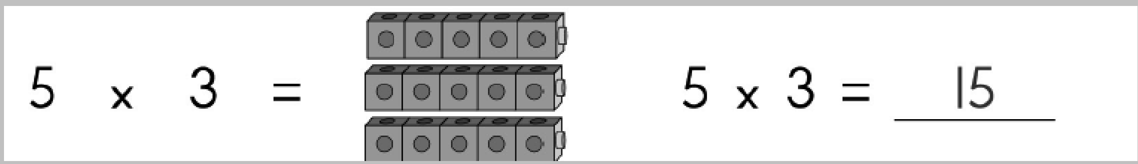
1 × 3 = 3	1 repeated 3 times = 3
2 × 3 = 6	2 repeated 3 times = 6
3 × 3 = 9	3 repeated 3 times = 9
4 × 3 = 12	4 repeated 3 times = 12
b × 3 = 3b	b repeated 3 times = b + b + b = 3b

Times as ‘groups of’

In other countries **the convention is the opposite.** × means that **the first number tells how many repeats, and the second number tells what is repeated.**
factor × factor = product
5 × 3 = 15 means 5 **groups of 3** gives 15.
[This is the same as 3 repeated 5 times].

In classrooms where African languages are dominant the convention of × as “repeat-repeat” or phinda-phinda is usually already well understood.

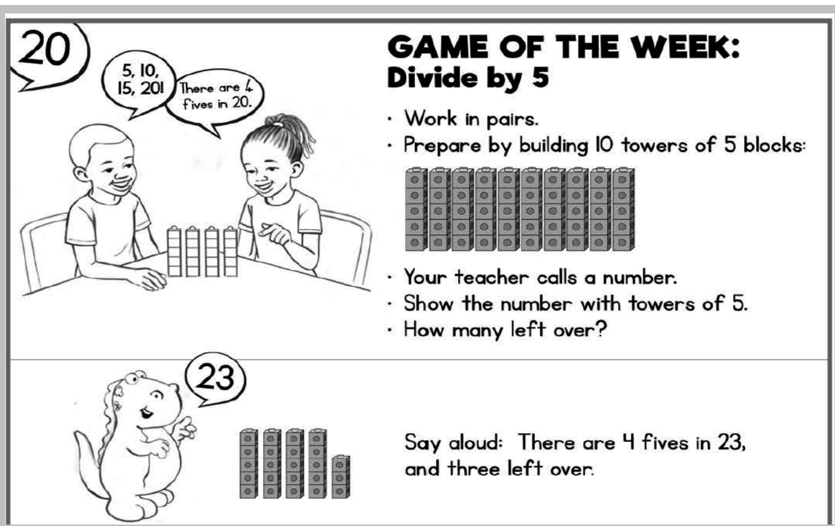
Example



5 × 3 = 5 **repeated** 3 times = 5 + 5 + 5
5 × 3 = 5 **phinda ka** 3 times = 5 + 5 + 5
5 × 3 = 5 **atiša ka** 3 times = 5 + 5 + 5
5 × 3 = 5 **atisa ka** 3 times = 5 + 5 + 5

In African language dominant classrooms, where children are learning mathematics bilingually, we think it is important that the English meaning of × matches the African language meaning of ×. In South Africa, we therefore recommend introducing **×** to mean **repeated by** or **phinda ka** or **atiša ka.**[1]

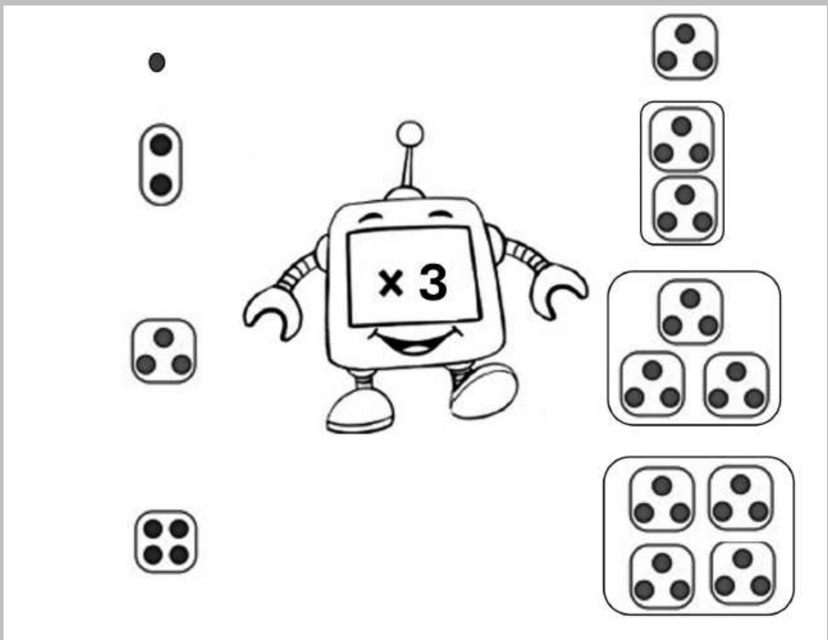
Activity for learners



Because there are so many South African texts that use **×** to mean **groups of,** it is worthwhile exploring the commutativity of multiplication soon. So children should realise that “5 repeated 3 times” is the same amount as “3 repeated 5 times”. 5 × 3 = 3 × 5**.**

1 In Sesotho **multiply by** is **atisa ka**. Although in common language ‘atisa’ refers to ‘increase’, 5 atisa ka 3 does NOT mean 5 + 3 which is 8. We reserve ‘5 **atisa ka** 3’ to **mean** ‘5 × 3’. Add is hlakatsa. In Sepedi **multiply** is **atiša**.

Diagram



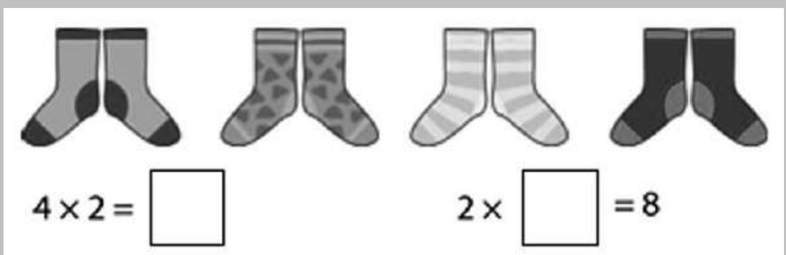
In this function, each 'one' in the input, becomes a group of 3 in the output.
Using the **'groups of'** convention the times tables for the function **× 3** is written like this:

1 × 3 = 3	1 group of 3 = 3
2 × 3 = 6	2 groups of 3 = 6
3 × 3 = 9	3 groups of 3 = 9
4 × 3 = 12	4 groups of 3 = 12
b × 3 = 3b	b groups of 3 = 3b = 3 repeated b times

It is important that teachers do not mark learners incorrect if they write 3 × 5, when 5 × 3, might be the teachers expected answer. You don't know if the child is thinking "5 **phinda phinda** 3" (5 repeated 3 times) or 5 **groups of** 3. Both children and teachers need to be flexible.

Activity for learners

Write two different multiplication equations for this picture



We can say: 4 **groups of** 2 socks = ...

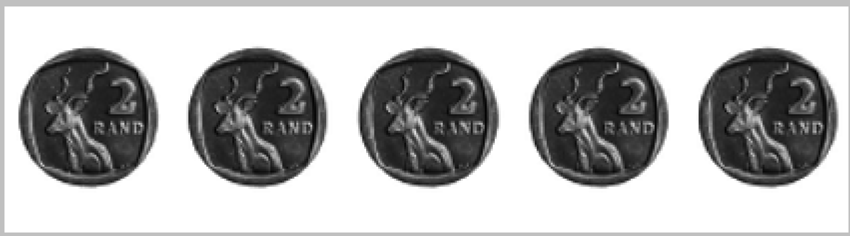
We write: 4 × 2 = ...

We can say: 2 socks **repeated ... times** =

We write: 2 × ... = 8

Activity for learners

Look at these R2 coins



We say: There are five R2 coins

We write: 5 × 2 = ...

We say: A R2 coin, repeated 5 times.

We write: 2 × ... = ...

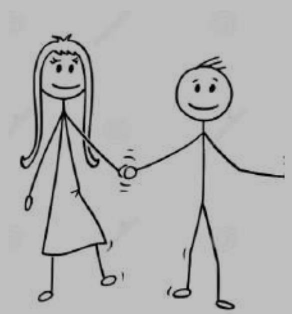
The meaning of the ÷ sign: Sharing and grouping/packing

When we do division we can either think about it as sharing, or as grouping.
It can help to think of grouping as packing. We pack boxes or bags with equal amounts.

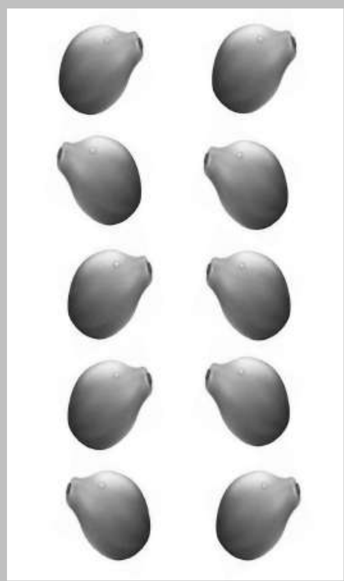
Pawpaws are a delicious fruit. They grow in tropical areas.

Example

Example A: Sharing 10 pawpaws



10 pawpaws can be shared equally by 2 people.This is **sharing.** We share 10 pawpaws equally into 2 groups.



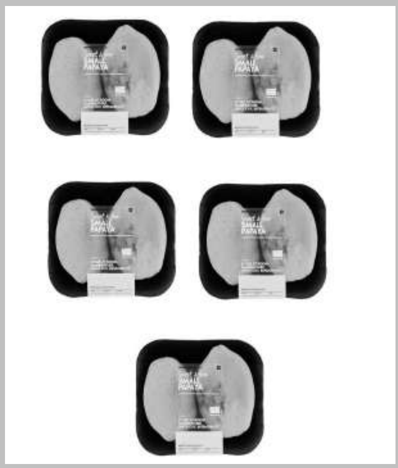
In the academic literature on maths education the sharing model for division is called **partitive**. You partition into a fixed number of parts. In a sharing situation "divide by 2" means **share equally** between 2. Make 2 equal groups.

Example

Example B: Packing 10 pawpaws

10 paw-paws can also be packed into boxes to sell them. There are two pawpaws in each box. This is **grouping** or **packing**. We pack 2 in each box. Each group has 2 paw-paws.

In the academic literature, the **packing (grouping)** model for division is called **quotative**. Divide by 2 means pack into boxes with 2 in each box. The unit repeated is 2s



It is easiest to understand sharing (partitive) and grouping (quotative) when we think about different problem contexts.

- **Sharing** involves word problems about **sharing** equally between people;
- **Grouping** involves word problems about **packing** into boxes or bags, where there is the same amount in each box or bag.

TASK A: Sharing 20 apples

There are 20 apples. We **share** the **apples equally between 4 people**.
How many apples does each person get? Draw a picture to show your reasoning.

TASK B: Packing 20 apples

There are 20 apples. Pack the apples into bags with **4 apples in each bag.**
How many bags do you need? Draw a picture to show your reasoning.

Learn the multiples of 10

All work on place value is about multiplying and dividing by 10s.
When we multiply by 10, we repeat by 10.
When we divide by 10 we share into 10 equal groups or we pack into boxes with 10 in each box. We count the 10s.

Children should first learn to skip count in 10s.

- Start at 10 for the multiples of 10. **Counting forwards in 10s from 10 is for multiplying and dividing.**
- Start at any number to learn to **add 10 each time.** This is the medium number song. **Counting in 10s forwards and backwards from any number is for adding and subtracting.**

Then they work on word problems or picture problems for multiplying and dividing by 10.

Activity for learners

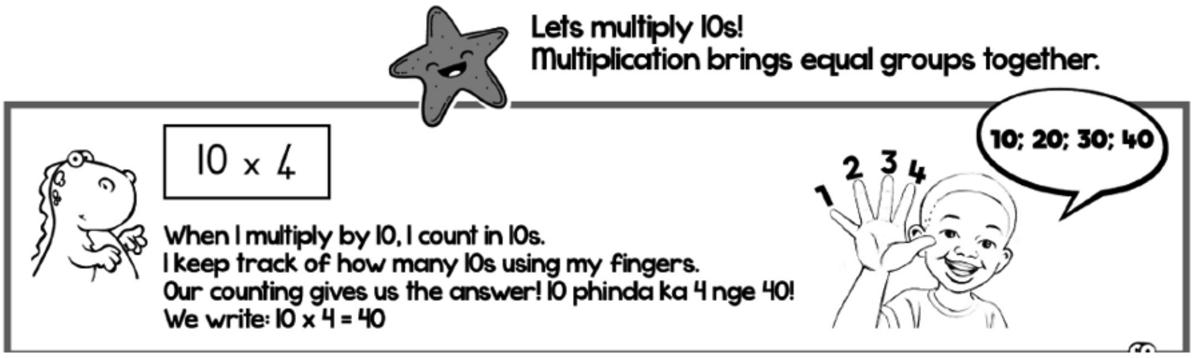
Find each number on the chart. How many tens?
How many more ones?

40	26	45
How many tens? ☐	How many tens? ☐	How many tens? ☐
How many ones? ☐	How many ones? ☐	How many ones? ☐

70	62	58
How many tens? ☐	How many tens? ☐	How many tens? ☐
How many ones? ☐	How many ones? ☐	How many ones? ☐

When we multiply by 10, we repeat by 10.
Children should **group count**. Count forwards in groups of 10. Keep track of the 10s on their fingers.

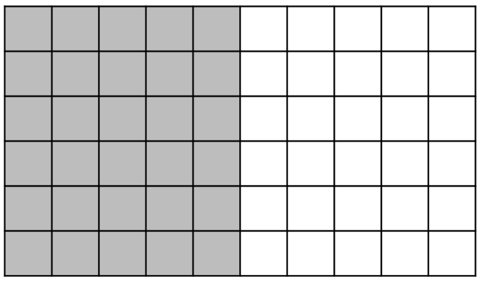
Example



To work out 4 repeated 10 times, they can add 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4…
But this is slow and difficult.
It is quicker to work out 10 repeated 4 times. 10 + 10 + 10 + 10 = 40
They can **group count**. Count forwards in 10s, 4 times. Keep track of the 10s on their fingers.

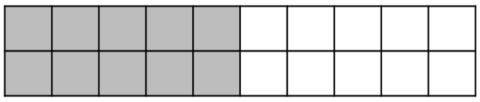
Example

Each row in the 100 chart has 10

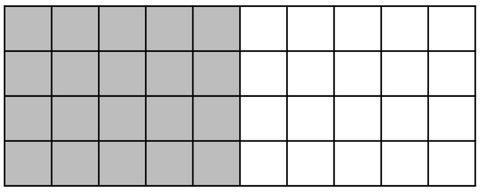


6 rows have 60
6 **× 10** = 60

Each row in the 100 chart has 10



2 rows have 20
2 **× 10** = 20



4 rows have 40.
4 **× 10** = 40

6 = 2 + 4
Therefore
6 **× 10** = (2 **× 10**) + (4 **× 10**)
I can break up 6 and multiply each part by 10

Breaking up a number and multiplying each part is important for multiplying big numbers.
This is called the **distributive property**. Multiplication distributes over addition.

When we divide by 10 we share into 10 equal groups or we pack into boxes with 10 in each box.
We count the 10s.

- To divide 50 by 10, they can **share** 50 **equally** between 10.
- To divide 50 by 10, they can **group count**. Count forwards in 10s, until they reach a target of 50. Keep track of the 10s on their fingers. 5 tens is 50.
  So 1 tenth of 50 is 5. 50 ÷ 10 = 5

There are faster ways to multiply and divide by 10, than using group counting.

As soon as children know (rapid recall) the '×10' and '÷10' number facts, they should use them.

- For 4 × 10 it is quickest to write 4, then write a 0.
- You can see if number can be divided by 10, because it ends with 0.
- 50 ÷ 10 is quickest if you just remove the 0 from 50.

It is your job as a teacher to make sure the children learn the ×10 and ÷10 number facts. Practice these often in **Lesson Starters.**

- **Fizz-pop.**
  - » The rule is times by 10. The rule is repeat 10 times. The rule is groups of 10 etc
  - » The rule is divide by 10 (use multiples of 10). The rule is share between 10.
    The rule is pack into boxes with 10 in each box. etc.
- **I am thinking of a number.** Ten times my number is 30. What is my number? Ten times my number is ... etc. My number divided by ten is 3. What is my number? ...etc.
- **Group counting.** We are counting forwards in 10s. Start at 10. Stop at 120.
- **Games in pairs:** 1,2,3 Repeat by 10! First use 1 hand, then use both hands

×10	
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80

Our number system uses groups of ten.

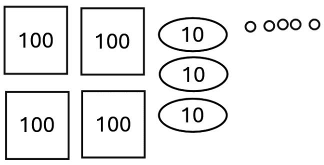
- We have ones. 10 ones are grouped to make 1 ten
- We have tens. 10 tens are grouped to make 1 hundred
- We have hundreds. 10 hundreds are grouped to make 1 thousand.
- We have thousands. 10 thousands are grouped to make 1 ten thousand, etc,

In a place value table we can use H for Hundreds, T for Tens and O for ones.

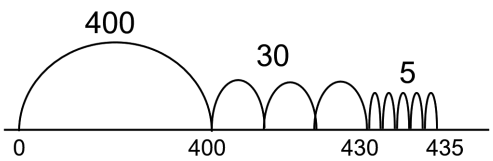
Place value table

Hundreds	Tens	Ones
4	3	5

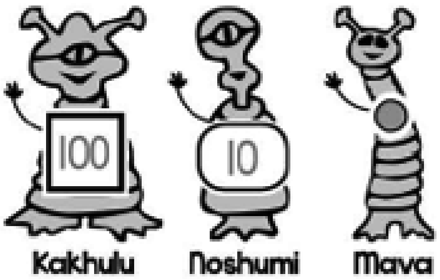
Number picture



Number line



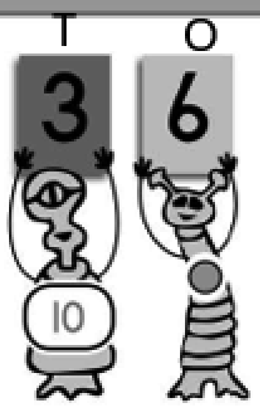
Numbers are organised into ones, tens and hundreds. Let me introduce you to Mivo, Shumi and Kakhulu!



Hi! I am Mivo. I carry the ones!

People ask me, "How many 1s?"

I carry 0 to 9 by myself.
10 is too much for me.
I need my friend Shumi.

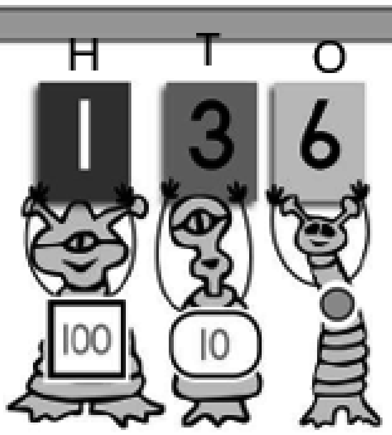


Hi! I am Shumi. I carry the 10s!

People ask me, "How many 10s?"

I always stand with Mivo. I carry the 10s.
Mivo carries the extra ones.

100 is too much for us.
We need our friend Kakhulu!



Hi! I am Kakhulu. I carry the 100s.

People ask me, "How many 100s?"

I always stand with Shumi and Mivo.

PrimTEd Std. NA 4.1
Demonstrate the relation between multiplication and division. Division is the inverse of multiplication. This is easy to imagine using a function machine.

Ideas for your classroom

Link division to multiplication
Multiplication brings equal groups together.

Activity for learners

10 cups fill one jug.

When we think about groups of 10, we can think about 10 cups filling one jug!

Multiplication Asks:			
2 , how many ?		4 , how many ?	
7 , how many ?		6 , how many ?	
9 , how many ?		3 , how many ?	

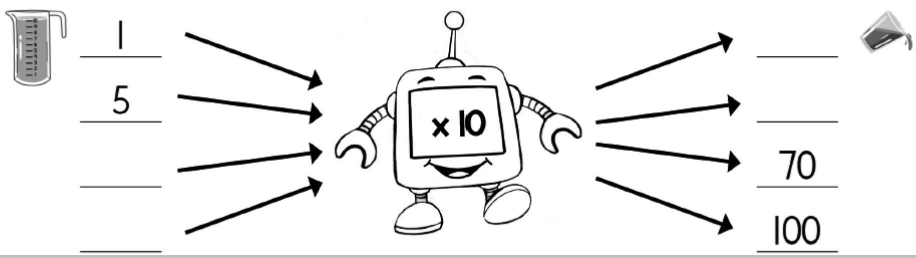
10 cubs fill one jug. 3 jugs, how many cups?
Group count forwards in 10s, 3 times. Keep track on your fingers: 10, 20, **30**
Division separates into equal groups.

Activity for learners

There are 10 CUPS in ONE JUG.

Division Asks:			
20 . How many ?		40 . How many ?	
70 . How many ?		60 . How many ?	
90 . How many ?		30 . How many ?	

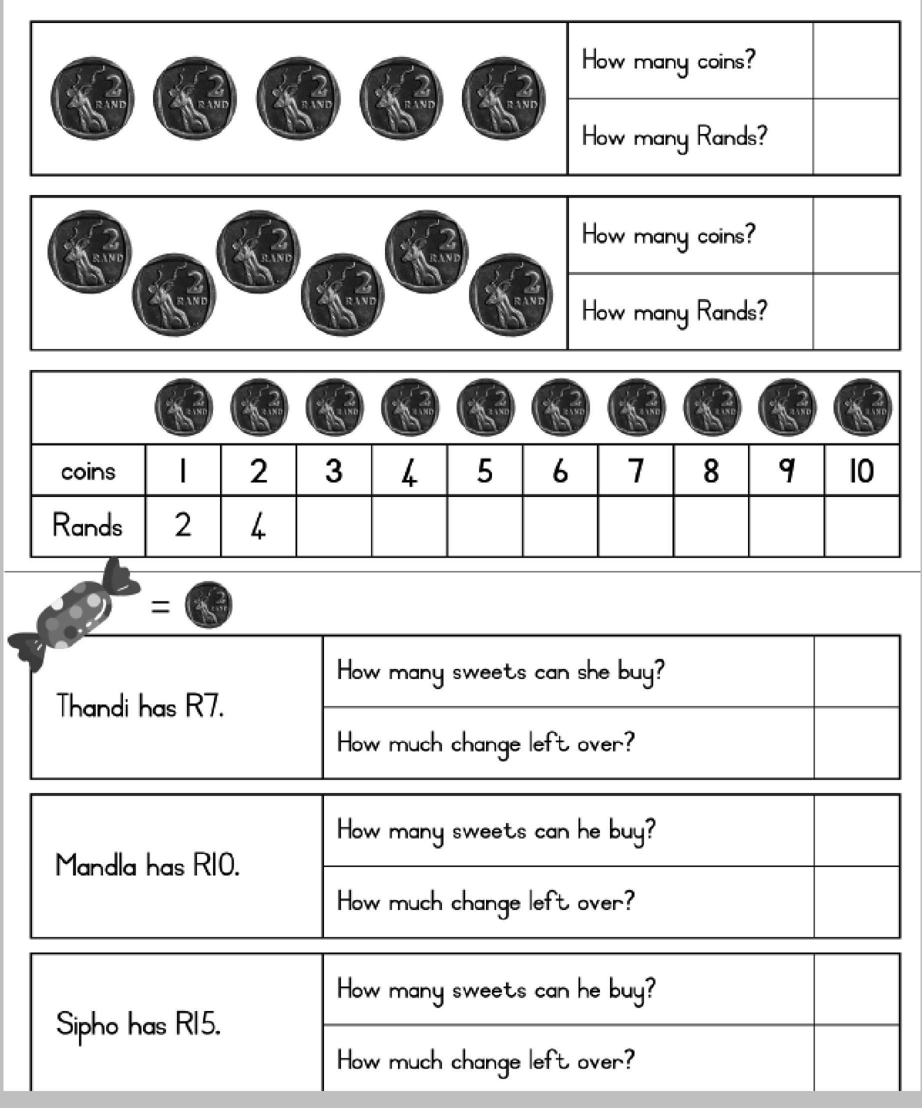
Funky is programmed to change the number of jugs into the number of cups. Complete the missing numbers.



10 cubs fill one jug. 40 jugs, how many cups?
Group count forwards in 10s, to reach a target of 40. Keep track on your fingers: 10, 20, 30, 40. That's 4 counts on my fingers.

Money is also a good context for multiplicative thinking.

Activity for learners



(5 R2 coins)	How many coins?	
	How many Rands?	

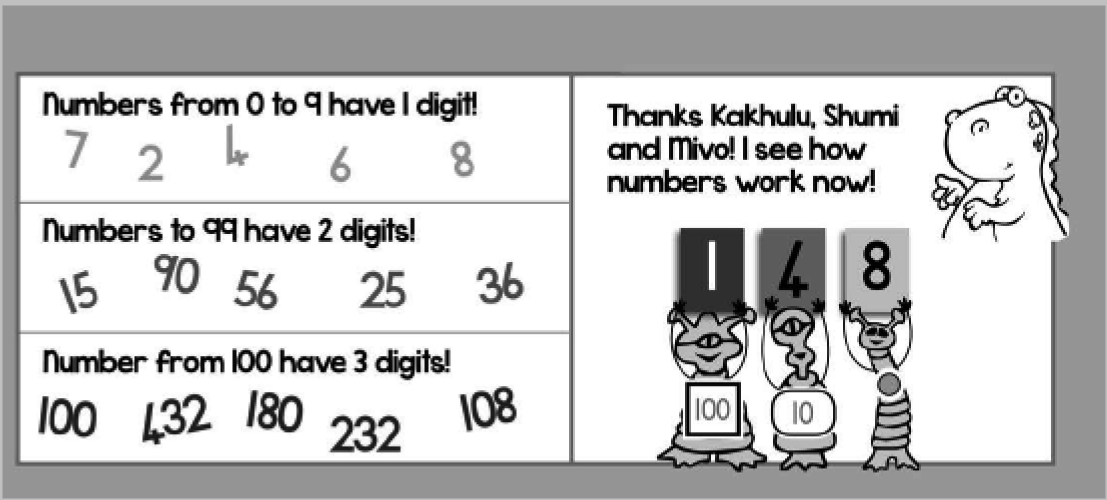
(6 R2 coins)	How many coins?	
	How many Rands?	

coins	1	2	3	4	5	6	7	8	9	10
Rands	2	4								

=

Thandi has R7.	How many sweets can she buy?	
	How much change left over?	
Mandla has R10.	How many sweets can he buy?	
	How much change left over?	
Sipho has R15.	How many sweets can he buy?	
	How much change left over?	

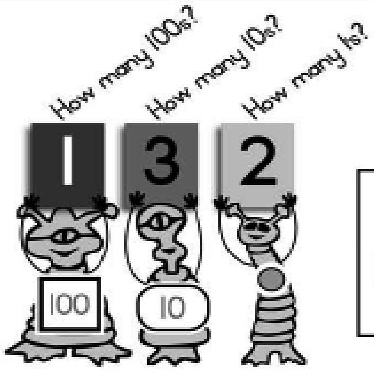
Activity for learners



-✂-- Activity for learners

Organise the number into 100s, 10s and 1s.
Write a number sentence.

132



132 = 100 + 30 + 2

147

100s?	10s?	1's?

147 = ____________________

164

100s?	10s?	1's?

164 = ____________________

181

100s?	10s?	1's?

181 = ____________________

192

100s?	10s?	1's?

192 = ____________________

We can also refer to 100s as **H**, 10s as **T** and 1s as **O**, in a place value table.

PRACTICE

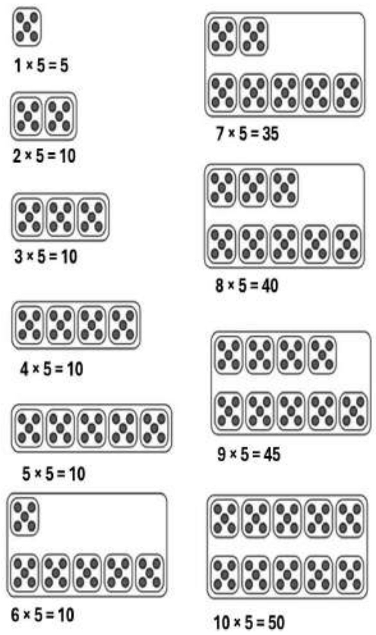
Tasks 1.3

PrimTEd Std GPM 1.
Plan effective learing experiences

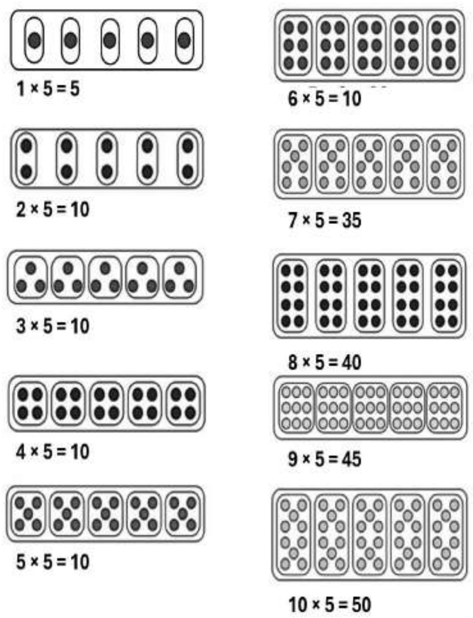
Unit fractions

1. The meaning of the × sign

POSTER A: 5 TIMES TABLE



POSTER B: 5 TIMES TABLE



Your head of department asks you to choose mathematics posters for the Grade 3 classes.

Which poster do you choose: Poster A or Poster B?
Give reasons for your choice.

2. Complete

2 × 5 = 5 × ...	4 × ... = 10 × 4	1 × ... = 7 × 1	0 × 5 = ... × 0
7 × 5 = 5 × ...	3 × ... = 10 × 3	10 × ... = 8 × 10	4 × 2 = 2 × ...

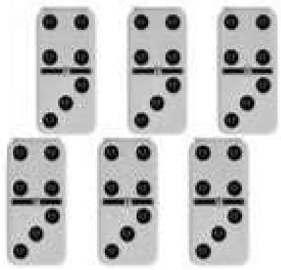
This property of multiplication is called the ____________________ property.

3. Solve sharing and grouping problems

- For each division problem, identify whether it is a sharing or a grouping problem.
- Solve the problems. Show your work.

Circle which problem type	Solve the problem, show you working out
A: A teacher gave out 24 pens to 8 children in her class. Each child got the same number of pens. How many pens did each child get? Circle which problem type: **Sharing / Grouping or packing**	
B: A baker made 30 cookies and placed them in boxes; 3 in each box. How many boxes of cookies did he fill? **Sharing / Grouping or packing**	
C: A gardener had 28 seeds to plant in pots, and planted 7 in each. How many pots did she plant? **Sharing / Grouping or packing**	
D: A class of 64 children was divided into relay teams. There were 4 children in each team. How many relay teams were created? **Sharing / Grouping or packing**	
E: A shopkeeper was arranging 24 books for a display, and placed them in 6 rows with equal numbers of books in each. How many books were in each row? **Sharing / Grouping or packing**	

4. Number talk with dominoes.

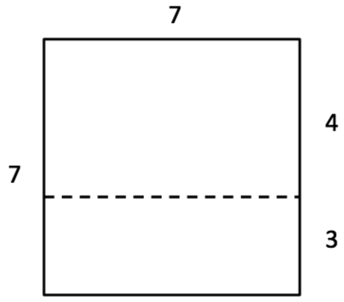


How many dots?

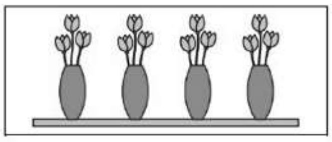
____

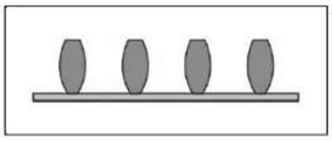
Show a few different ways to work it out.

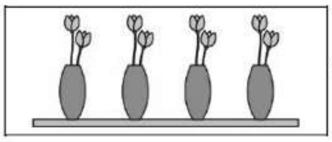
Fill in the area of a rectangle.

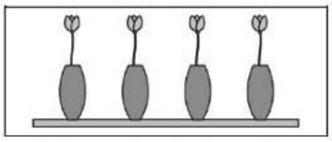


5. Draw a line to match the picture to the multiplication cards









repeat	phindaphinda	atisa	groups of
1 repeated 4 times 1 × 4	1 phinda ka 4 1 × 4	1 atisa ka 4 1 × 4	4 groups of 1 4 × 1
3 repeated 4 times 3 × 4	3 phinda ka 4 3 × 4	3 atisa ka 4 3 × 4	4 groups of 3 4 × 3
0 repeated 4 times 0 × 4	0 phinda ka 4 0 × 4	0 atisa ka 4 0 × 4	4 groups of 0 4 × 0
2 repeated 4 times 2 × 4	2 phinda ka 4 2 × 4	2 atisa ka 4 2 × 4	4 groups of 2 4 × 2

6. How do you say: '3 repeated 4 times' in an African language?

7. How do you say: '3 groups of 4' in an African language?

Tasks 1.3

Multiplicative thinking reading

SCAN ME

Math Is FigureOutAble. Retrieved from https://www.mathisfigureoutable.com/blog/development

Harris, P. (2020). *The development of mathematical reasoning: Counting, additive thinking, multiplicative thinking and beyond.*

Listen to Pam Harris's podcast. Follow her on X

Reading

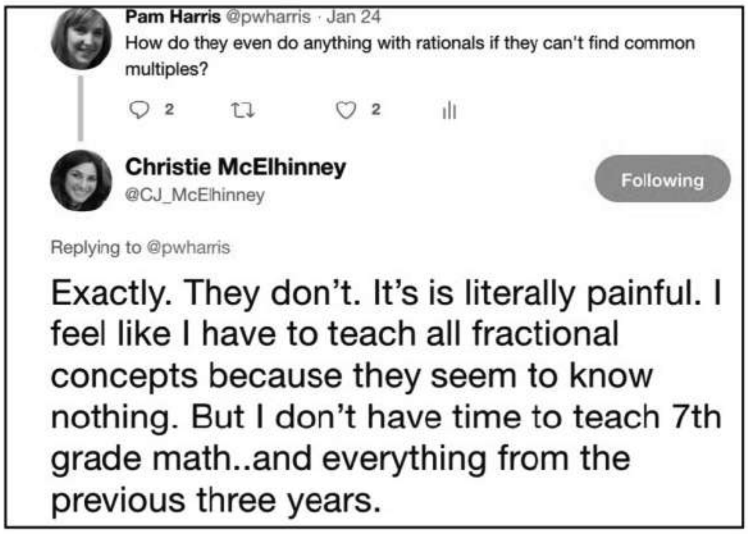
The Development of Mathematical Reasoning

Jun 06, 2020

Harris, P. (2020.). *The development of mathematical reasoning: Counting, additive thinking, multiplicative thinking and beyond*. Math Is FigureOutAble. Retrieved from https://www.mathisfigureoutable.com/blog/development Math Is F

Introduction

Check out this exchange I recently had on Twitter. I was discussing frustrations of middle school teachers:

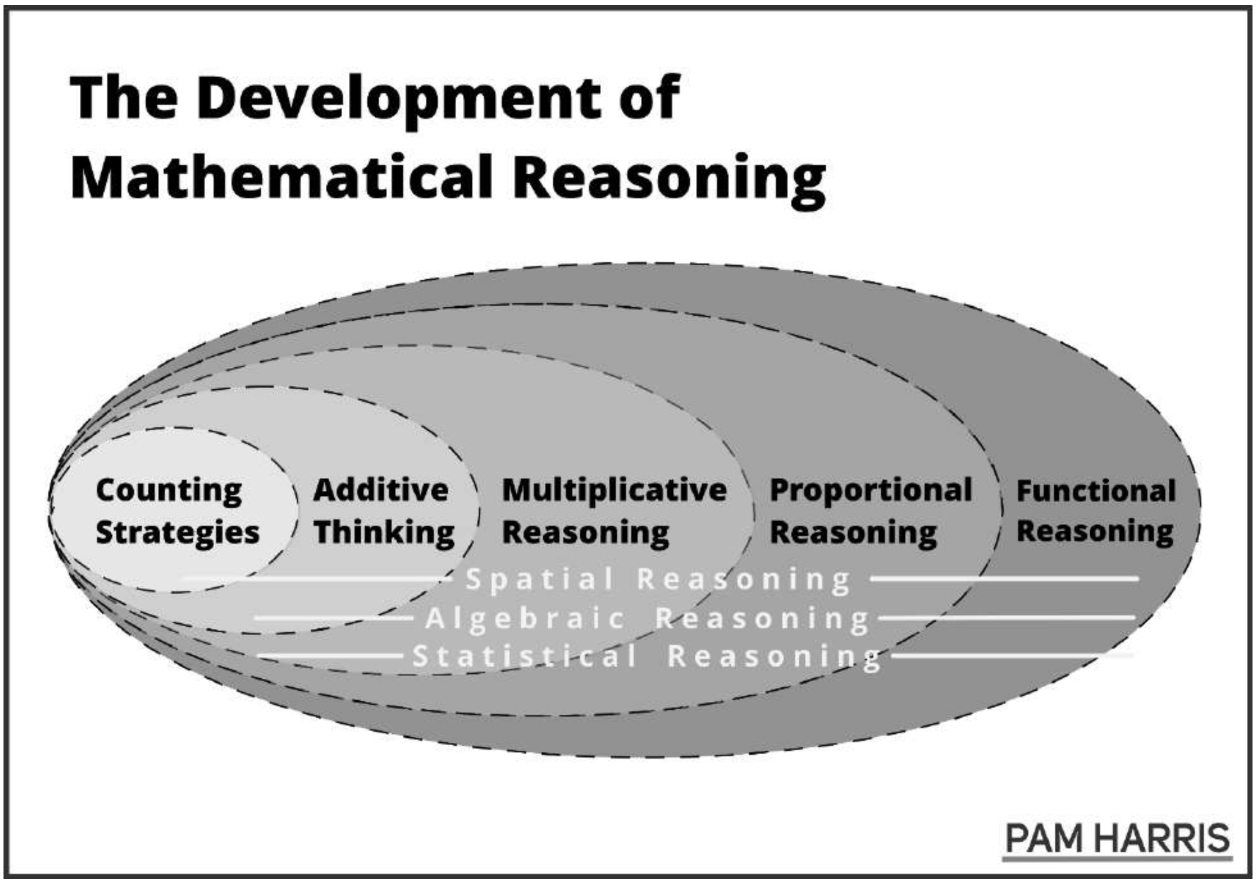


Have you ever felt like this Tweet, that you don't have the time to teach your content and all of the content your students should have learned before you? Why do we have students who seemed to do fine last year, but don't have what they need to succeed now? How is it that so many students have *so many gaps*?

Reading

- Do you teach **third** grade and find that your students don't know **addition** facts?
- Do you teach **middle school** and find that your students don't know their **multiplication** facts?
- Do you teach **algebra** and find that your students don't understand **ratio**?

I have thought a lot about this. I have thought and researched and read and worked with teachers and students at all grade levels. And I think I have found something that can help. This post is intended to start that conversation.
I invite you to consider this graphic that represents the development of mathematical reasoning.



Notice how these types of reasoning build on each other. Each domain of thinking and reasoning is based on the previous and is more complicated, more sophisticated, where more things are happening simultaneously than the previous.

This graphic is meant to suggest that the teacher's role is to help students change the way they think, in increasingly sophisticated ways. Too often, well-meaning teachers help students get answers to questions, but because it's easier they often use methods with reasoning in the level(s) before. Students get answers but don't increase/improve the *way* they think. Then students are stymied when they are expected to solve problems in the next level involving more layers of complexity. They haven't developed the ability to think with sophisticated mathematics, they only know how to get answers. We, as teachers, need to understand the development of mathematical reasoning so we can help our students think and reason in ways that allow them to continue to grow and learn.

In this post, I'll give examples of what it means to reason in these increasingly sophisticated ways.

Reading

Counting Strategies

What does it mean to use counting strategies to solve problems?
Counting strategies involve counting by 1s. Here are examples of solving problems involving the four operations counting by 1s.

Counting Strategies

27 + 8	27, 28, 29, 30, 31, 32, 33, 34, 35. This could be done with fingers, tally marks, pictures, beads on a rack, on a number line. The part that makes it a counting strategy is the counting by 1s.
27 − 8	Student says 27. Then puts up a finger and says 26, puts up another finger and says 25... puts up an eighth finger and says 19.
7 × 8	Count out 8 (tallies, beans, etc.) into a pile. Then do that 6 more times so you have 7 piles, each with 8 in them. Then start at the beginning and count them all by 1s.
40 ÷ 8	Draw 8 circles. Deal out 40 tallies, one at a time to each circle. Count the number of tallies in each circle: 1, 2, 3, 4, 5.

These are all *counting strategies*, even though the student is finding answers to addition, subtraction, multiplication, and division problems. The student's brain is thinking in terms of counting by 1s.

Additive Thinking

What does it mean to solve problems using additive thinking?
Additive thinking means students solve problems using bigger amounts of numbers than 1 at a time.

Adding Strategies

27 + 8	27 + 3 is 30, then add the leftover 5, 35.
27 + 49	27 and 50 is 77, but that's a bit too much, so I'll back up to 76.
27 − 8	27 subtract 7 is 20, subtract 1 more is 19.
52 − 27	52 subtract 30 is 22, but I subtracted too much, so adjust back up to 25.
7 × 8	Seven 8s. 8, 16, 24, 32, 40, 48, 56. This is called skip counting but it's not counting by 1s. It is adding one group of 8 at a time. That's additive thinking.
48 ÷ 8	48 − 8 = 40; 40 − 8 = 32; 32 − 8 = 24; 24 − 8 = 16; 16 − 8 = 8. How many 8's? 6. This is additive thinking, repeatedly subtracting 8s and keeping track of how many 8s.

Reading

These are all *additive strategies* even though in two of these examples, the student is finding answers to multiplication and division problems. If they are doing it one group at a time, that's additive.

Multiplicative Thinking

What does it mean to solve problems using multiplicative thinking?
Multiplicative thinking means students solve problems using bigger chunks than one group at a time.

Multiplicative Strategies

7 × 8	I know that seven 7s is 49. But I need eight 7s so I need one more 7. 49 and 7 is 56.
7 × 59	I'm going to think of this as fifty-nine 7s. I know that sixty 7s is 420. But I only need fifty-nine 7s, so 420 − 7 is 413.
48 ÷ 8	I know that 40 ÷ 8 is 5, so 48 ÷ 8 must be 6.
1287 ÷ 13	I know that 1,300 ÷ 13 is 100, but I need 13 less than 1,300, 1287 ÷ 13, so that's 99.

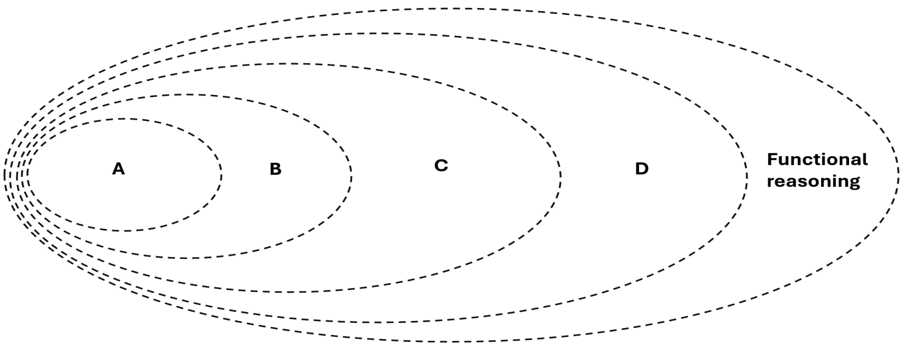
These are all *multiplicative reasoning* because the students are working with bigger chunks than one group at a time.

SCAN ME

To read the rest of the blog, use the QR code at the top of the reading.

Retrieved from https://www.mathisfigureoutable.com/blog/development

Harris (2020) draws nested sets to show how mathematical reasoning develops. She gives examples from Grade R to Grade 12, to illustrate each type of thinking.



TASK

1. Compete labels for the type of thinking A – D.

	Type of reasoning/thinking
A	
B	
C	
D	
E	Functional thinking

What are the 3 types of thinking, that Harris (2020) shows as cross cutting (relevant for all other types of thinking?

2. We can't look at an example and be sure how a child will **solves** that problem. To be sure about their type of reasoning we need to see their strategy. They must show their working. Or they must explain their reasoning to be sure about how they were thinking. Harris shows the same problem 7 × 8 = … ca be solved using different types of thinking.

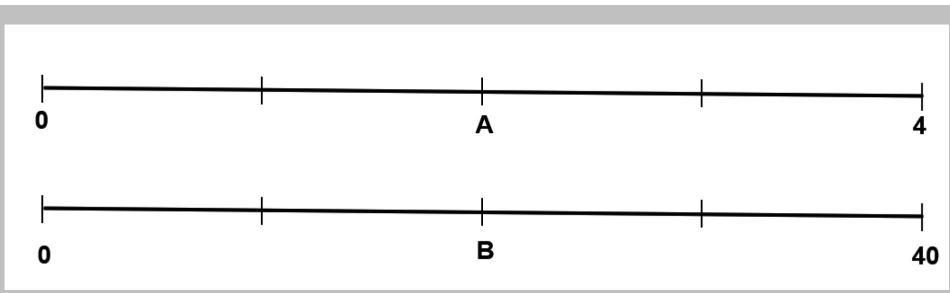
Complete each type by showing the child's reasoning from Harris (2020)

Type of thinking	**Problem**	**Child's reasoning**
Counting strategy	7 × 8	
Additive thinking	7 × 8	
Multiplicative thinking	7 × 8	

3. The examples that Harris (2020) gives are quite difficult, especially for proportional reasoning and for functional reasoning. Some primary school tasks can involve proportional reasoning and functional reasoning.

Complete the number lines. What is the value of A? What is the value or B?

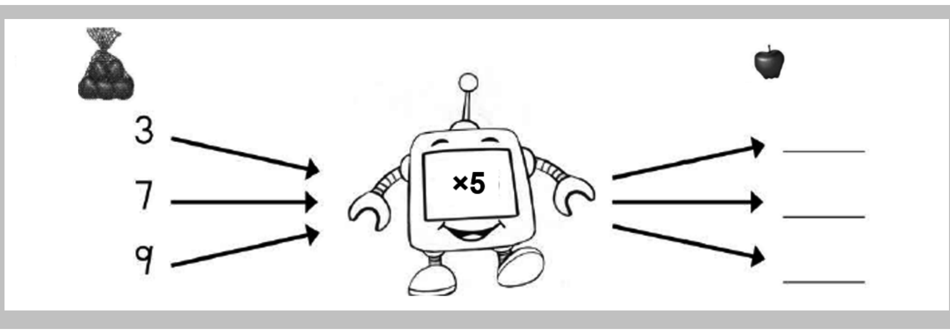
Example



Proportional reasoning:
Concerns the relationship of two quantities at the same time and the effect of scaling up (or down).

4. Apples are packed in bags with 5 apples in each bag. How may apples in 3 bags?
5. Funcky is programmed to change **the number of bags** to **the number of apples**.

Example



Functional reasoning:
Concerns a relationship which changes input values, to output values. For each input, there is only one possible output. The function is a rule which shows how to change the input, to get the correct output.

The diagram in Harris (2020) can help primary school teachers visualise the different types of thinking which we are building. Here are some primary school examples.

6. For each example, write down the type of thinking that this example is likely to develop.

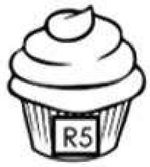
Types of thinking:
Counting strategies, additive reasoning, multiplicative reasoning, proportional reasoning, functional reasoning.

Example	Type of thinking/ reasoning	Example	Type of thinking/ reasoning
1		6	
2		7	
3		8	
4		9	Thami Ayanda
5		10	

SCAN ME

There is another helpful reading about teaching multiplication:
Wallace & Gurganes (2005)
'Teaching for mastery of multiplication' in
Teaching Children Mathematics

EXAMPLE 1



How much for 3 cupcakes?

EXAMPLE 2

Teacher: Double 5

Child: 10

Teacher: How did you work that out?

Child: “I just know that double 5 is 10”

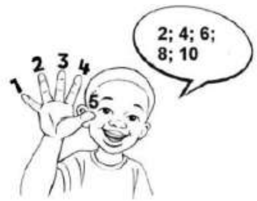
EXAMPLE 3

2 amagwinya cost R10.

How much for 5 amagwinya?

EXAMPLE 4

5 pairs of shoes. How many shoes?



EXAMPLE 5

29 + 9 = ...

I used round and adjust:

29 + 9 is the same as 30 + 8

EXAMPLE 6

27 + 31

I break up the 2nd number

Add the tens: 27, 37, 47,

Add the ones: 48

EXAMPLE 7

Hands	**1**	**2**	**3**	**4**	**5**
Fingers					

What is the relationship between Fingers and Hands?

Fingers = ____________________

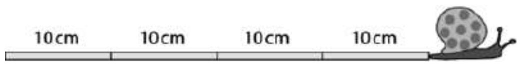
EXAMPLE 8

5 times 6 is 30.

“I know that 5 times 5 is 25

So I add 1 more 5 and I get 30.”

EXAMPLE 9

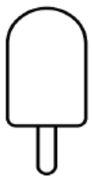


How far did the snail move?

Thami: 10 + 10 + 10 + 10 = 40

Ayanda: 4 tens are 40.

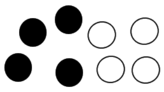
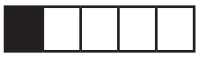
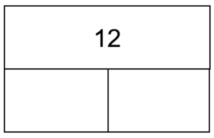
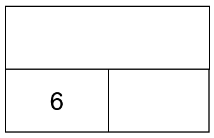
EXAMPLE 10



1 ice sucker cost R15.
How much will 100 ice suckers cost?
You have R100. How many ice suckers can you buy?

Tasks 1.3.3

Two min tango – division resulting in fractions.

1	Name the fraction:  1 _____ of the circles are black	11	Name the fraction:  1 _____ of the squares are black
2	40 ÷ 10 = ______.	12	1 tenth of 40 = _______.
3	Whole-equal parts  I half of 12 = _______.	13	Whole-equal parts  6 = 1 half of _______.
4	Share 7 marbles equally between 2 children. Each child gets _____ marbles. There is 1 left over.	14	Share 7 amagwinya equally between 2 children. Each child gets _____ and _____ amagwinya.
5	'30 divided by 10' is the same as 'Find 1 ______ of 30'.	15	There are 20 apples. 1 half of the apples = ______ apples.
6	'10 divided by 2' is the same as 'Find 1 half of _____'.	16	'10 divided by 5' is the same as 'Find 1 _____ of 10'.
7	**Divide with a remainder** 9 divided by 2 = _____ and _____ left over	17	**Divide with a fraction** 9 divided by 2 = _____ and 1 _____
8	The whole line has a length of 1. A is in the middle.  Length A = 1 ______.	18	The whole line has a length of 1. It has been divided into 10 equal parts.  Length A = 1 ______.
9	A rope in 10 metres long. It is cut into 5 equal pieces. Each piece is _______ m long.	19	 This full jug can fill 5 cups. 1 fifth of this jug can fill ______ cup(s).
10	Peaches are packed with 10 in a box. _______ boxes are needed for 25 peaches. The last box is _______ full.	20	10 children share 25 peaches equally. Each child gets ___________ peaches.

Working on these tangos strengthens your ability to:

- Name the fractional part.
- Realise that 'find the fraction' involves division.
- Reason about division with remainders and about division with fractions.

Summary 1.3

Draw a mind map of what you have learnt so far about teaching division and multiplication in Foundation Phase. Make sure you include:

- The meaning of × sign. Multiply as 'repeat by' (phinda ka) and multiply as 'groups of'
- The meaning of ÷ sign. Division as sharing and division as grouping/packing.
- Examples from counting and measuring pathways

Whether a division problem has a remainder, or a fraction depends on the problem context.
Give two division word problems for 11 divided by 2. One example must require a remainder. The other example must require a fraction.

UNIT 2: LEARNING THE TIMES TABLES

There are 100 multiplication facts for multiplying the numbers 1 to 10. We learn these by making connections between new and known facts. We build the connections using these properties:

1. **Commutative property.** Multiplication can be done in any order
2. **Division is the inverse of multiplication.** For every multiplication fact, there is a family of 4 division, and 4 multiplication facts which are all equivalent
3. **Distributive property.** You can break up a number using addition or subtraction and multiply each part.
4. **We keep track of multiplication facts using various representations**

This is a 10 by 10 multiplication and division table.
In total there are 1 hundred facts to learn.
The 3rd row from the bottom is the 3 times table.
The 5th row from the bottom is the 5 times table.

The second number

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	65	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

The first number

3 x 5 = 15 is in

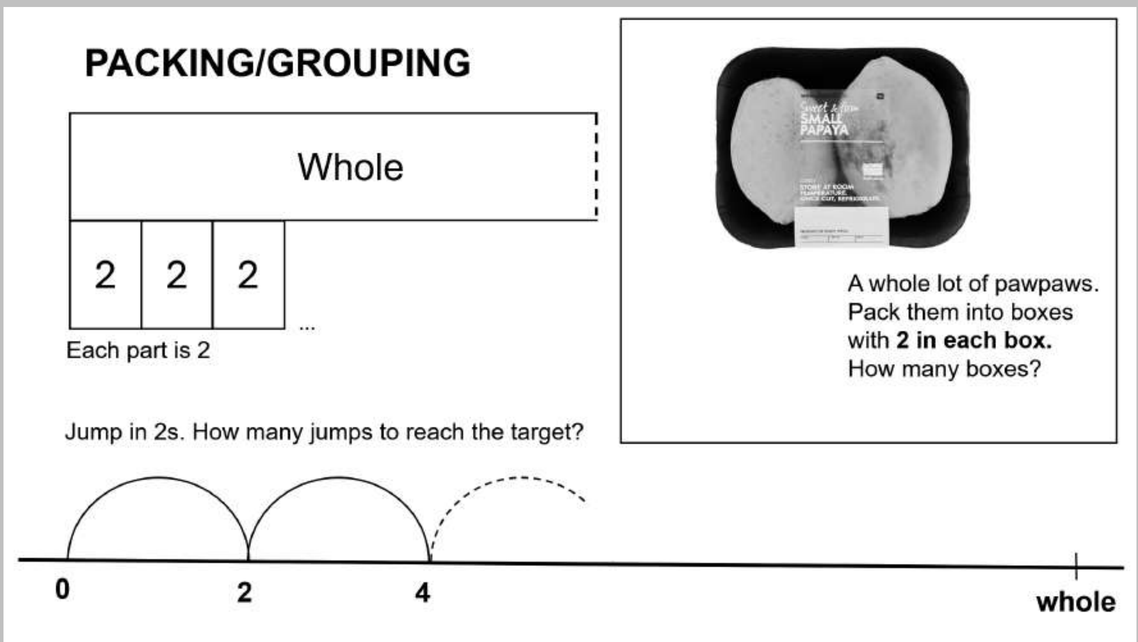
- the column starting with 3 (at the bottom), and
- the row starting with 5 (on the left)

As you work through the tasks, colour in the facts that you learn.

- In Umthamo 1.1 you worked with ×5. Colour in the multiples of 5.
- In Umthamo 1.2 you worked with odd and evens, doubling and halving and division giving rise to fractions. Colour in the doubles.
- In Umthamo 1.3, you focused on ×10 and place value. Colour in the multiples of 10.

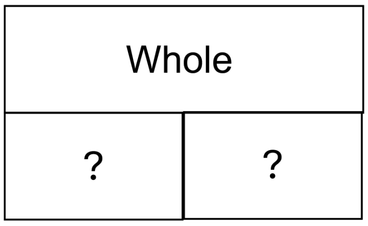
Key ideas

1. We can count efficiency by group/skip counting
   - We can count in 2s
   - We can count in 10s
   - We can count in 5s
2. The **×2 table** are the doubling facts.
   a. The multiples of 2 are the doubles.
   b. The multiples of 2 are the even numbers.
   c. Factor × factor = product. The products in the ×2 table are even numbers.
   d. You know whether a number is **divisible by 2** if it is even. It must have 0, 2, 4, 6 or 8 in the ones column. It must end with 0, 2, 4, 6, or 8.
3. **Halving is the inverse of doubling**. You can use doubling to solve a halving or divide by 2 problem. “Half of 6 = ____” is the same as “Double ____ = 6”. There is a family of related multiplication and division problems.
4. There are two ways to think about
   a. multiplication (× sign): **groups of _____, or repeated ____ times**.
   b. division (÷ sign): **sharing equally between _____, or grouping/packing into groups of _____.**

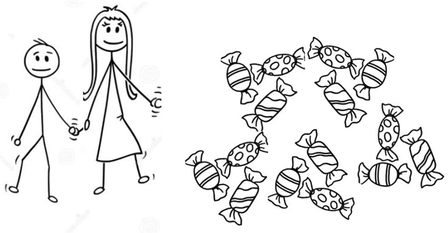


Key ideas

SHARING

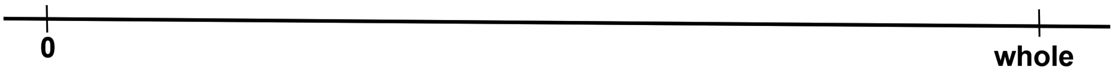


2 equal parts



A whole lot of sweets. **Share them equally between 2 people.** How many sweets does each person get?

2 equal jumps to the whole. How big is each jump?.



5. **Multiplication and division are much easier if you have a real context.**
6. You learn multiplication and division facts by making connections:
   a. **Multiplication is commutative**. The order does not matter.
   b. **Division is not commutative**. The order does matter.
   c. For every multiplication fact, there is a **family of 8** related multiplication division facts.
   d. **Multiplication is distributive**. You can break up a number by adding or subtracting, and then multiply each part.
   e. **Division is distributive.** You can break up a number by adding or subtracting, and then divide each part.
7. Learning the **10 times table** is easy.
   a. We use the 10 times table for **place value.**
   b. Factor × factor = product. The multiples of 10 are the products in the 10 times table.
   c. Factor × factor = product. The products in the ×10 table have 0 ones. They have 0 in the ones columns. They end in 0.
   d. You know whether a number is **divisible by 10** if it is a multiple of 10. It must have 0 ones. So it has 0 in the ones column. It must end with 0.
8. Learning the **5 times table** connects with the 2 times tables and the 10 times table
   a. Products in the 10 times table are double the products in the 5 times table.
   b. Products in the 5 times table are half of the products in the 10 times table
   c. You know whether a number is **divisible by 5 if it has 5 or 0 ones.** It must have 5 or 0 the ones column. It must end with 5 or 0.
9. We cannot divide by 0. Sharing ‘my number’ between 0 people does not make sense. Finding how many zeros are in ‘my number’ does not make sense.

UMTHAMO 2.1

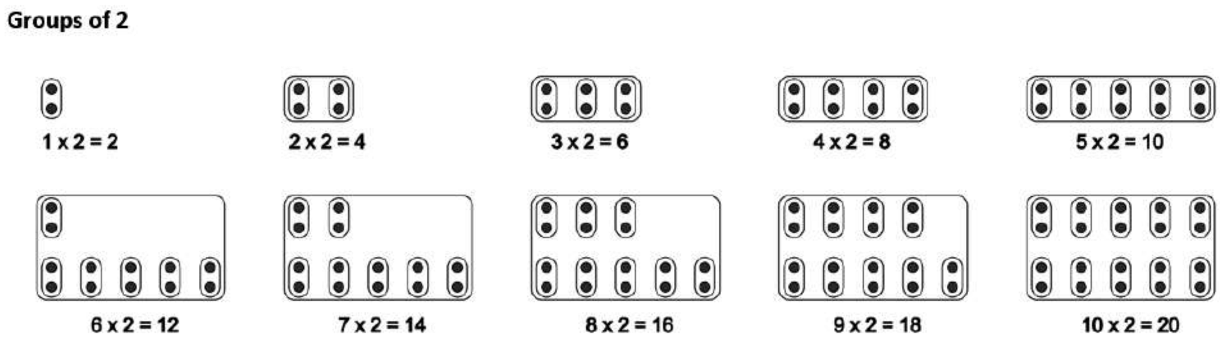
Multiplicative structures with 2s, 10s and 5s

Twos, evens and the doubles

1. The ×**2 table** are the doubling facts.
   a. The multiples of 2 are the doubles.
   b. The multiples of 2 are the even numbers.
   c. Factor × factor = product. The products in the ×2 table are even numbers.
   d. Multiples of 2 are even and end in 0, 2, 4, 6 or 8
2. **Halving is the inverse of doubling**.
   a. You can use doubling to solve a halving or divide by 2 problem.
   b. "Half of 6 = ____" is the same as "Double ____ = 6".
      There is a family of related multiplication and division problems.
3. The products/ multiples in the ×**10 table** end in 0.
   a. We use the multiples of 10 for our base-ten number system and place value
   b. Multiples of 10 are especially easy, if we say the tens in African languages.
4. The products/ multiples in the **x5 table** end in 0 or 5
   a. Multiply by 5 is the same as multiply by 10 then halve.
5. **Division is the inverse of multiplying**. You can solve a division problem, using multiplication facts.
   a. 20 divided by 5 = _____, is the same as _____ multiplied by 5 = 20
   b. 30 divided by 10 = _____, is the same as _____ multiplied by 10 = 30
   c. There is a family of related multiplication and division problems.

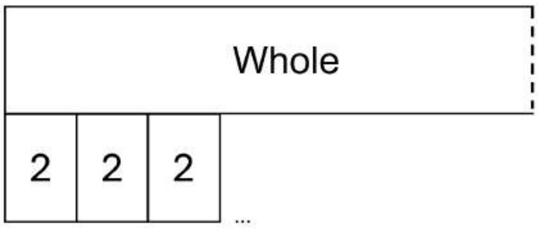
Learning trajectory for doubling and halving

Multiples of twos are learnt about by group counting in twos.
These are the even numbers. They are also the doubles.
Here is the two-times table shown as "____ groups of 2".



The unit is a “a group of two”, or a pair, a couple. We repeat that unit.

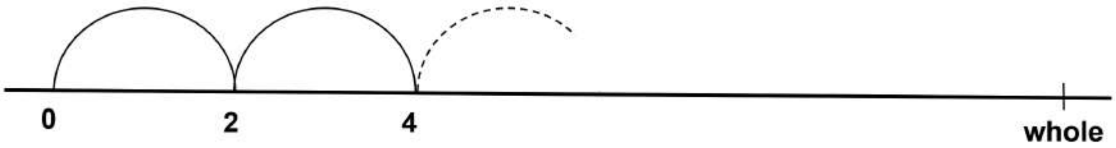
PACKING/GROUPING



Each part is 2

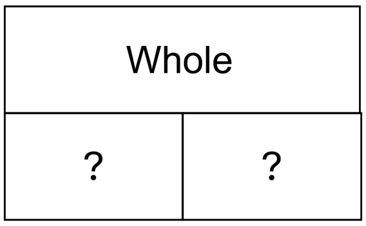
A whole lot of pawpaws.
Pack them into boxes with **2 in each box.**
How many boxes?

Jump in 2s. How many jumps to reach the target?

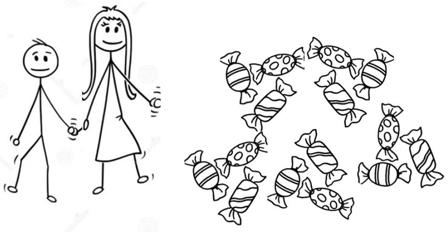


The two-times table can also be shown as “____ repeated 2 times”.

SHARING

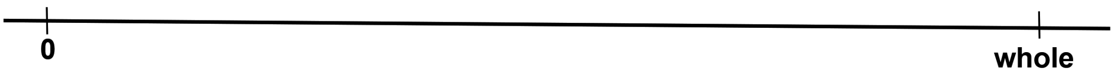


2 equal parts



A whole lot of sweets. **Share them equally between 2 people.**
How many sweets does each person get?

2 equal jumps to the whole. How big is each jump?.



When we divide into 2 equal parts, each part is 1 half.

This is a **learning trajectory for the 2 times-table**:

1. Start with **picture problems, talk problems and word problems**.
   Give a real multiplicative context.
   a. Arrange objects (like counters or stones) into little piles or groups of 2.
   b. Circle groups of 2 objects in pictures of many objects.
   c. Arrange cubes into towers of 2.
   d. Share equally into 2 groups.

2. **Learn the 2s song**: Count forwards in 2s. Start at 2. These are the multiples of 2. They are the even.
3. **Use the 2s song to solve real problems**. These can be doubling and halving word problems or picture problems.
4. **Model multiplying and dividing by 2 with the manipulatives**.
   a. The game of the week is towers or 2. Each tower has 2 cubes.
   b. The game of the week is break a tower into 2 equal groups
5. **Keep track of the counts, when singing the 2s song**. Keep track of both units and involved in the times-table. Be presented with, and then use, any of these representations to keep track of both units:
   a. Use your **fingers**
   b. Use **dot pictures**
   c. Use a **number line**. Jump forwards in 5s. Start at 0.
   d. Use a **horizontal table**
   e. Use a **vertical t-table**
   f. Keep track with a **function machine**
6. **Learn the 2 times table as rapid recall number facts**.
7. Explore **divisibility rules and factors** (including prime factorization)

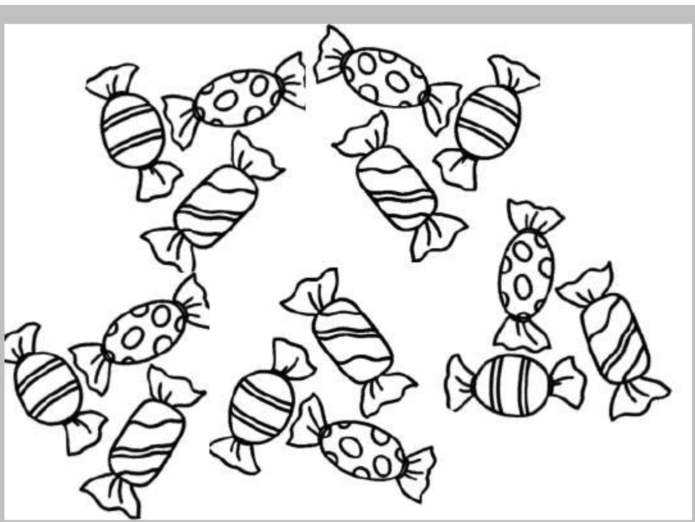
STEP 1:

Picture problems

You can give children some counters and ask them

- To make 2 equal groups (sharing).
- To make pairs (groups of 2)

Activity for learners



Circle each pair of sweets.

How many 2s?

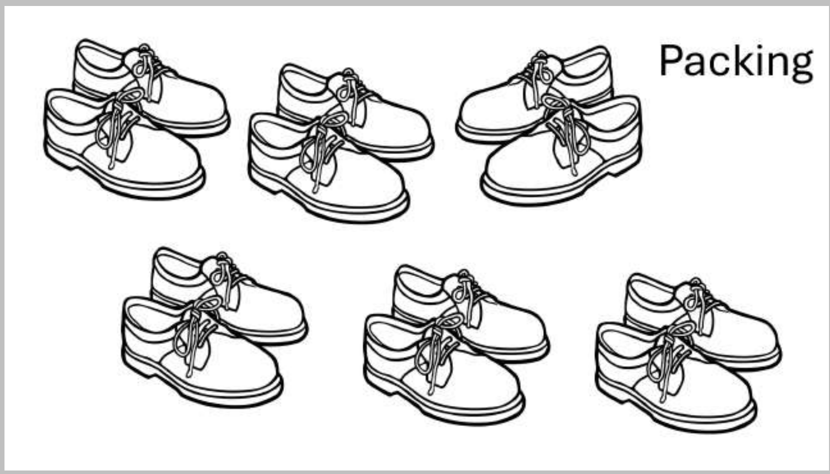
Separate the sweets into 2 equal groups.

How many in each group?

Notice that with this example about sweets there are not clear pairs (groups of 2). The process of packing into 2s does not have an underlying structure or reason.

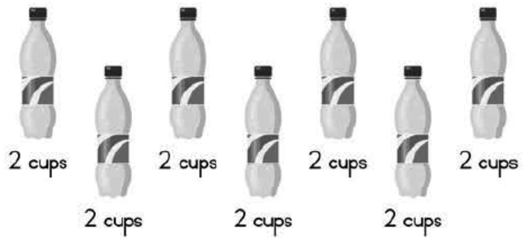
But with the example of shoes – there is an underlying reason. Shoes always come in pairs. Choosing a context, which has an underlying structure of pairs, helps children make sense of the two ways of thinking about division.

Activity for learners



2 cups fill one bottle.

When we think about groups of 2, we can think about 2 cups filling one bottle!

 2 cups, 2 cups, 2 cups, 2 cups, 2 cups, 2 cups, 2 cups	How many bottles?	
	How many cups?	

This example is on the measurement pathway. We are using capacity.
Notice that these picture problems start with a specific example. Here are 8 bottles. How many cups? We specialise – with a specific example.

TASK

Make a picture problem about dividing by 2. DO NOT use sweets, shoes or 2 cups filling 1 bottle. Choose your own context, Ask children questions about your context.

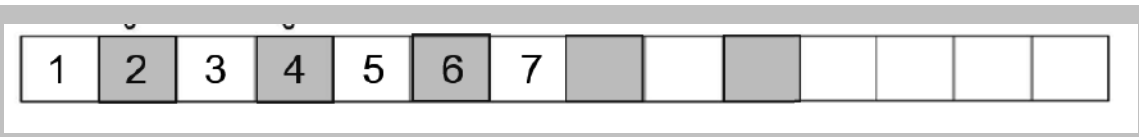
STEP 2:

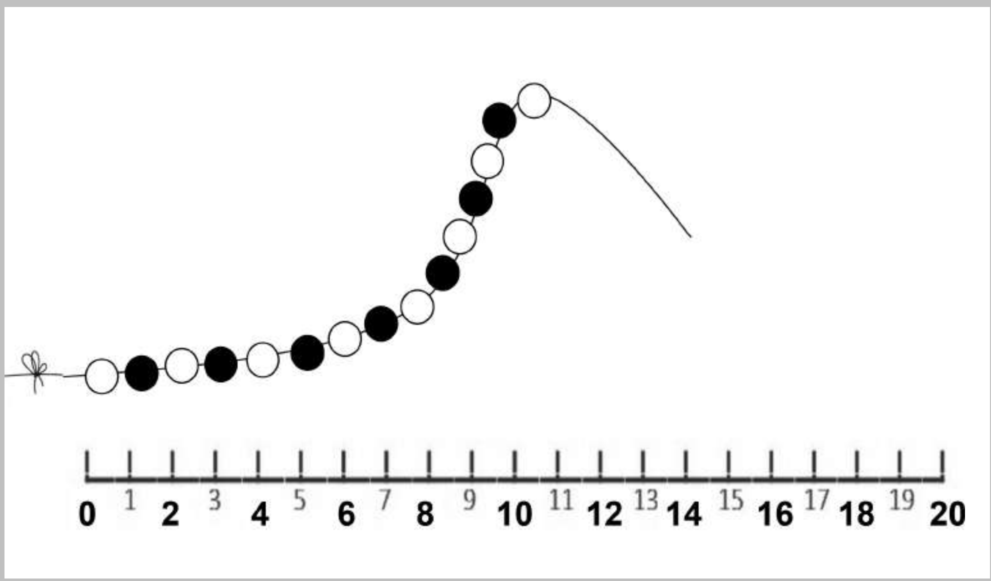
Learn the 2s song...First the evens, then the odds

Children learn the 2s song (the even numbers) from what they know about the 1s song (the small number song). We can start by counting in 1s, clap on the 2s: Click, clap, click, clap, click, clap.

Give children paper with the numbers on them. They line up from smallest to biggest. The children with even numbers stay standing. The children with odd numbers sit down. To learn the evens song, we say the first number as a whisper, and the next number loudly.

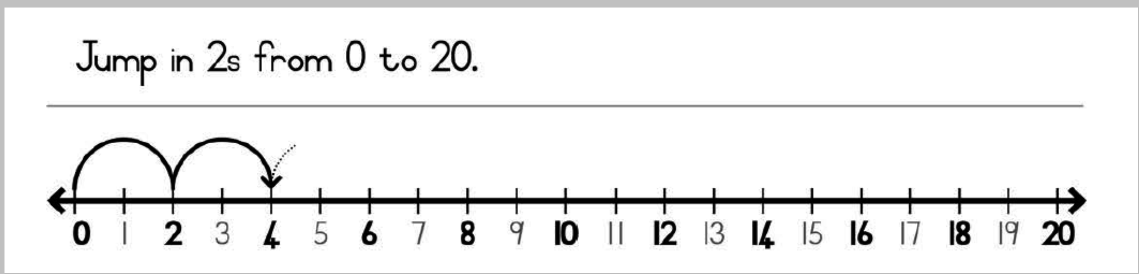
-✂-- Activity for learners





A necklace pattern of black-white-black white helps to show the evens song. The black beads marks or shows the end of the group of 2 beads. Children can start to jump on the number line. Skip the odd numbers, jump on the even numbers.

-✂-- Activity for learners



As children start to learn the 2s song, they can write it down, and complete the count in twos pattern by writing the numbers.

Gradually you remove the odd numbers, that are between the even numbers. You are left with the evens, the doubles, or the multiples of 2.

Activity for learners

STEP 3:

Use the counting song in 2s to solve real problems

It is always better for children to have a **real context** to think about when repeating the unit (equal groups of 2).

- 1 person has **2 eyes.** Therefore 2 people have ..., 3 people have ...
- 1 sweet costs **R2.** Therefore 2 sweets cost ...
- 1 pair has **2 socks.** Therefore so 2 pairs have ...

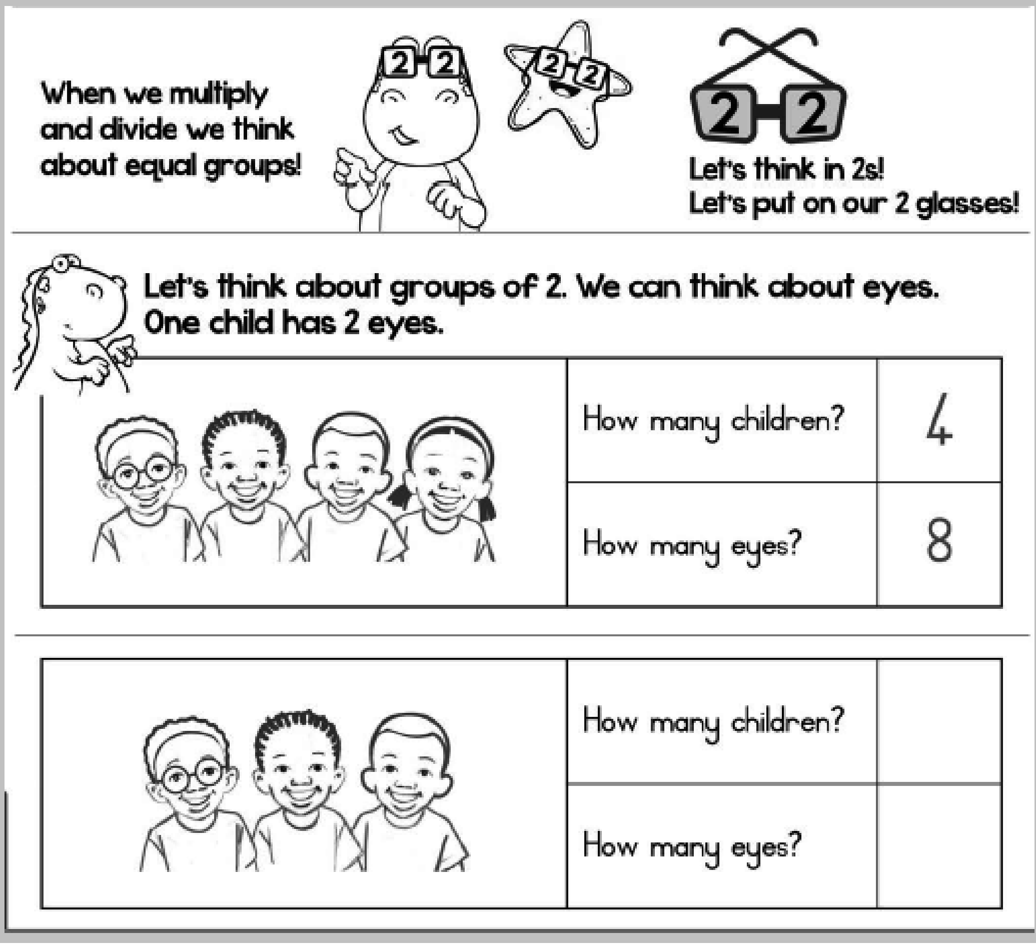
Multiplication Asks:			
4 , how many ?		2 x 4 =	
5 , how many ?		2 x 5 =	
7 , how many ?		2 x 7 =	

8 ÷ 2

When we divide, we separate into equal groups.
How many 2s in 8? I count in 2s. My target is 8.
I keep track of my counts using my fingers.
My fingers give me the answer. 8 ÷ 2 = 4.

Division Asks:			
14 , how many ?		14 ÷ 2 =	
10 , how many ?		10 ÷ 2 =	
8 , how many ?		16 ÷ 2 =	

Activity for learners


When we multiply and divide we think about equal groups!

Let's think in 2s!
Let's put on our 2 glasses!

Let's think about groups of 2. We can think about eyes.
One child has 2 eyes.

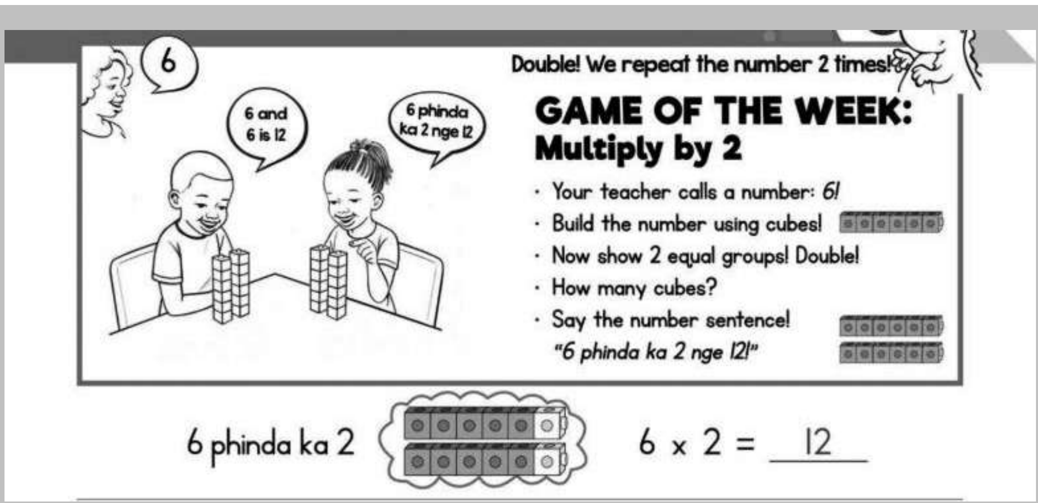
	How many children?	4
	How many eyes?	8

	How many children?	
	How many eyes?	

STEP 4:

Model doubling and halving using manipulatives

Activity for learners

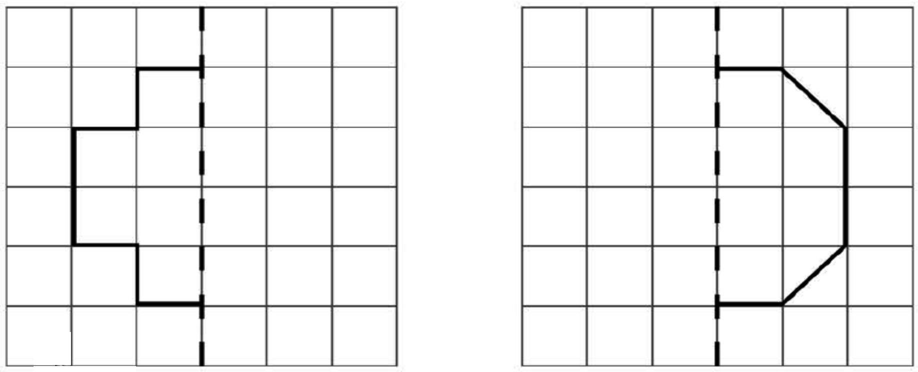


This doubling task uses double as 'phinda ka 2' or 'repeat by 2'. This is how we most commonly think of doubling. You take a number, and repeat that number. Double 6 is 6 + 6. This idea of repeating can be used for symmetry. A mirror line is useful idea for repeating a number when doubling.

Doubling and halving is also visual. There is a mirror line or line of symmetry. When you are given half the shape, you can draw the other half.

Activity for learners

Draw the other half of the shape.



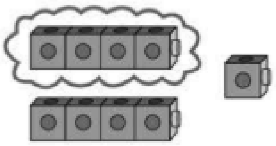
Encourage children to count the squares in half the shape. How many squares in the whole shape?

Activity for learners

GAME: Divide by 2

- Your teacher calls a number: 9!
- Build a tower!
- Break the tower into two equal halves!
- Do you have any cubes remaining?
- Say the number sentence.
- *9 hlula ka 2 nge 4 with one left over!*

9 hlula ka 2 =



$9 \div 2 =$ 4 and one left over

Build towers to solve:

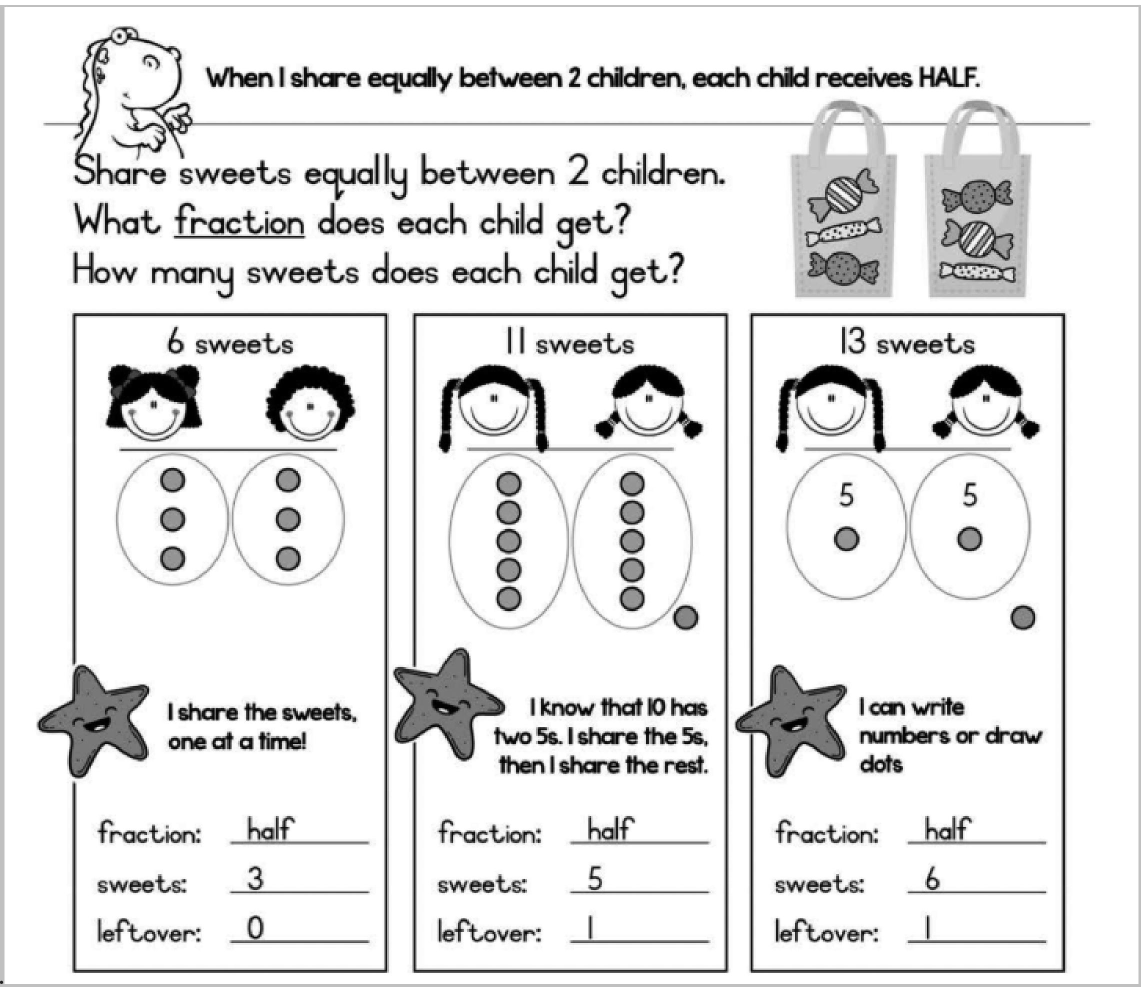
6 hlula ka 2 = ____	7 hlula ka 2 = ____
10 hlula ka 2 = ____	11 hlula ka 2 = ____

This example uses halving (dividing by 2 or hlula ka 2). This is a sharing example – share the cubes equally between 2 people. How many does each person get? The idea of left overs becomes clear, when you try and share an odd number of cubes equally between 2 people.

This idea of sharing equally between two is extended in this next task. The tasks helps children connect **sharing equally between two 2 people**, with each person **getting half**. This builds the vocabulary of

- **Halve:** the verb, meaning the process to share equally between 2 and
- **Half:** the noun, meaning the part/amount which has been shared into 2 equal groups

Activity for learners



But it is also important for children to experience halving as creating groups of 2...
Halving is also - How many 2s?

Why is this packing / grouping way of thinking about division by 2 important?

Groups of 2 are important because when children start to write down division with bigger numbers, they work with the idea of "how many 2s?"
Our written methods for division use the grouping/packing way of thinking about division.

Can you find half of 0? Can you divide 0 by 2?

Learning about the twos has lots of vocabulary to help children talk about the mathematics: even, odd, double, halve, half, share equally between 2, repeat 2 times, twice, equal groups of 2, 2 in each group.

Lets talk maths

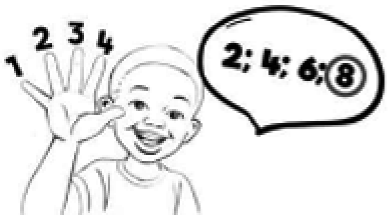
Let's Talk Math in English!

NgesiXhosa sithi:	In English we say:
Xa siphinda kabini inani, siphindaphinda inani kabini.	When we DOUBLE a number, we repeat a number 2 times.
Xa sisabela abantwana aba-2 ngokulinganayo, umntwana ngamnye ufumana ihafu!	When we share equally between 2 children, each child receives half!
Xa sisabela abantwana aba-2 INANI ELINGUMNQAKATHI ngokulinganayo, kusoloko kukho u-l oshiyekayo.	When we share an ODD number between 2 children, there is one left over.

STEP 5:

Keep track of the counts

When children are secure counting in twos starting at 2, they need to start to keep track of their counts on their fingers.



A real problem context helps children think about how many groups (the repeats), and how many are in each group (the unit).

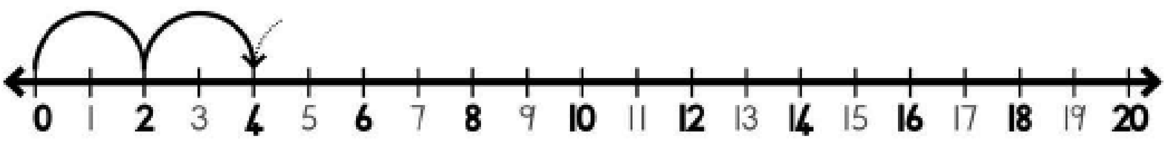
TASK

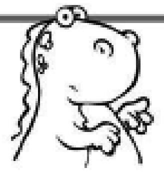
Write down at least 3 problem contexts which are excellent for working with 2s.

As a teacher, you should also use the **measurement pathway** for number.
Here is group counting in 2s on the **number line.**

Activity for learners

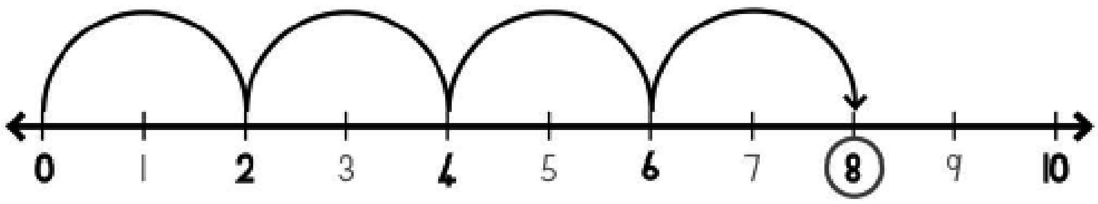
Jump in 2s from 0 to 20.





We can multiply and divide 2 on a number line.
We make equal jumps. We count our jumps!

2 × 4	8 ÷ 2
2 phinda ka 4 = **We make equal jumps of 2!** **We jump 4 times!**	**8 hlula ka 2 =** **We jump in 2s. Our target is 8.** **How many jumps to reach 8?**



Solve by looking at the number line.

A number line is helpful to keep track of both counts.
There are **always two counts with multiplication and division:**

- **Keep track of the number of jumps** (on your fingers or the jumps on the number line)
- **Keep track of the group counting** (say the counting song in twos, or see where you land on the number line).

Children can also use their fingers to keep track of the **number of counts,** and children can use their voices to keep track of the **group counts.**

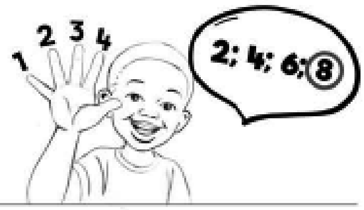
When we multiply:

- We know **the unit** that is repeating.
- We know **how many times to repeat the unit**.
- We are looking for the **total count** (where we land on the number line).

Activity for learners

2 x 4

When we multiply we repeat equal groups. When I multiply 2s, I count in 2s. I keep track of how many twos by using my fingers. The last number I say is the answer. $2 \times 4 = 8$



Multiplication Asks:			
4 , how many ?		$2 \times 4 =$	
5 , how many ?		$2 \times 5 =$	
7 , how many ?		$2 \times 7 =$	

When we divide:

- We know **the unit** that is repeating.
- We know **where we must land**. We have a target to reach (the total count)
- We are looking for **how times we must repeat the unit, to reach the target.** So how many jumps?

Activity for learners

8 ÷ 2

When we divide, we separate into equal groups. How many 2s in 8? I count in 2s. My target is 8. I keep track of my counts using my fingers. My fingers give me the answer. $8 \div 2 = 4$.

Division Asks:			
14 , how many ?		$14 \div 2 =$	
10 , how many ?		$10 \div 2 =$	
8 , how many ?		$16 \div 2 =$	

Children should also work on problems where they repeat 2s. They count in twos and keep track of the counts. Both the bicycles and wheels problem, and the shoes and children problem involve a structure of repeating 2s. These problems involve **repeating 2s.**

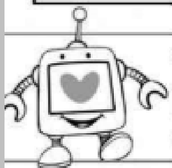
STEP 6:

Learn the doubles and halves (the 2 times table)

Over time children should stop counting in twos, and just **know their doubles by rote**. For example they should know that double 3 is 6. This becomes a rapid recall, known fact.

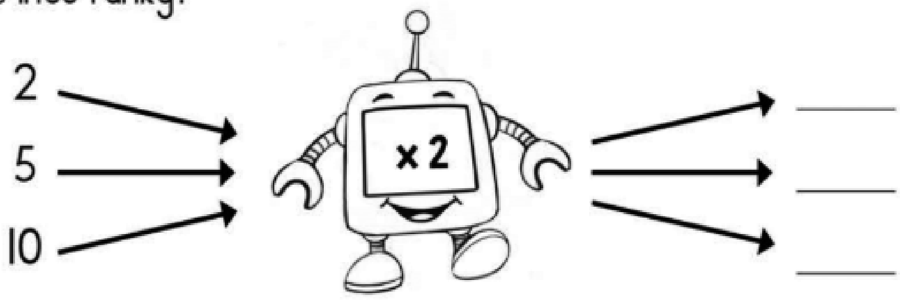
- Multiples of 2 are the doubles. These are the even numbers. Multiples of 2 end in 2, 4, 6, 8 or 0.
- So, you know a number is divisible by 2 if it is even. It must end in 2,4,6 8 or 0.

Activity for learners

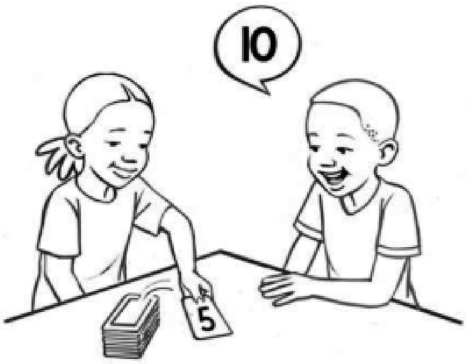


This is a friend of Zibalo called, Function. He is a Maths Robot. His friends call him Funky. He can be programmed to produce different math functions.

Function is now programmed to multiply 2s. When you put a 3 in to Funky, it produces 6! What will happen when we put each of these numbers into Funky?



Activity for learners



Fast Maths with Cards: DOUBLE

- Play in pairs with one pile of cards.
- Use cards 0 to 5 and the 10 card. Mix up.
- Flip over one card.
- DOUBLE!
- Do it again! Faster.

When you are ready, include the 6 to 9 cards!

Can you double 0?

Tens and teaching place value

You focused on the tens in Umthamo 1.3. We will repeat only the main ideas for each step here.

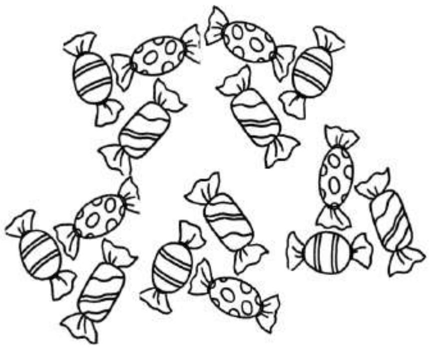
The 10 times table is easy.

- Multiples of 10 end in 0.
- So, you know a number is divisible by 10 if it ends in 0.
- We use the multiples of 10 for our base-ten number system
- We use the multiples of 10 for place value
- Multiples of 10 are especially easy, as we say the tens in African languages.

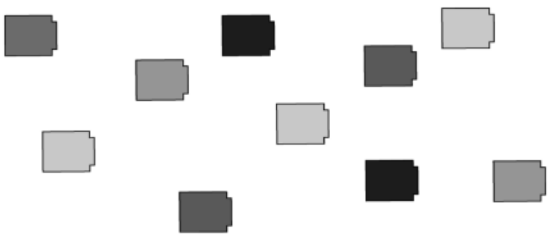
It is important to realise that when we are teaching place value,
we are teaching about the multiples of 10.

STEP 1:

Picture problems



Circle 10 sweets.
Make a bag of 10 sweets.
How many 10s?
Separate the sweets into
10 equal groups.
Hoe many in each group?



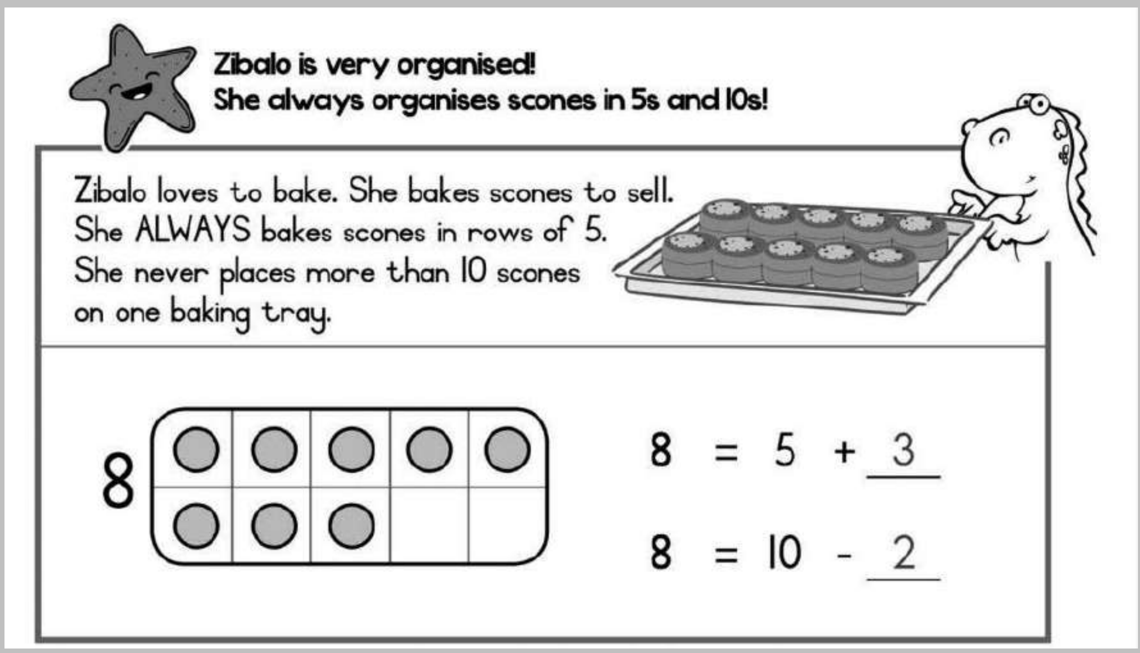
Count out 10 cubes.
How many ways can you find to arrange
them into equal groups?

As our number system is base ten, children need lots of experience with picture problems involving circling 10 ones, to make 1 ten. Again, this is best done using a real context.
You can use ten-frames, or trays of scones.

TASK

Draw some discrete objects. Pose a question so that children group the objects into 10s.

Example



Notice in this activity, that learners think about a number as a row of 5 plus some ones. They also think about a number as two rows of 5, so making ten. The activity encourages thinking in 5s, and thinking in 10s. So

- they think about 8 as 5 plus something...
- they think about 8 as 10 minus something...

Later (late in Grade 1 and early in Grade 2), they start working with place value.

At first they draw and circle the ones, to make 1 ten. Notice that they are dividing by 10.

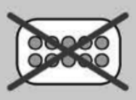
- 17 divided by 10 is 1 ten and 7 ones left over.
- With each number bigger than 10, they make 1 ten, and see how many left over ones they have.

TASK

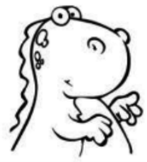
Draw 34 scones. Circle groups of 10 scones to show how they go on Zibalo's tray.

Now draw a number picture for 34. With a number picture you always know that there are 10 scones on 1 tray. You should just write the 3 tens. You should not draw 30 circles!

Zibalo, do not draw EVERY SCONE!
For every full pan of 10 scones, write:



10



Zibalo likes this suggestion.
She draws 16 like this:

16 (10) ○○○○○ ○

Help Zibalo record her scones.

13 (10) ○○○ $13 = 10 + \_\_\_$

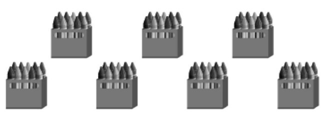
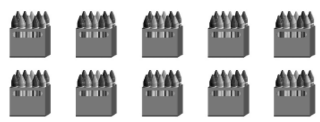
Soon they should not draw all the ones on a tray of 10 scones. They just write 10.

This allows them to start thinking about 10 as a concept on its own. It is 1 ten and they do not have to think about 10 scones, or 10 ones on the tray very time. They just know there are 10. They just write 10.

Working with young children, it is always easier to use a real context for an equal group (a unit) that is repeating. Here 1 box with 10 crayons is used as the unit that repeats.

Activity for learners

When we think of groups of 10, we can think about crayons in a box.
There are 10 crayons in a box!

	How many boxes?	
	How many crayons?	
	How many boxes?	
	How many crayons?	
	How many boxes?	
	How many crayons?	

	
1	10
2	20
3	30
4	
5	
6	
7	
8	
9	
10	

Notice that the children first work on real questions: How many boxes? How many crayons. Then they work on a T-table for boxes and crayons. This time there are no pictures for them to look at.

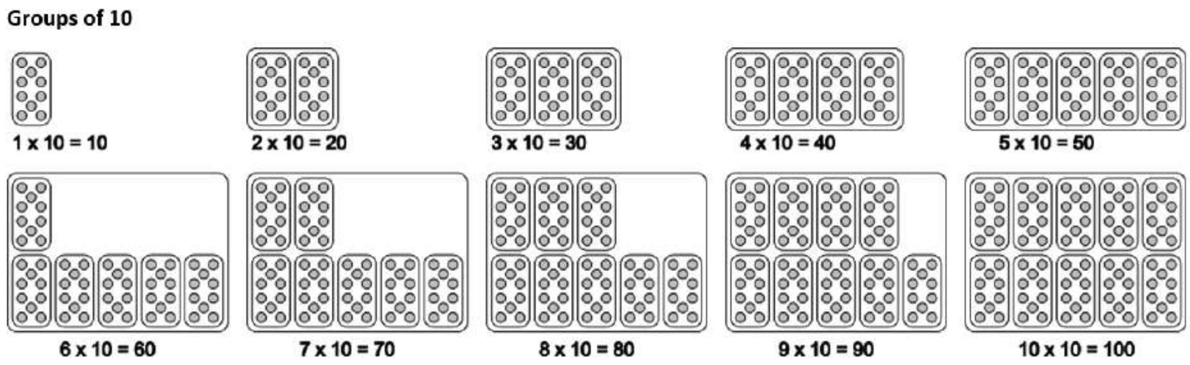
STEP 2:

Learn the medium number song – the 10s song…
Learn about place value

Multiples of ten are first learnt about by group counting in tens.
These are the friendly tens that Zibalo likes to visit when we are adding.
Multiples of ten are easy to say in African languages: ten (there is) 1, ten (there are) 2, ten (there are) 3, ten (there are) 4, ten (there are) 5, ten (there are) 6, ten (there are) 7, ten (there are) 8, ten (there are) 9, ten (there are) 10.

10	Li**shumi**	10	the ten
20	ama**shumi** amabili	ama-10 ama-3	tens, there are 2
30	ama**shumi** amathathu	ama-10 ama-3	tens, there are 3
40	ama**shumi** amamane	ama-10 ama-4	tens, there are 4
50	ama**shumi** amahlanu	ama-10 ama-5	tens, there are 5

In English they are more difficult: ten, twenty, thirty, forty,…
The unit is a "a group of ten", or 1 ten. We repeat that unit.
Here is the ten times table for "____ groups of 10".



Notice what is staying the same, and what is changing.
The equal groups of 10 stay the same. The number of those groups (how many repeats) changes. Recall that in some countries, the phrasing "4 groups of 10" is used, rather than "4 repeated 10 times".

STEP 3:

Use the medium song, the counting song in 10s to solve real problems

It is always better for children to have a real context to think about.

- 1 box has **10 crayons**, 2 boxes have …
- 1 roll has **10 chewing gums**, 2 rolls has …
- 1 box has **10 candles**, so 2 boxes have …
- 1 jug holds **10 cups**, so 2 jugs holds…

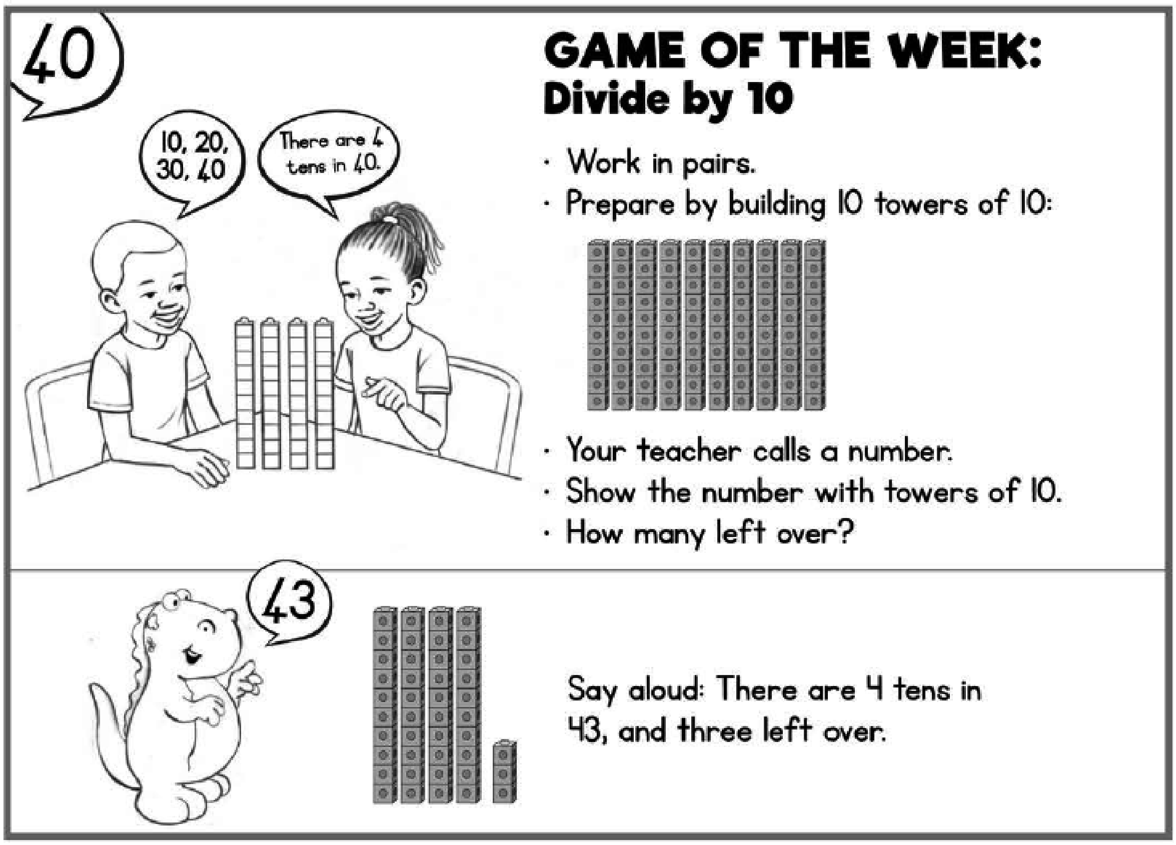
STEP 4:

Model multiplying and dividing by ten using manipulatives and picture problems

You should not give children just one multiplication problem. You decide on a context with equal groups (in this case groups of 10), and ask them lots of questions about that same context. They think about the equal groups of 10, and how many there are with different number of repeats. Any multiplication word problem can become a context for multiplication and division.

- You start with the core word problem: Each box holds 10 candles. There are 4 boxes. How many candles?
- Now ask: **What if?**
- **What if** there were 2 boxes? Or 5 boxes, or 3 boxes? Learners can be coached to create a T table and explore that patterns
- Now you can ask **what if** division questions: **What if** I had 20 candles. How many boxes could I fill? **What if** I had 60 candles and so on.

Activity for learners

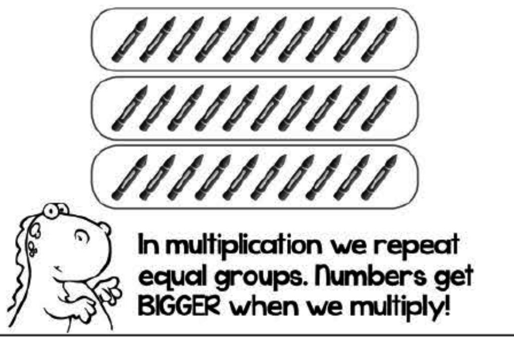
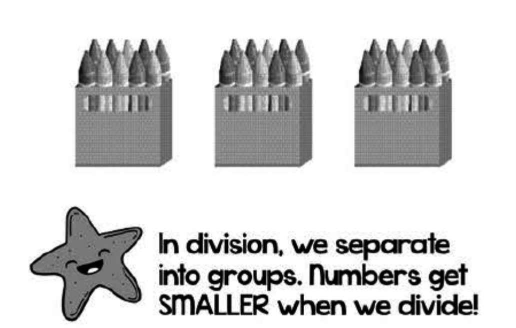


Notice that when making groups of 10, children are dividing by 10. This is the process that they need to understand **place value. 43 is 4 tens and 3 ones.** 43 ÷ 10 = 4 and 3 left over. In Unit 3 we will discuss when we start to use fractional parts, rather than left over for division situations.

STEP 5:

Keep track of the counts

When children are secure counting in tens starting at 10, they need to start to keep track of their counts on their fingers.

Multiplication	Division
	
10 crayons x 3 boxes 10 x 3 = 30 crayons	30 crayons. How many boxes? 30 ÷ 10 = 3 boxes

Here **10 teacups in 1 set**, is used as the equal group that repeats.

Activity for learners

There are 10 teacups in one set.

	How many sets?	
	How many teacups?	
	How many sets?	
	How many teacups?	
	How many sets?	
	How many teacups?	

1	10
2	20
3	30
4	
5	
6	
7	
8	
9	
10	

Notice that the children first work on real questions: How many boxes? How many crayons. Then they work on a T-table for boxes and crayons. This time there are no pictures for them to look at.

STEP 6:

Learn the 10 times table

Over time **children should stop counting in tens, and just know their multiples of ten**.
For example they should know that 5 tens are 50. This becomes a rapid recall, known fact.
Multiplying by 10 is one of the easiest times tables to learn. It is easy to see in the 100 square.

91	92	93	91	95	96	97	98	99	100	Ten tens is 100
81	82	83	84	85	86	87	88	89	90	Nine tens is 90
71	72	73	74	75	76	77	78	79	80	Eight tens is 80
61	62	63	64	65	66	67	68	69	70	Seven tens is 70
51	52	53	54	55	56	57	58	59	60	Six tens is 60
41	42	43	44	45	46	47	48	49	50	Five tens is 50
31	32	33	34	35	36	37	38	39	40	Four tens is 40
21	22	23	24	25	26	27	28	29	30	Three tens is 30
11	12	13	14	15	16	17	18	19	20	Two tens is 20
1	2	3	4	5	6	7	8	9	10	One ten is 10

Explore how to count forward by 10s on the number chart!

When children have explored counting in 10s and the multiples of 10 in the hundred square, they may notice an important pattern:

- To multiply a whole number by 10, write a 0 to the right of the final digit. So you write a 0 in the ones place. So 7 repeated 10 times is 70
- To divide a multiple of 10 by ten, you remove the 0 from the ones column.
- To divide a whole number by 10, you remove the digit in the ones place, and that is the remainder.
- **10 times as many** is the same as **multiplying by 10**

Can you multiply zero by 10?
Can you find 1 tenth of zero?

Lets talk maths

Let's Talk Math in English!

In isiXhosa we say:	In English we say:
	Equal groups.
	One child has 2 ears. 5 children have 10 ears.
	Five groups of two is ten. (5 groups of 2 is 10.)
	There are 5 twos in 10.
	One bucket has 10 litres. 4 buckets have 40 litres.
	Four groups of ten is forty. (4 groups of tens is 40)
	There are 4 tens in 40.

STEP 7:

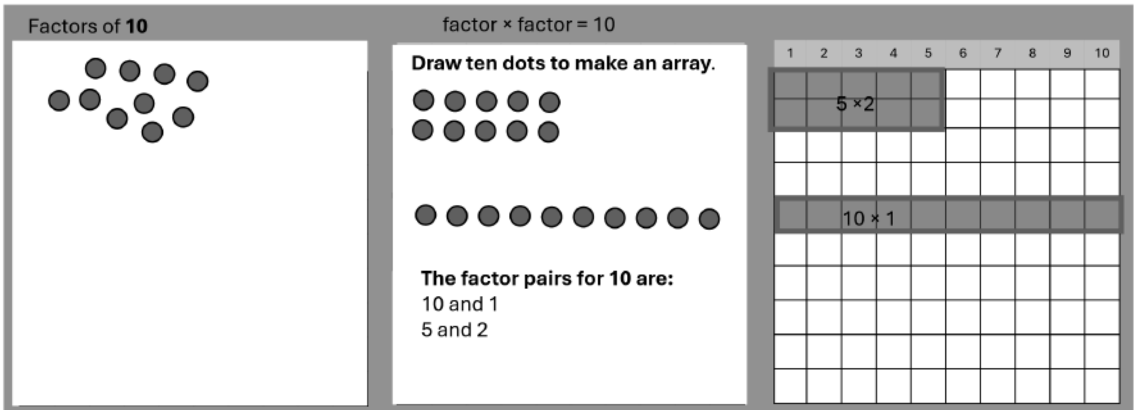
Factors of 10 and divisibility rule for 10

First, let's think about just 1 group of 10.

1 group of 10 can be split using multiplication. 1 group of 10 can be split into **factor pairs**.

10 can be broken down into different **factor pairs.**

1 ten kilogram bag of maize meal.



Here is a set of 10.
It is unstructured

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.

10 can make **2** different sized rectangles. The rectangles could be vertical or horizontal.
Therefore, 10 is not prime.
10 is a composite number

Multiple = factor **×** *factor*
10 = 5 × 2. So 5 and 2 are factors of 10. And...
10 = 10 × 1. So 10 and 1 are factors of 10.

$$10 = 10 \times 1$$

$$10 = 5 \times 2 \times 1$$

10 can be written as a product of prime numbers. 10 = 5 × 2 × 1.

Example

Now, let's think about groups of 10.

These are the multiples of 10.
For example, 6 groups of 10 can be split into factor pairs, and 10 is factor.
6 groups of 10 = 6 × 10 = 60.
So 6 and 10 are factors of 60.
60 can be broken down into different **factor pairs.**

6 ten kilogram bag of maize meal.

Example

How do you know, just by looking, whether a number is a multiple of 10?
My number is a multiple of 10, if it can be divided by 10, without leaving a remainder.

- 60 is a multiple of 10 because 60 ÷ 10 = 6 **rem 0**.
  - » 6 × 10 = 60, therefore 6 and 10 are factors of 60.
  - » 60 is a multiple of 10.
  - » Using place value, 60 = 6 tens and **0 ones**.
  - » 60 has 0 in the ones place. The last digit is 0.
- 62 is NOT a multiple of 10 because 62 ÷ 10 = 6 rem 2
  - » ? x 10 = 62. Oh dear! There is no whole number which we can multiply by 10 to get 62.
  - » 62 is NOT a multiple of 10.
  - » Using place value, 62 = 6 tens and 2 ones.
  - » 62 has 2 in the ones place. The last digit is 2.

TASK

Which of these numbers can be divided by 10, and leave no remainder?
Colour in the multiples of 10

23	7	85	15
70	78	84	146
28	80	42	160
30	40	95	100

Complete:

I can always see that a number is divisibly by 10 because

____

IDEAS FOR YOUR CLASSROOM WITH 10'S

Useful contexts for 10s:

Any groups of 10 each time;
Pack 10 crayons or candles in a box
Each child has 10 fingers;
Each child has 10 toes;
A set of 10 teacups;
10 cups fill 1 jug
A 10 litre bucket
10 kg bag of flour
A 10 minute break

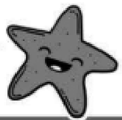
1 ten kg bag of maize meal

Activity for learners

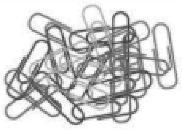
Fast Maths with Dice

- Play in pairs with as many dice as possible.
- Roll the dice.
- Organise into tens to calculate the total.

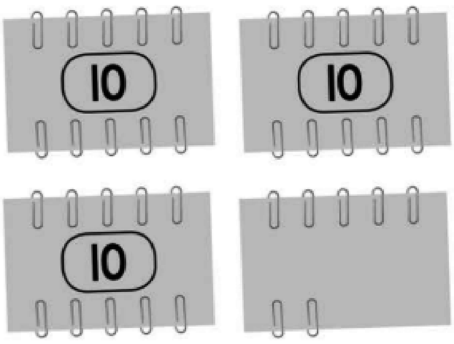
There are four 10s and two more. That is 42!



Zibalo loves to organise numbers into 10.



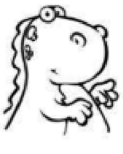
Use paper clips and small pieces of paper from Cut and Play - p 115.



GAME OF THE WEEK:

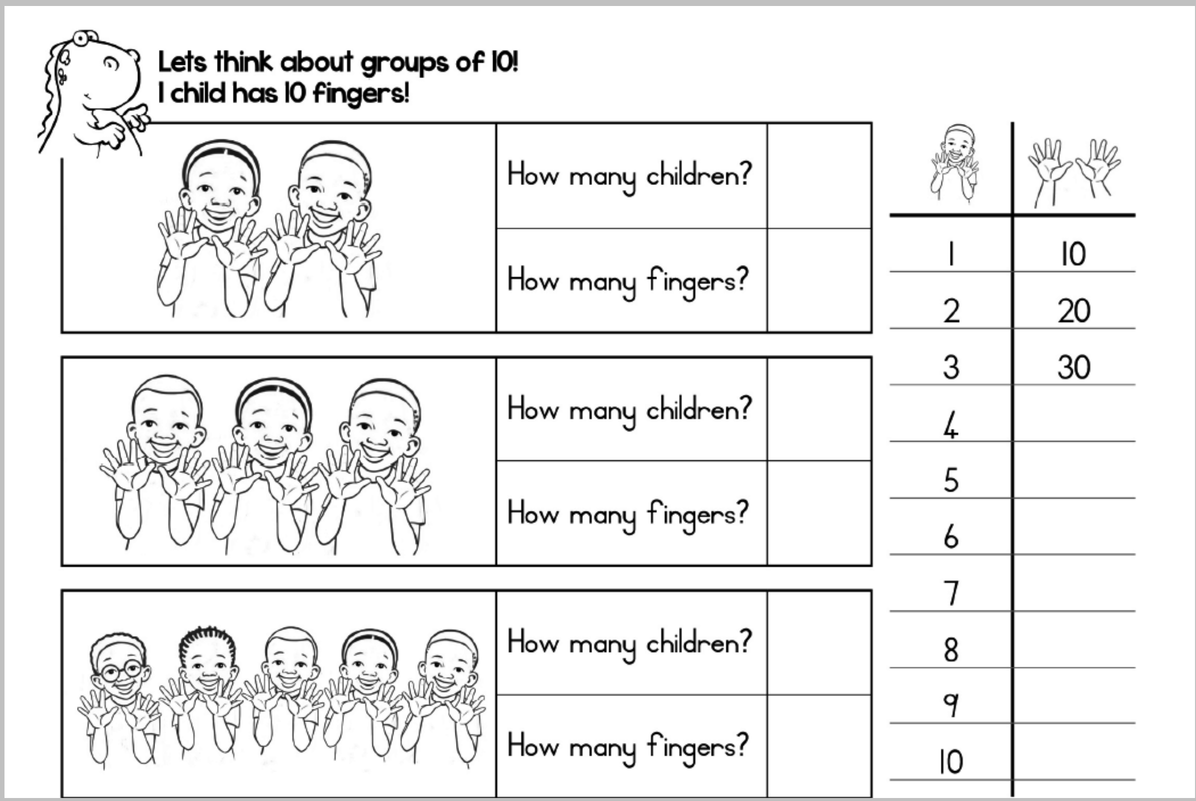
Show so I can see quickly!

- Cut out rectangle cards. See Cut and Play, page XX.
- Take a big handful of paper clips.
- Organise the paper clips so that you can quickly see how many.
- Watch Zibalo!



Zibalo places 10 clips on each 10-card.
She shows 2 groups of 5.
When a 10 cards is complete, she writes the number 10!
Can you see how many paper clips she has?
Three groups of 10 and 7 more! That is 37!

-✂-- Activity for learners



Lets think about groups of 10!
1 child has 10 fingers!

(2 children)	How many children?	
	How many fingers?	
(3 children)	How many children?	
	How many fingers?	
(5 children)	How many children?	
	How many fingers?	

1	10
2	20
3	30
4	
5	
6	
7	
8	
9	
10	

-✂-- Activity for learners

Tens times bigger/smaller on the Gattegno chart

You need: 2 players, Gattengo chart

- One player points to a number
- The other player has to use the chart to calculate ten times the number
- You can do numbers that are not on the chart like 12

Variation:

- Calculate 100 times the number
- Calculate the number divided by 10; or by 100

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

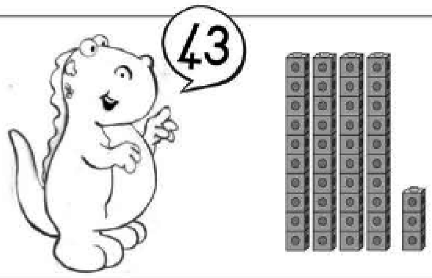
Gattegno chart

-✂-- Activity for learners

MULTIPLY 10s!

- Play in pairs, with one pile of cards.
- Mix up cards 1 to 10.
- First learner flips a card.
- The second learner multiplies by 10.
- Do it again! Faster!

-✂-- Activity for learners



Say aloud: There are 4 tens in 43, and three left over.

Show the number using your towers of 10. Complete the missing numbers.

36	There are 3 10s in 36, and 6 left over.
24	There are ____ 10s in 24, and ____ left over.
37	There are ____ 10s in 37, and ____ left over.
42	There are ____ 10s in 42, and ____ left over.
63	There are ____ 10s in 63, and ____ left over.
58	There are ____ 10s in 58, and ____ left over.
85	There are ____ 10s in 85, and ____ left over.

Fives

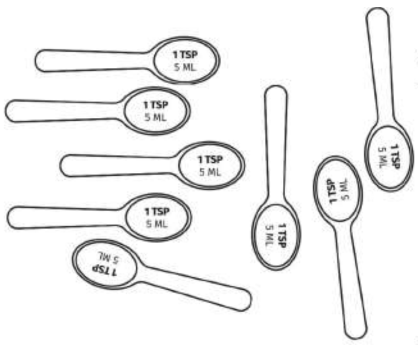
You focused on the fives in Umthamo 1.1. We will not repeat it here.
The 5 times table is easy, if you know the 10s and 2s.

- Multiples of 5 end in 0 or in 5.
- So, you know a number is divisible by 5 if it has 0 or 5 ones. It ends in 0 or 5.

As with learning the 100 addition facts, children must make connections.

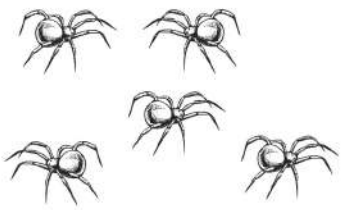
Multiplication is commutative. Division is not commutative

First, multiplication is commutative. It can be done in any order.



8 teaspoons, how many milli litres?

____							
5	5	5	5	5	5	5	5



5 spiders, how many legs? ____

____				
8	8	8	8	8

I know 5 repeated 8 times = 40. Therefore, I know 8 repeated 5 times = 40.

I know 5 × 8 = 40. Therefore, I know 8 × 5 = 40.

If you know one multiplication fact, you immediately know another one too.

Activity for learners

Pose a multiplication word problem for:
3 repeated 5 times.
Draw a whole- equal part diagram.

Pose a multiplications problem for
3 groups of 5.
Draw a whole-equal part diagram.

A family of 8 equivalent number sentence

Second, for every multiplication fact, there is a family of equivalent number sentences.

One way of thinking about the relationship between 5, 8 and 40

40							
5	5	5	5	5	5	5	5

5 repeated 8 times
8 groups of 5
How many 5s, fit into 40?
Share 40 into 8 equal groups..

Family of 8 number sentence

5 × 8 = 40
8 × 5 = 40
40 ÷ 8 = 5
40 ÷ 5 = 8
and the reverse
40 = 5 × 8
40 = 8 × 5
5 = 40 ÷ 8
8 = 40 ÷ 5

Another way of thinking about the relationship between 5, 8 and 40

40				
8	8	8	8	8

8 repeated 5 times
5 groups of 8
How many 8s, fit into 40?
Share 40 into 5 equal groups...

Family of 8 number sentence

5 × 8 = 40
8 × 5 = 40
40 ÷ 8 = 5
40 ÷ 5 = 8
and the reverse
40 = 5 × 8
40 = 8 × 5
5 = 40 ÷ 8
8 = 40 ÷ 5

A family of multiplication and division facts

5 × 8 = 40	40 ÷ 8 = 5
8 × 5 = 40	40 ÷ 5 = 8
40 = 5 × 8	5 = 40 ÷ 8
40 = 8 × 5	5 = 40 ÷ 8

Division is the inverse of multiplication. Multiplication and division fit together.

The distributive property

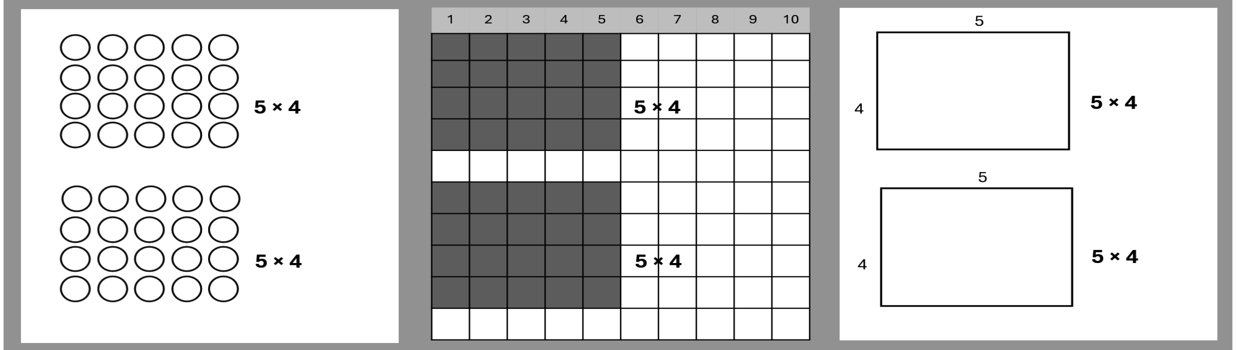
Third, you can break up a number and multiply each part (distributive property).
You can imagine 5 repeated 8 times as picture problem, or as an array, a grid or as the area of a rectangle.

- In the **picture problem** each hand has 5 fingers.
  You can arrange the 8 hands in many different ways.
- In the **array**, 1 row has 5 dots. This is repeated 8 times.
  You can break up the 8 rows in many different ways.
- In the **grid,** 1 row has 5 blocks. This is repeated 8 times.
  You can break up the 8 rows in many different ways.
- In the **area of a rectangle**, the length is 5 cm.
  The breadth is 8 cm. you can split the rectangle into 2 (or more) parts in many different ways.

8 = 4 + 4 or 8 = 5 + 3 = 6 + 2 etc...When you beak up the 8 you multiply each part by 5, and you always get the same result.

Here we break up 8 into 4 and 4.

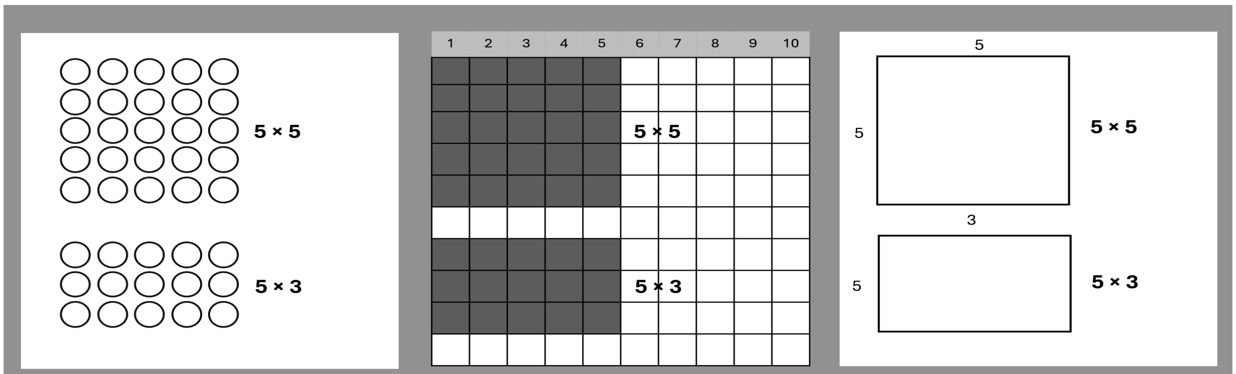
8 = 4 + 4. Therefore, 5 × 8 = 5 × (4 + 4) = (5 × 4) + (5 × 4).



Array
Grid
Area of rectangles

Here, we break up 8 into 5 and 3.

8 = 3 + 5. Therefore, 5 × 8 = 5 × (3 + 5) = (5 × 3) + (5 × 5)



Array
Grid
Area of rectangles

To work out 8 × 5, we can break up 8 in any way, multiply each part by 5, add the products together, and still get the same answer. No matter how we break up the 8, we always get a product of 40.

In high school, we generalise this.

- 8 can be broken up in any way. Let's call the one part n and the other part m. If I am multiplying by 5, then I can first multiply n by 5, then multiply m by 5, then add them.
- If 8 = n + m, then **8** × 5 = (**n + m**) × 5 = (**n** × 5) + (**m** × 5)
- In high schools we write the coefficient (the number) before the variable (the letter), and so we write 5(n + m) = 5n + 5m

Using a clue board

The clue-board works because of the distributive property of multiplication over addition: You can break up a number and multiply each part.

You can create new numbers to add to the clue-board using the distributive property of multiplication

Example

This is a ×5 clue board

×5

Hands	Fingers
1	5
2	10
4	20
8	40
10	50

"Start with 1. **Double, double double. Stop and think**: 10"

"Start with 5. **Double, double double. Stop and think**:

5 × 10

You create a ×5 clue board like this:

Draw the T for the table. Write ×5 at the top.

It is helpful to think of a context where there are groups of fives. Eg hands and fingers.

For the left hand side:

Start with 1

Double 1 to get 2. **Double** 2 to get 4. **Double** 4 to get 8

Those are the doubling facts.

Stop and think.

The other easy number fact is x10

So write 10.

For the right had side:

This is the 5 times table.

1 repeated 5 times is 5.

Double 5 to get 10. **Double** 10 to get 20. **Double** 20 to get 40

Stop and think.

The other easy number fact is ×10

So 5 x 10 = 50

3 = 1 + 2 therefore 3 × 5 = (1 × 5) + (2 × 5)

5 = 1 + 4 therefore 5 × 5 = (1 × 5) + (4 × 5)

7 = 1 + 2 + 4 therefore 7 × 5 = (1 × 5) + (2 × 5) + (4 x 5)

If there are 0 hands, how many fingers?
If there are 0 fingers, how may hands?

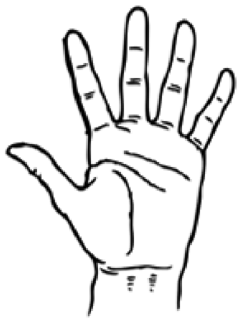
Factors of 5 and divisibility rule for 5

factor **×** *factor = multiple*

First, let's think about just 1 group of 5.

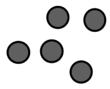
1 group of 5 can be split using multiplication. 1 group of 5 can only be split into one **factor pair**. 5 = 5 × 1

5 cannot be broken down into different **factor pairs.** Five is a prime number.



1 hand has 5 fingers

Factors of **5**



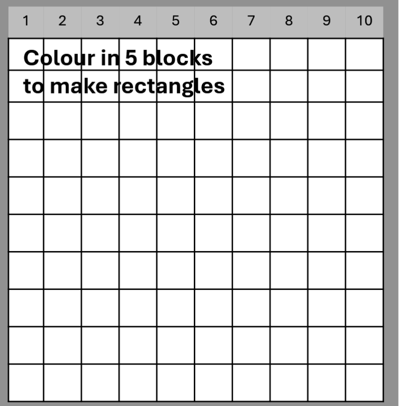
Here is a set of 5. It is unstructured

factor × factor = 5

Draw 5 dots to make arrays.

The factor pairs for 5 are:
___ and ____
___ and ____

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.



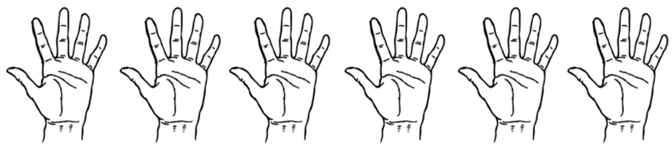
5 can make ___ different rectangles.
Therefore, 5 is prime / composite.
4 can make a square. True/false

Multiple = factor **×** *factor*

5 = 5 × 1. So 5 and 1 are factors of 5.
5 can be written as a product of prime numbers. 5 = 5 × 1.
5 is a prime number.

Now, let's think about groups of 5. These are the multiples of 5.

For example, 6 groups of 5 can be split into factor pairs, and 5 is factor.
6 groups of 5 = 6 × 5 = 30.
So 6 and 5 are factors of 30.



6 hands, with 5 fingers on each hand.

factor **×** *factor = multiple*

30 can be broken down into different **factor pairs.**

30 = 5 × 6 30 = 10 × 3 30 = 15 × 2 30 = 1 × 30

How do you know, just by looking, whether a number is a multiple of 5?
My number is a multiple of 5, if it can be divided by 5, without leaving a remainder.

- 30 is a multiple of 5 because 30 ÷ 5 = 6 **rem 0**.
  - » 6 × 5 = 30, therefore 6 and 5 are factors of 30.
  - » 30 is a multiple of 5.
  - » Using place value, 30 = 3 tens and **0 ones**.
  - » 30 has 0 in the ones place. The last digit is 0.
- 35 is a multiple of 5 because 35 ÷ 5 = 7 **rem 0**.
- 32 is NOT a multiple of 5 because 32 ÷ 5 = 6 rem 2
  - » ? × 5 = 32. Oh dear! There is no whole number which we can multiply by 5 to get 32.
  - » 32 is NOT a multiple of 10.
  - » Using place value, 32 = 3 tens and 2 ones. Using our 5s glasses, 32 = 6 fives and 2 ones.
  - » 32 has 2 in the ones place. The last digit is 2.

IDEAS FOR YOUR CLASSROOM WITH 5's

Activity for learners

Which of these numbers can be divided by 5, and leave no remainder?
Colour in the multiples of 10

23	7	85	15
70	75	84	146
28	80	42	165
30	40	95	101

Complete:

I can always see that a number is divisibly by 5 because

____
____
____
____

Useful contexts for 5s:

Any groups of 5 each time;

Pack 5 in a box

Each hand has 5 fingers;

Each foot has 5 toes;

There are 5 schools days in each week

We count the minutes on an analogue clock in 5s.

A teaspoon holds 5 mℓ

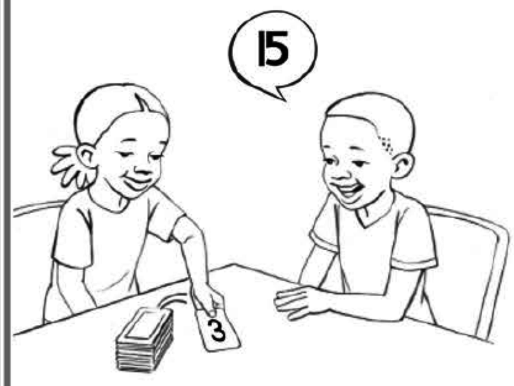
5 kg of maize meal

A dice and domino pattern of 5 is quick to see.

Hands	Fingers
1	5
2	10
4	
8	
10	

There are 5 fingers one each hand

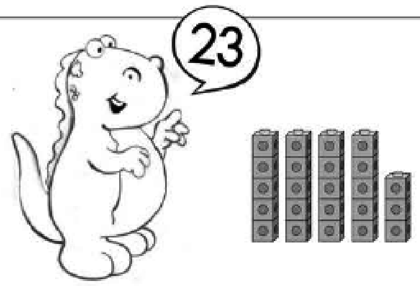
Activity for learners



MULTIPLY 5s!

- Play in pairs, with one pile of cards.
- Mix up cards 1 to 10.
- First learner flips a card.
- The second learner multiplies by 5.
- Do it again! Faster!

Activity for learners



Say aloud: There are 4 fives in 23, and three left over.

Show the number using your towers of 5. Complete the missing numbers.

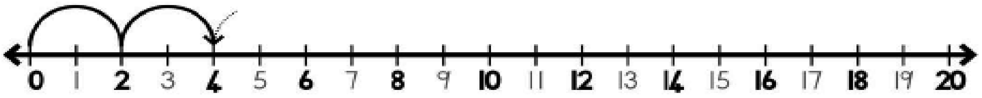
16	There are 3 5s in 16, and 1 left over.
18	There are ____ 5s in 18, and ____ left over.
25	There are ____ 5s in 25, and ____ left over.
27	There are ____ 5s in 27, and ____ left over.
17	There are ____ 5s in 17, and ____ left over.
20	There are ____ 5s in 20, and ____ left over.
24	There are ____ 5s in 24, and ____ left over.

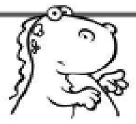
Tasks 2.1

Number lines and problem contexts

1. Use a number line for multiplication and division.

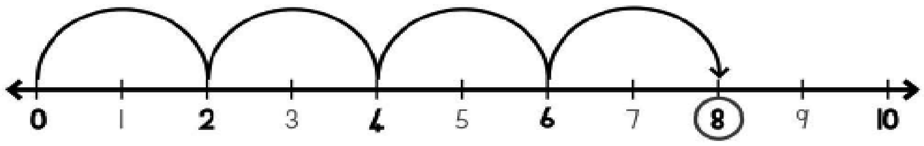
Jump in 2s from 0 to 20.



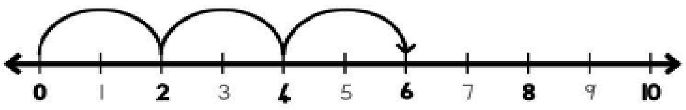


We can multiply and divide 2 on a number line.
We make equal jumps. We count our jumps!

2 × 4	8 ÷ 2
2 phinda ka 4 = **We make equal jumps of 2!** **We jump 4 times!**	**8 hlula ka 2 =** **We jump in 2s. Our target is 8.** **How many jumps to reach 8?**

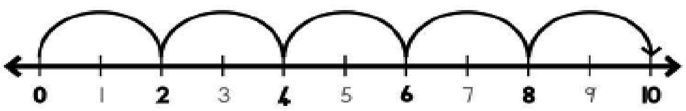


Solve by looking at the number line.



$2 \times 3 = \square$

$6 \div 2 = \square$

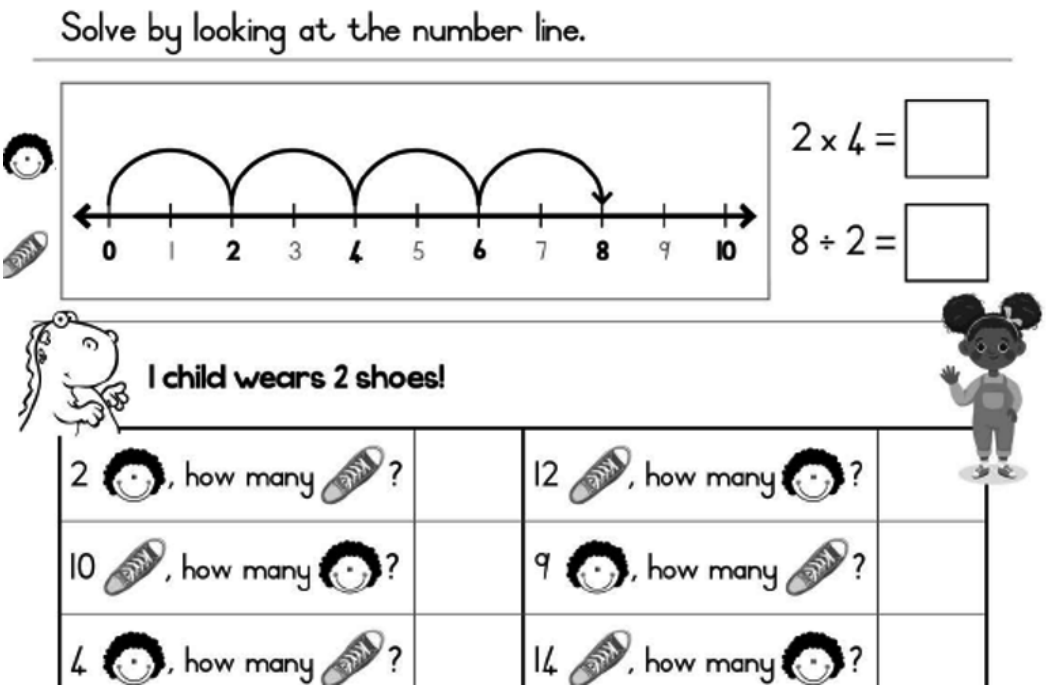


$2 \times 5 = \square$

$10 \div 2 = \square$

2. Show how you will draw 5 repeated 4 times = _____ , on a number line.

3. Show how you will draw 2 × 7 = _____ , on a number line.



Solve by looking at the number line.

0 1 2 3 4 5 6 7 8 9 10

$2 \times 4 = \square$

$8 \div 2 = \square$

I child wears 2 shoes!

2 , how many ?		12 , how many ?	
10 , how many ?		9 , how many ?	
4 , how many ?		14 , how many ?	

4. **Now draw your own number line**.

Show how you will draw 8 ÷ 2 = ______, on a number line.
Remember 8 ÷ 2 = ______, is the same as 2 × ______ = 8.

____

5. Show how you will draw 12 ÷ 2 = ______, on a number line.

Remember 12 ÷ 2 = ______, is the same as 2 × ______ = 12.

____

6. Write the **family of number sentences** for 2 × 5 = 10

____

____

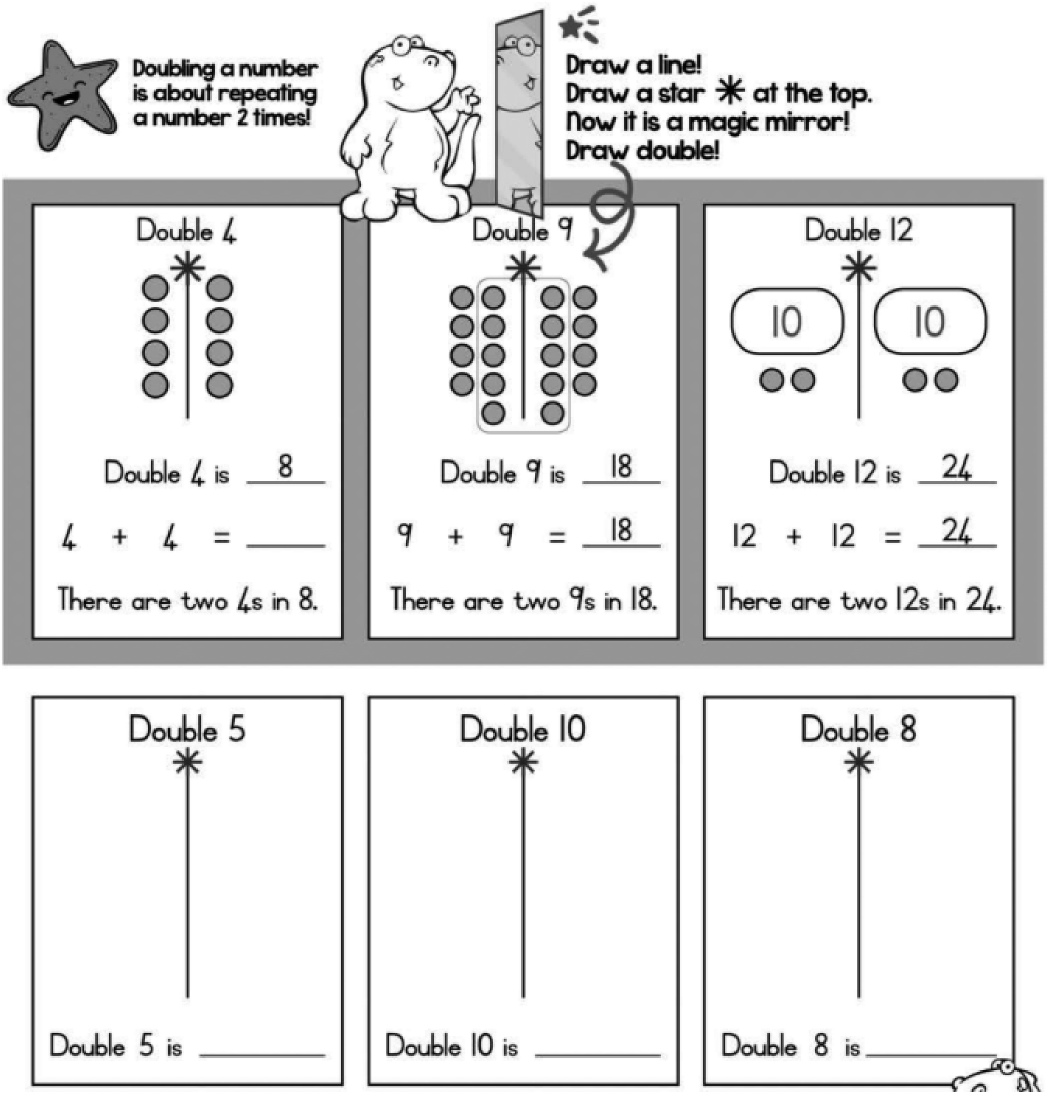
____

____

7. A teacher gave this picture problem for ×2 multiplying and dividing:
   1 bicycle has 2 wheels…

	How many bikes?	
	How many wheels?	

Make a similar picture problem but use another context for ×2.



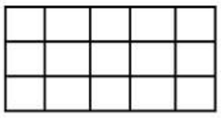
8. Use the magic mirror, to show how to repeat a number (double the number).

9. A teacher made a card for her children to practice ×5 in different contexts

Make your own card for ×10.
Use counting and measurement contexts.

Card 110

1. How many tiles?



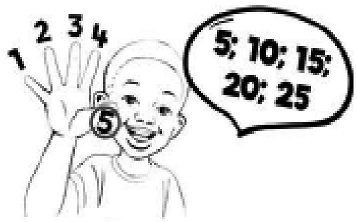
2.
How many millilitres?

3.

Teaspoons	**Millilitres**
1	5
2	
4	
8	
10	

4.
I must take 20 mℓ of medicine. How many teaspoons must I take?

10. Asiya can group count forwards in 5s.

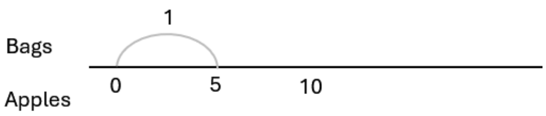


But she loses track of her counts for numbers bigger than 5. To help Asiya we **create a real context**. Let's think about **bags of apples**. Each bag has 5 apples.

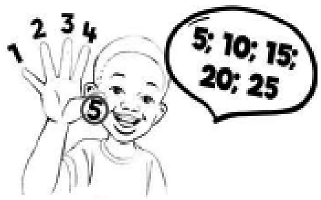
a. Draw a T-table to show Asiya how she can keep track of her counts, for 6 × 5 = ______

b. Draw a number line to show Asiya how she can keep track of her counts, for 4 × 7 = ______

Bags	Apples
1	5



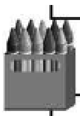
11. Raymond can group **count forwards in 5s.**



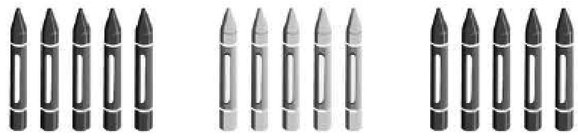
He needs help thinking about division.
He needs to work out 40 ÷ 5.

a. Show him how to draw a T-table.
b. Show him how to draw a number line.

12. Here is a problem context for division.



There are 10 crayons in a box.

	How many crayons?	
	How many boxes?	
	How many left over?	
	How many crayons?	
	How many boxes?	
	How many left over?	
	How many crayons?	
	How many boxes?	
	How many left over?	

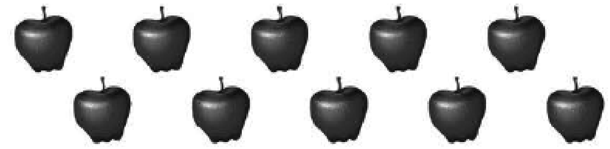
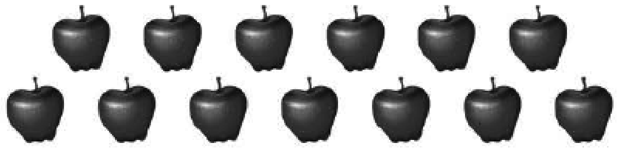
a. Answer the questions. You can circle to show equal groups.
b. Is this context on the counting pathway or on the measurement pathway?

____

c. Draw a function machine that shows the function to get from crayons to boxes.

13. Here is a division problem context.

Pack 5 apples in 1 bag.

	How many apples?	
	How many bags?	
	How many left over?	
	How many apples?	
	How many bags?	
	How many left over?	
	How many apples?	
	How many bags?	
	How many left over?	

a. Answer the questions. You can circle to show equal groups.

b. Is this context on the counting pathway or on the measurement pathway?

____

c. Draw a function machine that shows the function to get from crayons (input) to boxes (output).

d. Pose a new problem which uses a measurement context for this function.

14. At least 10 children in your class are struggling with multiplication and division by 10. It helps children if you give them a real context to think about.

Create a problem context for the ×10 function and the ÷10 function.

15.

- *'Use a tick or a cross to show whether each pair of equations is correct or not.'*

		Correct (✓) or incorrect (✗)
A. $2 \times 5 = 10$	$10 \div 5 = 2$	
B. $5 \times 2 = 10$	$10 \div 2 = 5$	
C. $10 = 2 \times 5$	$5 \div 10 = 2$	
D. $10 = 5 \times 2$	$5 = 10 \div 2$	

'Explain why the pairs of equations marked ✗ are not correct.'

Correct or incorrect

0 divided by 10 = 0
10 divided by 0 = 0

0 divided by 5 = 0
5 divided by 0 = 5

16.

- *'Jasmine is answering this question:*
10 children share 40 stickers between them.
How many stickers does each child get?'
'Jasmine writes:'

$4 \times 10 = 40$

so

$10 \div 40 = 4$

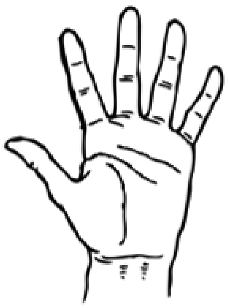
'Is she correct? Explain.'

Tasks 2.1.3

Timed tango × 5 and × 10

Two minute tango × 5

1	_____ = 5 × 6	**11**	6 × 5 = ______
2	5 × ______ = 50	**12**	40 ÷ 5 = ______
3	5 × 1 = ______	**13**	11 × 5 = ______
4	______ × 5 = 30	**24**	______ = 25 ÷ 5
5	______ × 5 = 35	**15**	______ ÷ 5 = 9
6	______ = 7 × 5	**16**	5 × ______ = 60
7	5 × 5 = ______	**17**	15 ÷ 5 = ______
8	12 × 5 = ______	**18**	1 fifth of 60 = ______
9	Factorise: 40 = 5 × ______	**19**	Factorise: 35 = 7 × _____
10	Which is a multiple of 5? 5; 13; 27; 50; 8	**20**	Which is a factor of 5? 5; 13; 27; 50; 8



Create a clue board
for hands and fingers

I know a number is divisible by 5 when...

The factor pairs of 5 are ....
Therefore 5 is prime/composite.
5 written as a product of prime numbers is _____________.

By practicing the x5 tango you will master:

- Find 1 fifth is the same as divide by 5
- The multiples of 5 are the 5s song starting at 5: 5, 10, 15, 20, 25, 30.
- That numbers are divisible by 5 is they have 0 or 5 in the ones column.
- They end in 0 or 5.

Two minute tango × 10

1	______ = 10 × 5	**11**	______ = 80 ÷ 10
2	10 × 9 = ______	**12**	100 ÷ 10 = ______
3	______ × 10 = 90	**13**	10 × ______ = 110
4	12 × 10 = ______	**24**	50 ÷ 10 = ______
5	10 × ______ = 110	**15**	______ × 10 = 40
6	______ = 8 × 10	**16**	10 × 9 = ______
7	______ × 10 = 50	**17**	1 tenth of 60 = ______
8	4 × 10 = ______	**18**	______ ÷ 10 = 10
9	Factorise: 70 = 10 × ______	19	Factorise: 120 = ______ × 12
10	Which is a multiple of 10? 5; 13; 27; 50; 8	**20**	Which is a factor of 10? 5; 13; 27; 50; 8

10 kg of maize meal

You don't need a clueboard for ×10!
List other good contexts for ×10:

I know a number is divisible by 10 when...
The factor pairs of 10 are ....
Therefore 10 is prime/composite.
10 written as a product of prime numbers is ____________.

By practicing the x10 tango you will master:

- Multiplying and dividing by 10
- Factorizing: product = factor x factor
- Knowing the 'find 1 tenth' is the same as divide by 10
- The multiples of 10: The tens song starting at 10: 10, 20, 30, 40, ...
- That numbers are divisible by 10 if they have 0 ones.

SUMMARY

SUMMARY 2.1

A teacher at school says her Grade 3 children just don't know their timetables:

"I just don't know what to do with these kids!. Their parents don't help them!"

Write down how she could help the children in her class to learn their 2, 5 and 10 times tables.

UMTHAMO 2.2

LEARNING 2S, 4S AND 8S

Key ideas

1. The **2, 4 and 8-times tables are closely related** to each other.
   - 4 is double 2, and 8 is double 4.
   - Multiples of 4, are double the multiples of 2
   - Multiples of 8, are double the multiples of 4
2. **Products in the 2, 4 and 8 times tables share the same factors**.
   - The factor pairs for 2 are 1 and 2. 2 is a prime number.
   - The factor pairs for 4 are 1 and 4, and 2 and 2. 4 is a composite number.
   - The factor pairs for 8 are 1 and 8, and 2 and 4. 8 is a composite number.
3. You can **factorise** a number to simplify the multiplication.
   Factorise means split into smaller factors.

For example:

$7 \times 4 = 7 \times 2 \times 2$. Multiply by 4 is the same as double, then double.
$7 \times 8 = 7 \times 2 \times 2 \times 2$. Multiply by 8 is the same as double, then double, and double again.

4. When you multiply two factors, you can **factorise** one or both factors to simplify the problem.
   - $\text{factor}_1 \times \text{factor}_2 = \text{product}$
   - $\text{factor}_3 \times \text{factor}_4 \times \text{factor}_2 = \text{product}$
5. Skip or **group counting in 4s**, is **counting the multiples of 4**.
   - **Adjacent even numbers** have a difference of 2.
   - **Adjacent multiples in the 4s song**, have a difference of 4.
     - The fours song is: 4, 8, 12, 16, 20 ... 8 and 12 are **next to** each other. So the difference between 12 and 8 is 4.
   - **Adjacent multiples in the 8s song,** have a difference of 8.
     - The eights song is: 8, 16, 24 ... 16 and 24 are **next to** each other. So the difference between 16 and 24 is 8.
6. **Divisibility rules:**
   - Products in the 2, 4 and 8-times table are always even numbers.
   - 100 is divisibly by 4. Therefore numbers bigger than 2-digits, are divisible by 4, if the last 2 digits are divisible by 4. So 8 744 is divisible by 4, because 44 is divisible by 4.

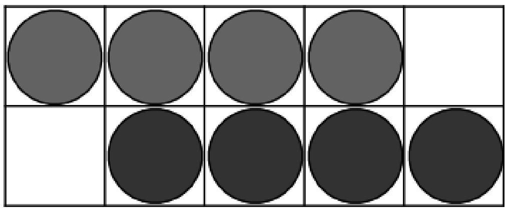
Children learn 2s, then 4s and then 8s.

7. You can divide zero by 2, 4 or 8. You get 0.
   But you can’t divide 2, 4 or 8 by zero. it is undefined.

Twos

You have done lots of work on twos in umthamo 2.1. We will not repeat that here. It is important to realise that doubling is the same as multiplying by 2.

Number talk with 2s



This number talk can be used as a reminder that there are two ways to thinking about multiplication:

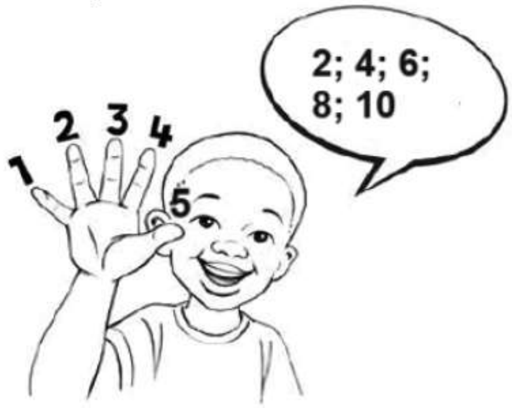
- 4 × 2 = 4 repeated 2 times = 4 + 4
- 4 × 2 = 4 groups of 2 = 2 + 2 + 2 + 2

Children may also see different patterns such as 1 + 6 + 1, or 1 + 3 + 3 + 1 etc.

Two ways to double and two ways to halve

Example

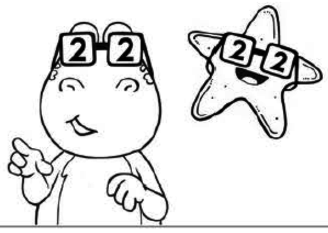
There are **two ways to double** and to **two ways to halve**
To double 5: Repeat 5 two times. **OR** Count forwards in 2s, for 5 counts.
To half 10: Share equally into 2 groups. **OR** Count forwards in 2s and reach a target of 10. How many counts?



One way to think about doubling or multiplying by 2, is repeating groups of 2.
6 × 2 = 6 groups of 2 = 2 + 2 + 2 + 2 + 2 + 2.

When you repeat two 0 times, you get 0.
When you repeat two 1 time, you get 2

When we multiply and divide we think about equal groups!



Let's think in 2s!
Let's put on our 2 glasses!

When we want children to think about doubling as groups of 2s, the we should use contexts that have a structure of pairs or 2s.

- They could think about R2 coins – those are always in 2s.
- They can think about 2 litre bottles. Or a bottle that is filled with 2 cups.

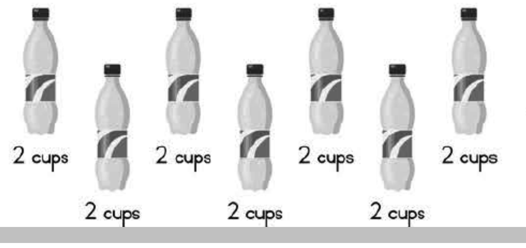
Example

We can think of a R2 coin! The value of 1 coin is 2 Rands!

How many coins?	
How many Rands?	R

2 cups fill one bottle.

When we think about groups of 2, we can think about 2 cups filling one bottle!



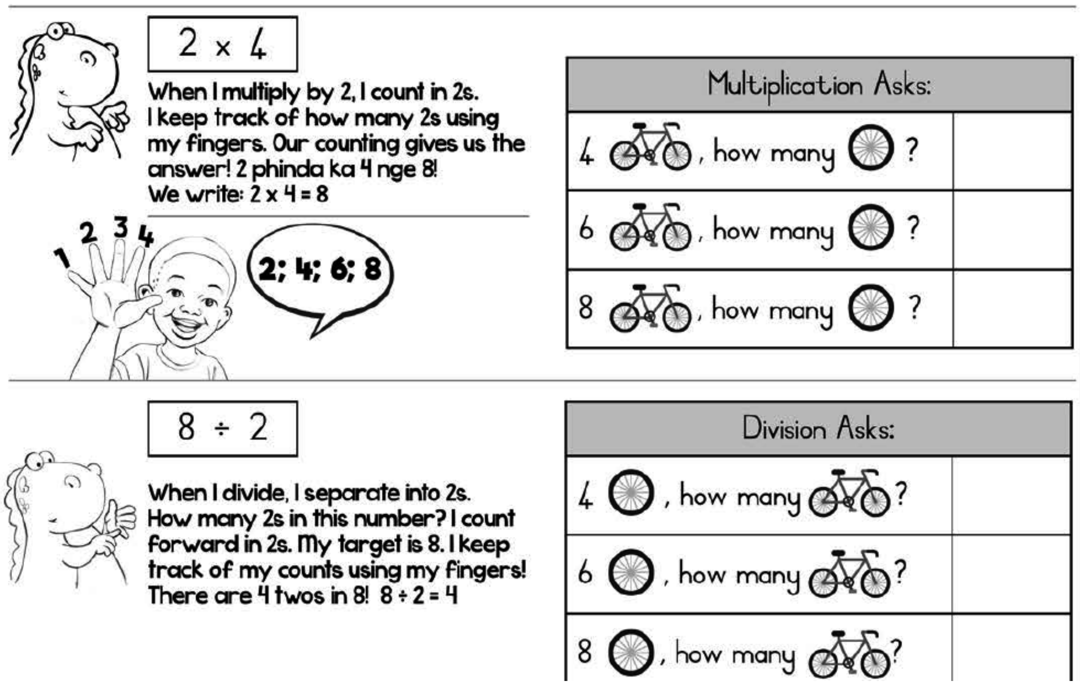
How many bottles?	
How many cups?	

TASK

Create another picture problem where there are groups of 2 that are repeated.

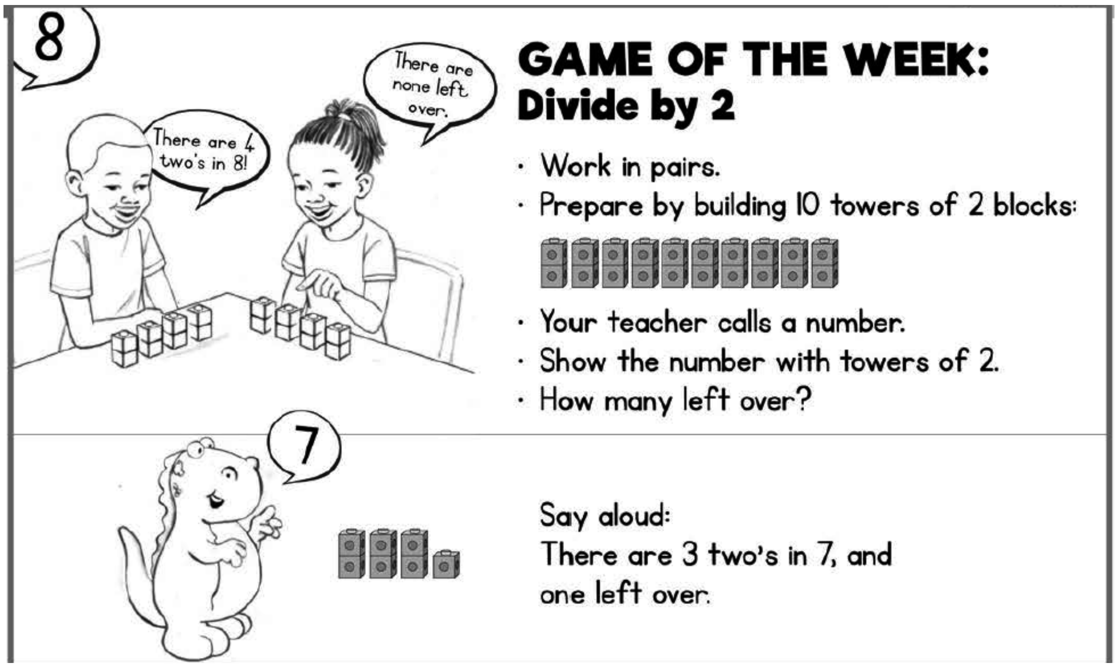
Linking halving to doubling

Working with groups of 2s, they can think about multiplying and dividing as inverses. A context of bicycles and wheels is a good context for multiplying and dividing by 2.



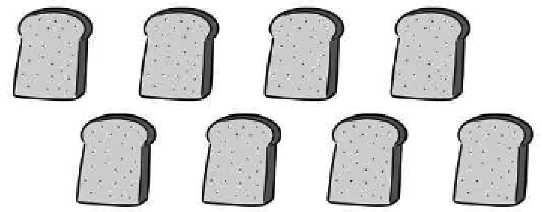
Children should also work with cubes. When dividing by 2, or when halving:

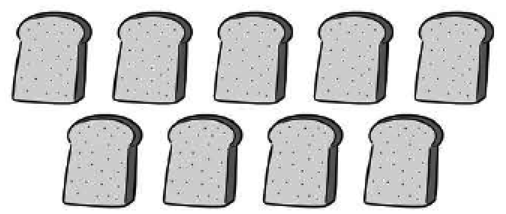
- They should not just make 2 equal groups.
- They should also make groups of 2s.



Again, **having a real context where the structure is 2s, is always helpful.**

A sandwich always has 2 sliced of bread. So thinking about slices of bread and sandwiches is a good context.

	How many slices of bread?	
	How many sandwiches?	
	How many slices left over?	

	How many slices of bread?	
	How many sandwiches?	
	How many slices left over?	

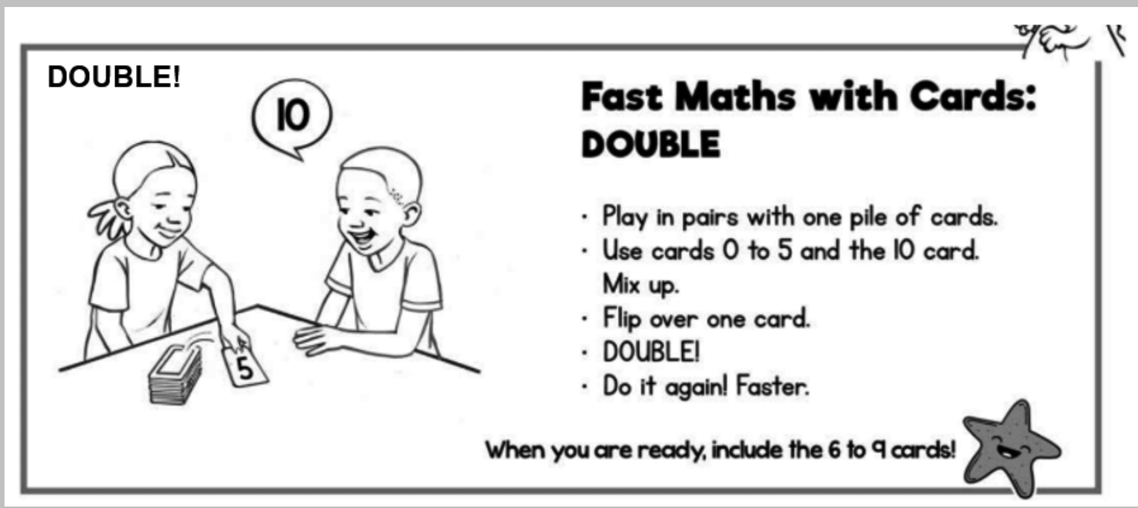
Remember to use vocabulary about fractions with working with doubling and halving:

- One slice is bread is 1 half of the sandwich.
- How many sandwiches can I make with 3 slices of bread?

More ideas for your classroom with twos

Activity for learners

Double with cards



The winner is the player that did the most calculations.
What do you notice about the multiples of 2?
Is zero a multiple of 2? Is zero an odd or an even number?

Fours

The fours song is closely connected to the twos song. So if children know the evens or the twos song, and they know how to double, then they can start to learn the fours song.

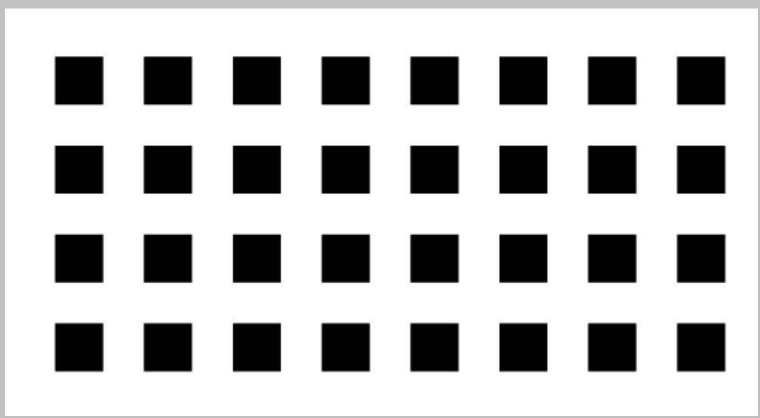
STEP 1:

Picture problems and number talks with 4s.

How many cupcakes? How did you work it out?
How many cupcakes in a full box?

TASK

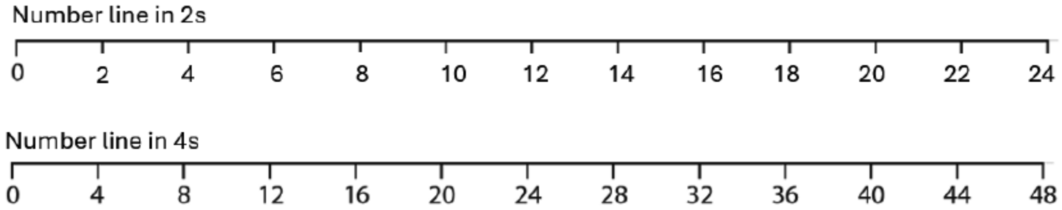
Describe how many squares you see in as may ways as you can, to a partner:



STEP 2:

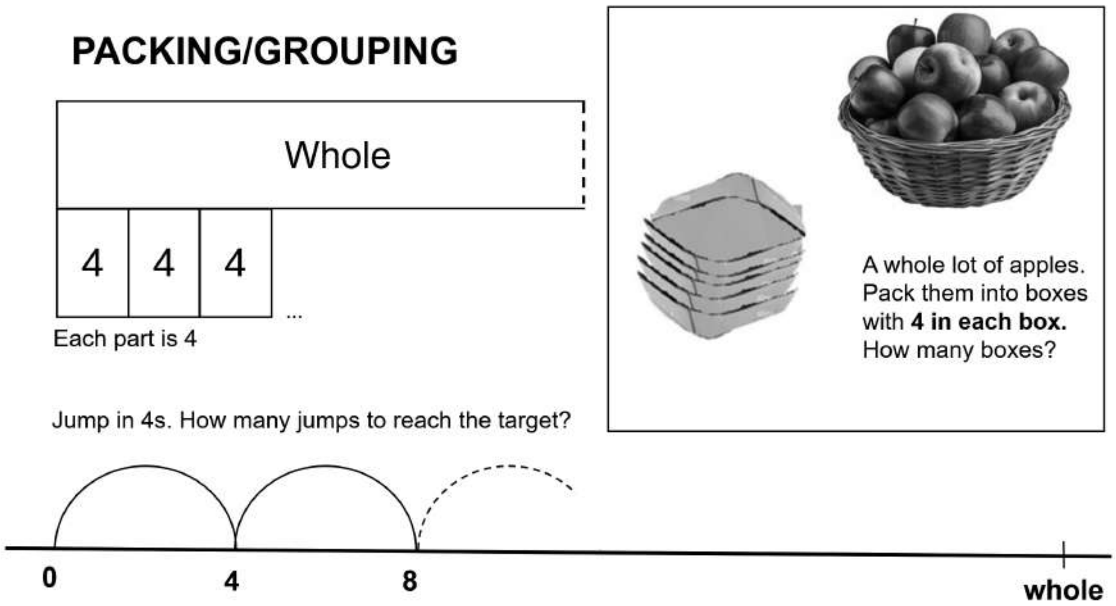
Learn the 4s song. Practice counting in 4s

It is helpful to think about the 4 times table as counting groups of 4. The unit that repeats is 4. This model is helpful for written methods when multiplying and dividing.
The 4s song is double the 2s song. Calculating ×4 is the same as 'double double'



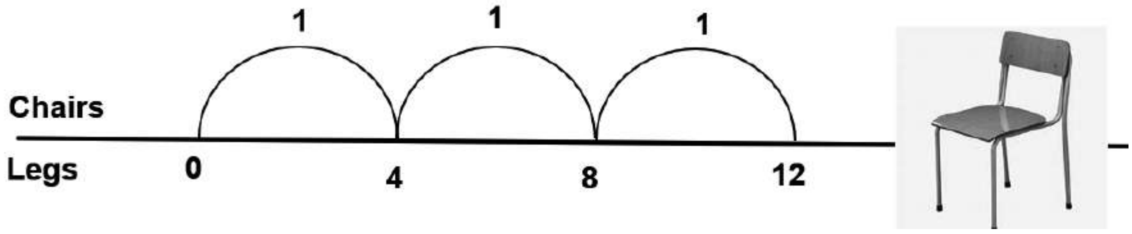
STEP 3:

Use the counting song in 4s to solve problems

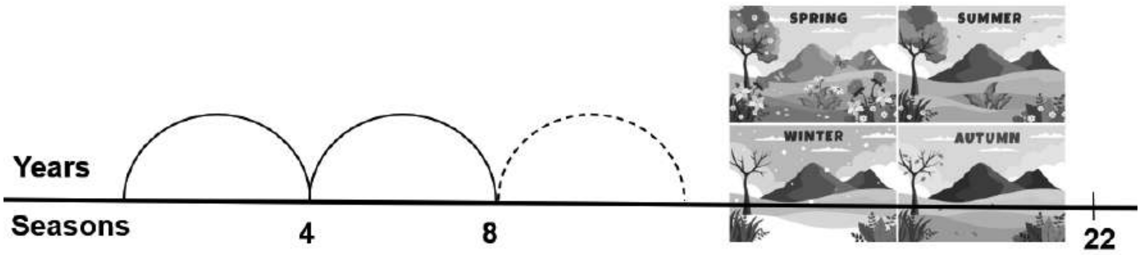


The multiplicative structure is a unit of 4, that is repeated.
Using the 4s counting song, children can work out multiplication and division problems.

- There are 4 chairs, how many legs?



- James has lived through 22 seasons. How old is James in years?



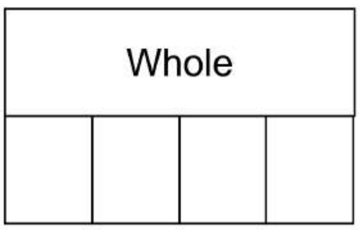
STEP 4:

Model multiplying and dividing by 4

Make sure that you use both meanings for division: Division in grouping/packing contexts, as well as division as sharing equally.

The multiplicative structure is 4 equal parts. When we make 4 equal parts we get 1 quarter.

SHARING



4 equal parts

A whole lot of apples. **Share them equally between 4 people.** How many apples does each person get?

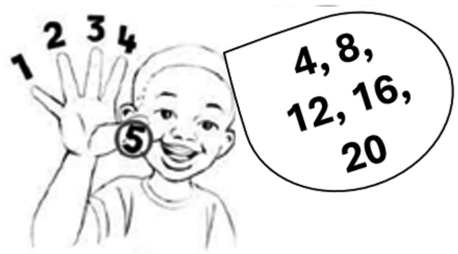
4 equal jumps to the whole. How big is each jump?.

When we divide into 4 equal parts, each part is 1 quarter.

STEP 5:

Keep track of the counts

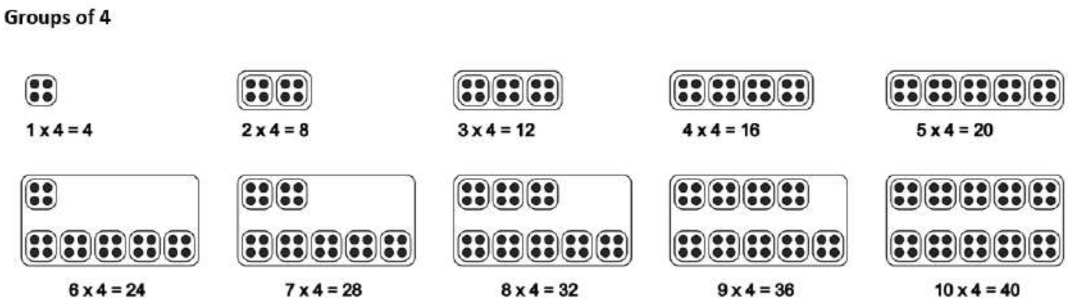
Children can keep track of the counts on their fingers, with T-tables, horizontal tables, number lines or function machines.



At first you give the children the way of recording. Later they should think about which way they want to record.

STEP 6:

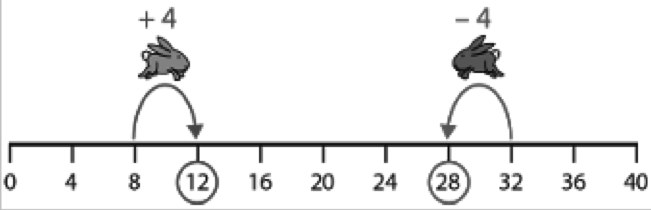
Learn the 4 times table



When keeping track of the counts, it is helpful to emphasise that **adjacent numbers** in the 4s song, have a difference of 4. The numbers **next to** each other are 4 units part.

Example

	× 4		
0	0		
1	4		
2	8		
3		↓ + 4	$3 \times 4 = 2 \times 4 + 4$
4	16		
5	20		
6	24		
7		↑ − 4	$7 \times 4 = 8 \times 4 - 4$
8	32		
9	36		
10	40		
11	44		
12	48		



If a child knows that 2 × 4 = 8.
Then to find the next multiplication fact, they just add 4.
3 × 4 = 2 × 4 + 4.
And the same holds when working backwards.
If you know 8 fours are 32, then 7 fours are 32 – 4 = 28

MULTIPLY 4s!

- Play in pairs, with one pile of cards.
- Mix up cards 1 to 10.
- First learner flips a card.
- The second learner multiplies by **4**
- Do it again! Faster!

In Intermediate Phase, when children are secure with the 4s song and the 4 times table. It is helpful to think about the 40s song.

STEP 7:

Factors of 4 and divisibility rules for 4

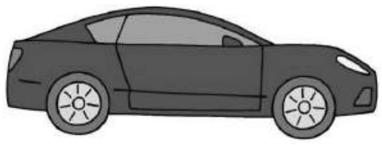
factor **×** *factor = multiple*
factor **×** *factor = product*

First, let's think about just 1 group of 4.
1 group of 4 can be split using multiplication. 1 group of 4 can only be split into two **factor pairs**.

4 = 4 × 1

4 = 2 × 2

Four is not a prime number.



1 car has 4 wheels

TASK

Factors of **4**

Here is a set of 4.
It is unstructured

factor × factor = 4

Draw 4 dots to make arrays.

The factor pairs for 4 are:
___ and ____
___ and ____

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.

1	2	3	4	5	6	7	8	9	10

Colour in 4 blocks to make rectangles

4 can make ___ different rectangles.
Therefore, 4 is <u>prime / composite</u>.
4 can make a square. <u>True/False</u>

TASK

Multiple = factor **×** *factor*

4 = 4 × 1. So 4 and 1 are factors of 4.
4 = 2 × 2. So 2 and 2 are factors of 4.

4 can be written as a product of prime numbers. 4 = ______ × _____ × 1.
2 and 4 are factors of 4.
4 is NOT a prime number.

Is 0 a multiple of 4?
Can you divided 0 by 4, and have no remainder?

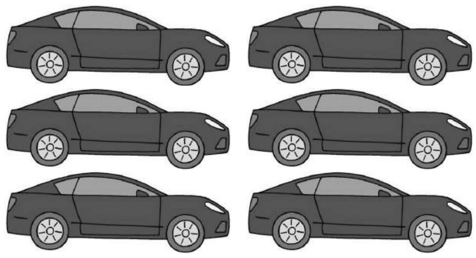
Now, let's think about groups of 4.

These are the multiples of 4.

For example, 6 groups of 4 can be split into factor pairs, and 4 is factor.

6 groups of 4 = 6 × 4 = _____.

So 6 and 4 are factors of 24.



6 cars, with 4 wheels on each hand.

factor **×** *factor = multiple*

TASK

24 can be broken down into different **factor pairs.**

24 = 6 × 4
24 = _____ × ______
24 = _____ × ______
24 = 1 × ______

How do you know, just by looking, whether a number is a multiple of 4?
My number is a multiple of 4, if it can be divided by 4, without leaving a remainder.

- My number CANNOT be a multiple of 4 if it is odd.
- Hundreds and thousands are always divisible by 4.
  - » 100, 200, 300 etc, ... 1000 are all multiples of 4
  - » 1 000, 2 000, 3 000 etc, ... 10 000 are all multiples of 4
- Therefore, using place value, you only need to **look at the tens and ones** to decide if my number is divisible by 4.
- **If the last 2 digits are divisible by 4**, then the whole number is divisible by 4.

TASK

Which of these numbers can be divided by 4, and leave no remainder?

Colour in the multiples of 4

23	7	200	15
203	75	84	27
28	16	24	165
30	3 640	95	101

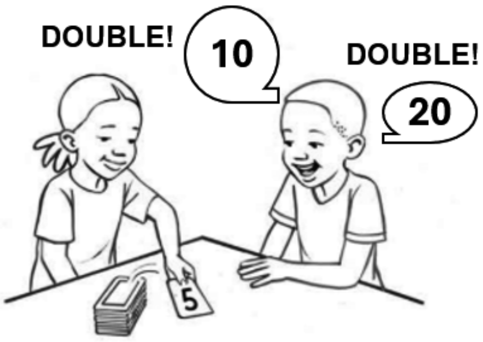
Complete:

I can always see that a number is divisibly by 4 because
____
____

IDEAS FOR YOUR CLASSROOM

Activity for learners

Double double



So 5 × 4 is 20.

Fast Maths with Cards:
DOUBLE THEN DOUBLE

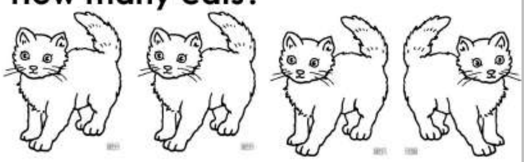
- Play in pairs with tone pile of cards.
- Use cards 0 to 5 and the 10 card.
- Mix up
- Flip over one card. **Times by 4!**
- DOUBLE! Then DOUBLE!
- Do it again! Faster

When you are ready, include the 6 to 9 cards!

Activity for learners

Card 1

1.
How many cats?



2.
How many cats eyes?

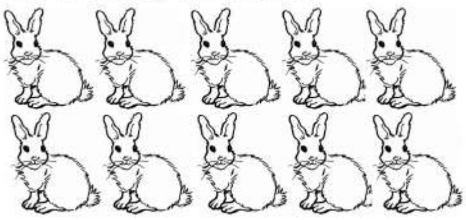
3.

Cats	Eyes
1	
2	
3	
4	8
5	

4.
There are 6 cats.
How many cats eyes are there?

Card 3

1.
How many bunnies?



2.
How many bunnies ears?

3.

Bunnies	Ears
1	
2	4
3	
4	
5	

4.
There are 6 bunnies. How many bunnies ears are there?

Activity for learners

Card 100

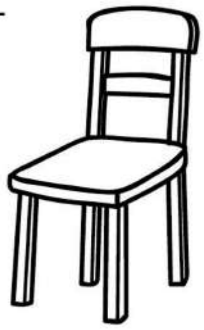
1.
Complete:
4, 8, 12, ____, ____, ____, ____,

2.
How many?

4 4 4 4 4

3

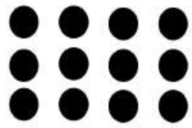
Chairs	Legs
1	4
2	8
4	
8	
10	



4.
There are 12 legs.
How many chairs are there?

Card 103

1.
How many dots?



2.
How many?

4 4 4 4 4

3.

Sweets	Cost
1	R4
2	R8
4	
8	
10	

4.
1 sweet costs R4.
How much will 10 sweets cost?

Activity for learners

How many can you make?

YOU NEED: 3 players; 3 dice; calculator; paper to keep record

Take turns:

- First player rolls the three dice
- Write down all the numbers that you can make from the numbers on the dice that are divisible by 4. You can make numbers from the digits on the dice
  - By adding
  - By using place value. So rolling 6 and 4 could be 10, or 64 or 46.
- Use a calculator to check that the numbers are divisible by 4.

6, 4 and 4

4 is divisible by 4
4+ 4 = **8** which is divisible by 4
4 + 6 = 10. Not divisible by 4

64 is divisible by 4
46 is not divisible by 4
446 is divisible by 4

The winner is the player that has made the most numbers after five rounds.

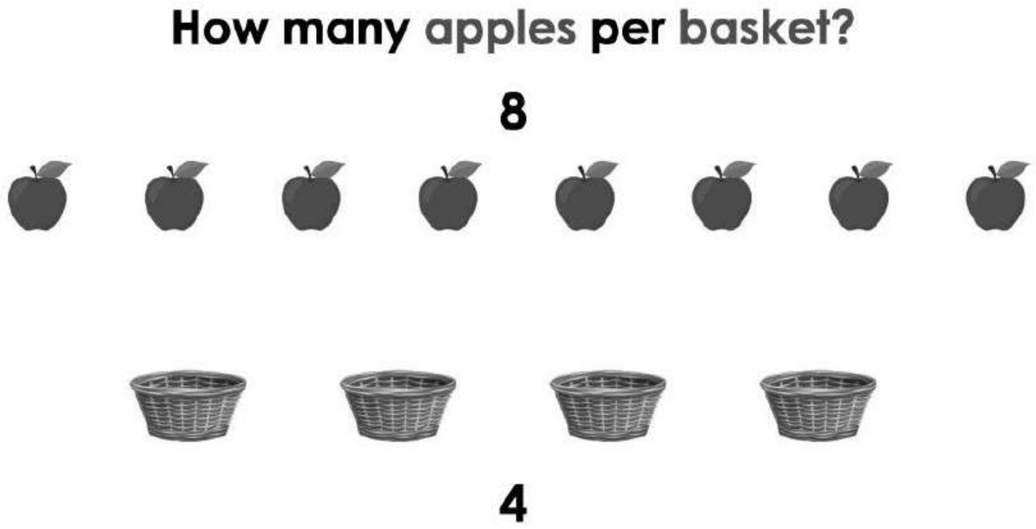
Eights

The eights are closely connected to the 2s and 4s.

STEP 1:

Picture problems and number talks with eights

The eight times table is double the four times table. So number talk that involve 8s and 4s (or 2s) are helpful for children to think about this relationship.



Notice the vocabulary of 'per'. Share the apples equally into the 4 baskets. How many apples in each basket? The apples and baskets example works, but there is nothing about the situation that forces a structure of 8s. There is nothing constraining the apples to always being 8. There is nothing special constraining the baskets to being 4. A better situation would be where there is a structure which is always in 8s

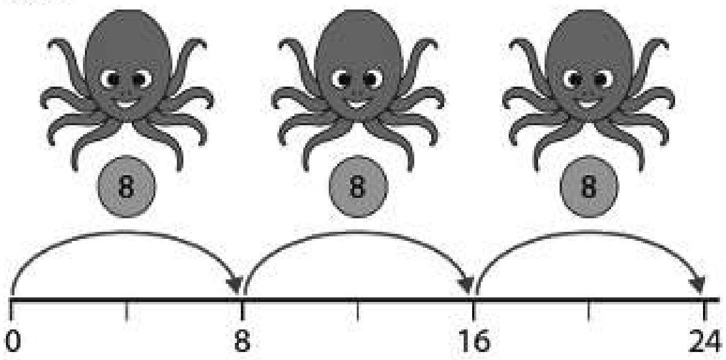
TASK

Write at least 3 different contexts where the structure is always in groups of 8.

This next number talk uses the whole-equal part structure of 1 octopus has 8 legs. The 8 structure (8 legs and 1 octopus. This is always the case). The 8 structure is part of the problem situation, Look at all the vocabulary you can use relating to this situation:

Example

'How many tentacles are there? Count in groups of eight.'



- *'Eight, sixteen, twenty-four. There are twenty-four tentacles.'*
- *'There are three groups of eight; there are twenty-four altogether.'*
- *'There are eight, three times; there are twenty-four altogether.'*

$3 \times 8 = 24$ $\qquad$ $8 \times 3 = 24$

- *'Three is a factor.'*
- *'Eight is a factor.'*
- *'The product of three and eight is twenty-four.'*
- *'Twenty-four is the product of three and eight.'*

The example of the octopus and its tentacles give children the vocabulary to talk about the groups – how many octopuses? And to talk about how many are in each group – each octopus has 8 tentacles.

Here is another number talk. This time we are using the measurement pathway and thinking about dividing by 8.

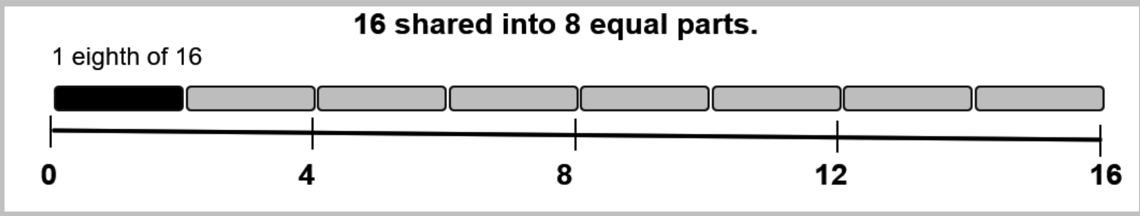
TASK

Finding 1 eighth is the same as dividing by 8.

Find 1 eighth of 16: ____________ $\qquad$ Find 1 eighth of 8: ____________

1 eighth of 24 is ____________ $\qquad$ 1 eighth of 80 is ____________

When we divide into 8 equal parts we get 1 eighth.

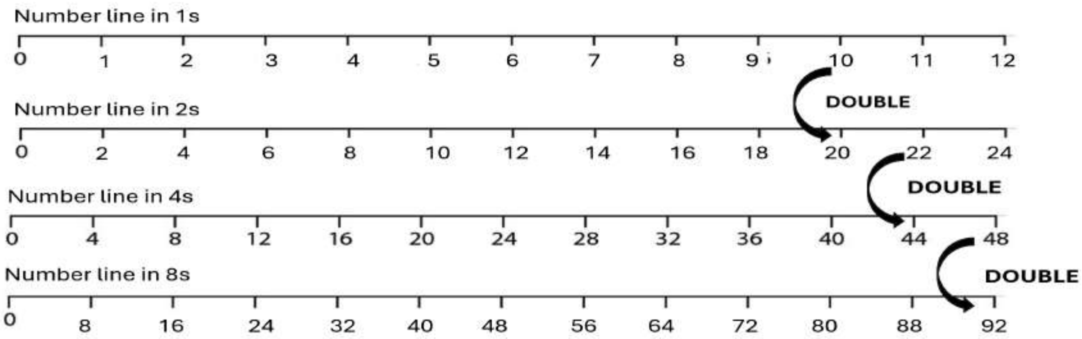


1 eighth of 16 is __________

STEP 2:

Learn the 8s song

The 8s song is connected to the 2s song and to the 4s song.
They all share common factors of 2.



STEP 3:

Using the 8s song to solve problems

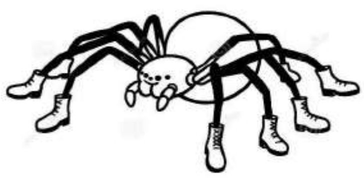
Investigation

Complete the pattern

8	16	24				

A factory makes shoes for spiders. The spider shoes are packed in boxes of 8.

Investigate how many boxes are needed for 1000 spider shoes



STEP 4:

Model multiplying and dividing by 8

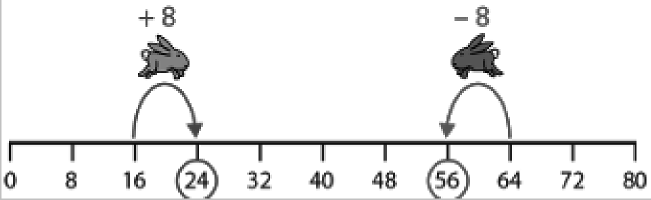
It is important for children to realise that multiples of eight that are next to each other, have a difference of 8. They can use this to work out new facts about the eights times table.
If I know that 2 × 8 = 16, then I can work out that 3 × 8 = 16 + 8,

Example

	×8
0	0
1	8
2	16
3	
4	32
5	40
6	48
7	
8	64
9	72
10	80
11	88
12	96

↓ + 8 $3 \times 8 = 2 \times 8 + 8$

↑ − 8 $7 \times 8 = 8 \times 8 - 8$



IF I know that 8 times 8 is 64,
THEN I can work out that 7 times 8 = 64 – 8.

STEP 5:

Keep track of the counts

You can use fingers, a horizontal table, a double number line, a t-table, or a clue board to keep track of the counts.

Complete:

TASK

Horizontal table

Boxes						
Spider shoes						

Number line

Boxes

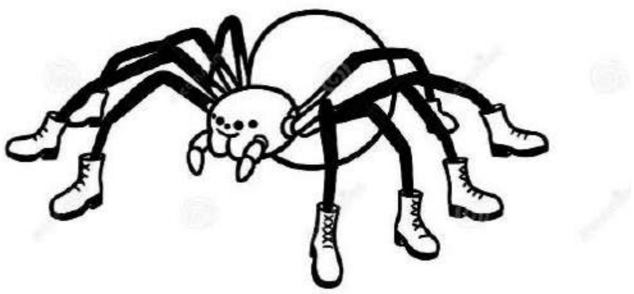
Spider shoes

T-table (1; 2; 3; 4; 5; etc)

Boxes	Spider shoes

Clue board (1; 2; 4; 8; 10)

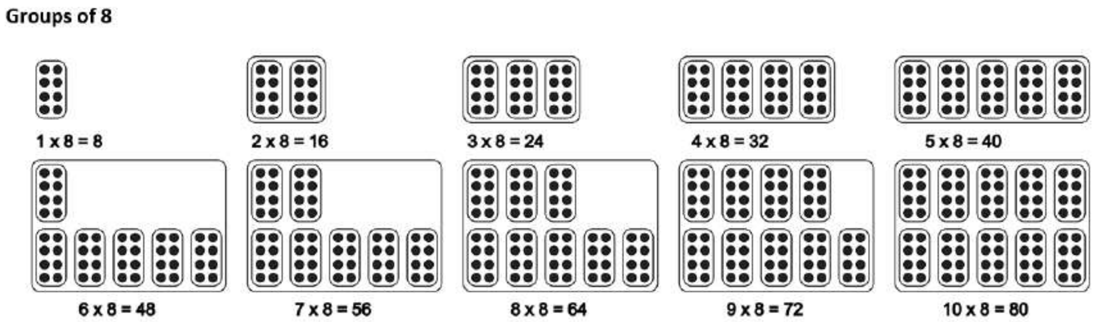
Boxes	Spider shoes



STEP 6:

Learn the 8 times table

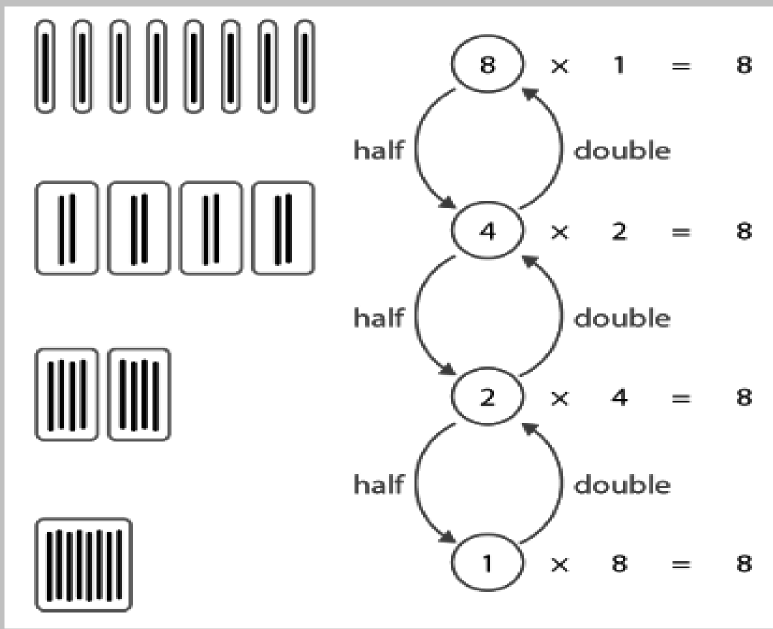
The eight times table can be thought about as repeating a group of eight each time.



The 8 times table connects closely with the 2 times table and with the 4 times table.
Look at the factor pairs of 8.

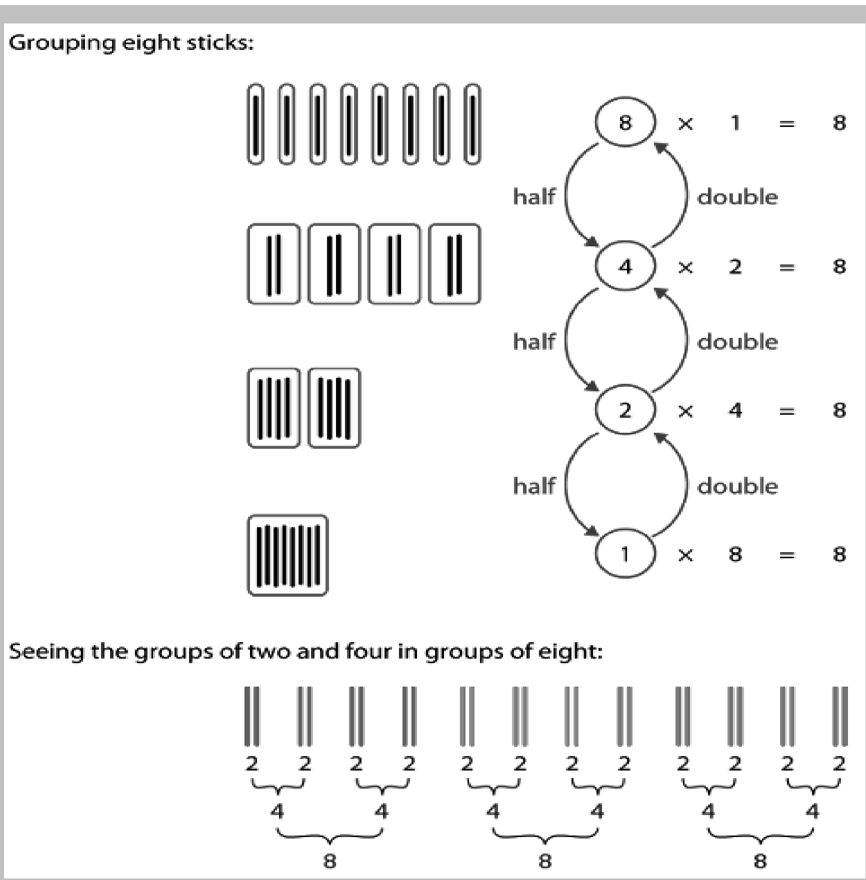
Example

Put on your 2s glasses, Can you see the groups of 2 in the collections of 8 sticks?



Now, put on your 4s glasses. Can you see the groups of 4 in the collections of 8s?
Now put on your 8s glasses. Can you see the groups of 8 in the collections of 8s?

Example



The factor pairs for 8:

8 = 8 × 1

8 groups of 1

8 = 4 × 2

4 groups of 2

8 = 2 × 4

2 groups of 4

8 = 1 × 8

1 group of 8

By the time children are learning the 8 times table, they are start to work on more formal written methods for multiplication and division.

Factors of 8 and divisibility rule for 8

factor × *factor* = *multiple*

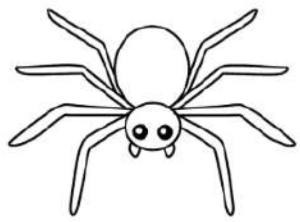
First, let's think about just 1 group of 8.

1 group of 6 can be split using multiplication. 1 group of 8 can only be split into two **factor pairs**.

8 = _____ × 1

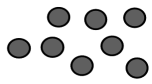
8 = _____ × _____

Four is not a prime number.



1 spider has 8 legs

Factors of **8**



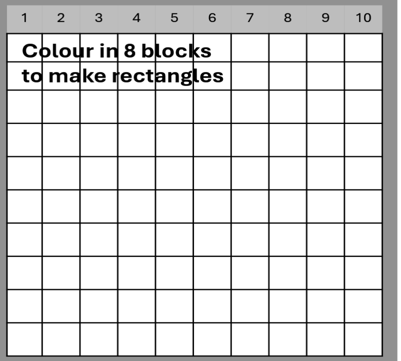
Here is a set of 8.
It is unstructured

factor × factor = 8

Draw 8 dots to make arrays.

The factor pairs for 8 are:
___ and ____
___ and ____

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.



8 can make ___ different rectangles.
Therefore, 8 is <u>prime / composite</u>.
8 can make a square. <u>True/false</u>

Multiple = factor × factor
8 = 8 × 1. So 8 and 1 are factors of 8.
8 = 2 × 4. So 2 and 4 are factors of 8.

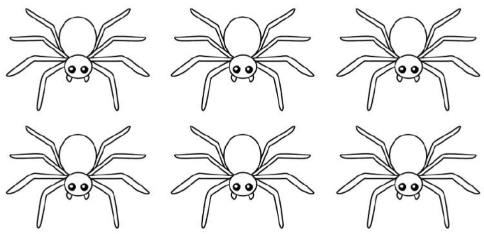
8 can be written as a product of prime numbers. 8 =______ × ______ × _____ × 1.
8 and 1 and 2 and 4 are factors of 8.
8 is NOT a prime number.

TASK

Now, let's think about groups of 8.
These are the multiples of 8.

For example, 6 groups of 8 can be split into factor pairs, and 8 is factor.

6 groups of 8 = 6 × 8 = _____.
So 6 and 8 are factors of 32.



6 spiders, with 8 legs each

TASK

factor × factor = multiple
32 can be broken down into different **factor pairs.**

32 = 32 × ________	32 = 4 × ________
32 = 16 × ________	32 = 2 × ________
32 = 8 × ________	32 = 1 × ________

How do you know, just by looking, whether a number is a multiple of 8?
My number is a multiple of 8, if it can be divided by 8, without leaving a remainder.

- My number CANNOT be a multiple of 8 if it is odd.
- My number CANNOT be a multiple of 8 if it is not divisible by 4.
  - » Therefore, using place value, you only need to **look at the tens and ones** to decide if my number is divisible by 4.
  - » **If the last 2 digits are divisible by 4**, then the whole number is divisible by 4.

- So, my number is divisible by 8, if it is even AND it is divisible by 4, AND if when I halve, halve and halve again I get a whole number.
  - Eg. 632 is even. It is divisibly by 4 as 32 is divisible by 4.
  - Half of 632 is 316. Half of 316 is 158. Half of 158 is 79. Therefore 632 is diviisble by 8.
  - Eg. 732 is even. It is divisibly by 4 as 32 is divisible by 4.
  - Half of 732 is 367. Half of 367 is 333 remainder 1, or 333,5. Therefore 732 is not divisible by 8.

TASK

Which of these numbers can be divided by 8, and leave no remainder?
Colour in the multiples of 8

24	7	200	15
203	75	84	27
28	16	24	165
130	3 640	95	112

Complete:
I can always see that a number is divisibly by 8 because ______________
____
____
____

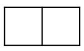
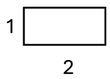
STEP 7:

Factors of 8 and division rules for 8

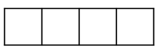
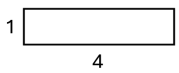
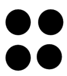
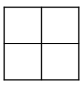
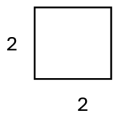
We can break up any number into its factor pairs. This is the same as

- making arrays with dots, or
- making grids with squares or
- making a rectangles to work out its area.

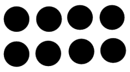
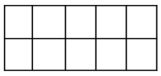
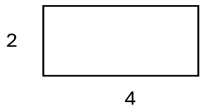
2 has only 1 way that it can be arranged into factor pairs.

Arrays	Grids	Areas of rectangles	Factor pairs Product = factor × factor
			2 = 1 × 2

2 only has 1 pair of factors: 2 and 1. Therefore, 2 is a **prime number**.
4 has more options for factor pairs. 4 is not a prime number. 4 is a **composite** number.
4 is a square number.

Arrays	Grids	Areas of rectangles	Factor pairs Product = factor × factor
			4 = 1 × 4
			4 = 2 × 2

8 also has 2 factor pairs. 8 is not a prime number. 8 is a **composite** number.

Arrays	Grids	Areas of rectangles	Factor pairs Product = factor × factor
			8 = 1 × 8
			8 = 4 × 2

TASK

24 = 3 x 8

Draw a whole part diagram to show 3 repeated 8 times is 24.

Draw a whole part diagram to show 3 groups of 8 is 24.

Draw a 3 x 8 array of dots.

Draw a 3 x 8 grid.

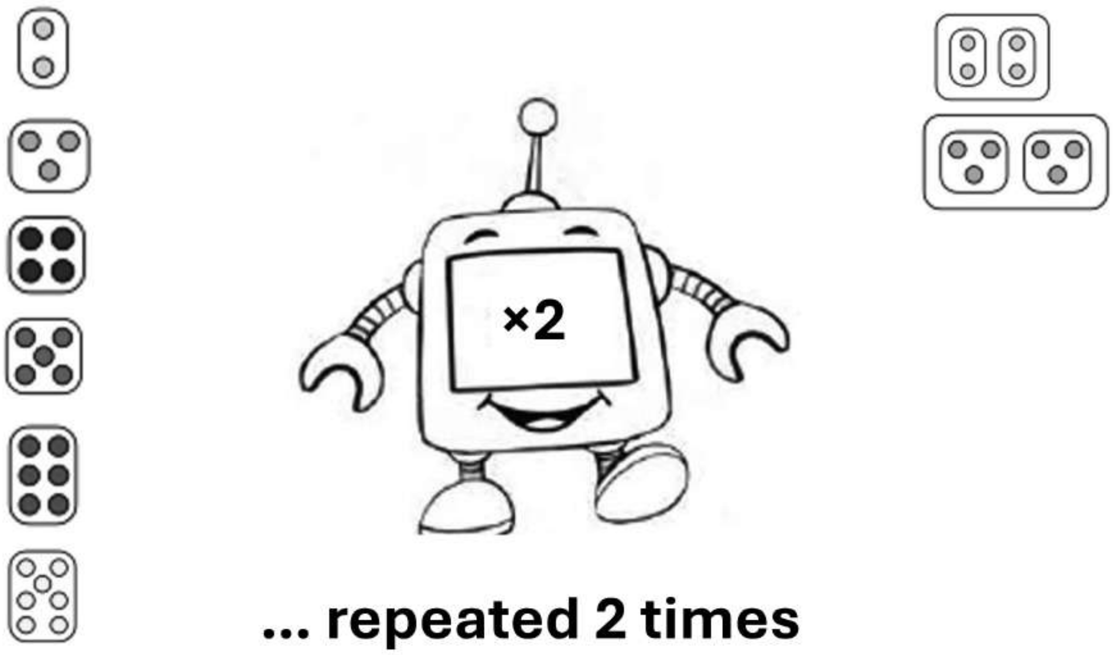
Write the 8 family of number sentence which are equivalent to 3 x 8 = 24

What are the factors pairs of 24?

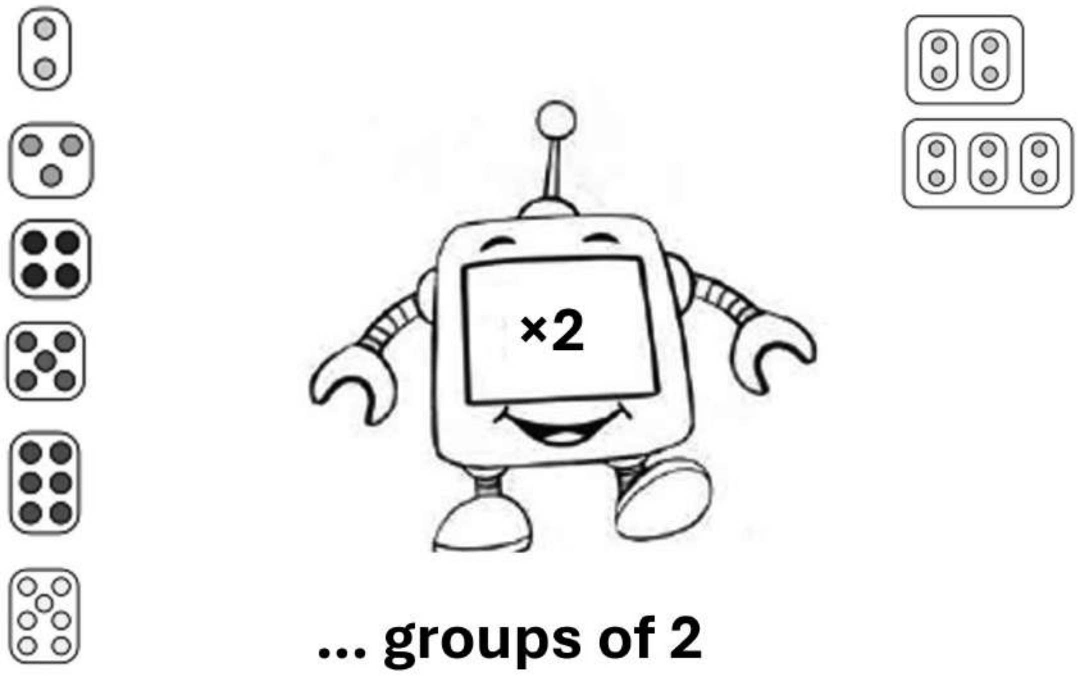
Tasks 2.2

Twos four and eights

1. Complete the drawing of funky, where '_____ × 2' means '_____ **repeated 2 times'.**



2. Complete the drawing of funky, where '____ × 2' means '_____ **groups of 2'.**



3. Sort these numbers according to whether they can be divided by 2 or not.

4	62	2	7	100	5	31	34	48	99	43

Can be divided by 2	Cannot be divided by 2

4. What is the divisibility rule for 2? How do you know if a number can be divided by 2 without a remainder?

5. Use the distributive property to break up a number and double each part.

Double 7

Double 27

Double 20 = ____

Double 7 = _____

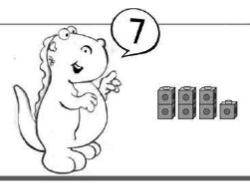
Double 27 = ______

Double 67

67 + 67 =

60 7 ____ _____

6. Practice seeing in 2s. Take a number and break it into towers of 2.



Say aloud:
There are 3 two's in 7, and one left over.

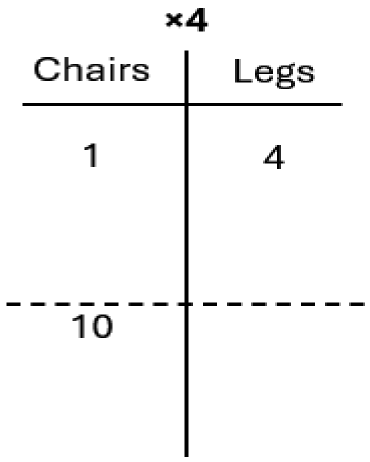
Show the numbers using your towers of 2. Complete the missing numbers.

4	There are 2 2s in 4, and 0 left over.
5	There are ____ 2s in 5, and ____ left over.
12	There are ____ 2s in 12, and ____ left over.
13	There are ____ 2s in 13, and ____ left over.
16	There are ____ 2s in 16, and ____ left over.
9	There are ____ 2s in 9, and ____ left over.
11	There are ____ 2s in 11, and ____ left over.

7. Think of a context for ×4... Chairs and legs

Example

Complete a ×4 clue board



You create a ×4 clue board like this.

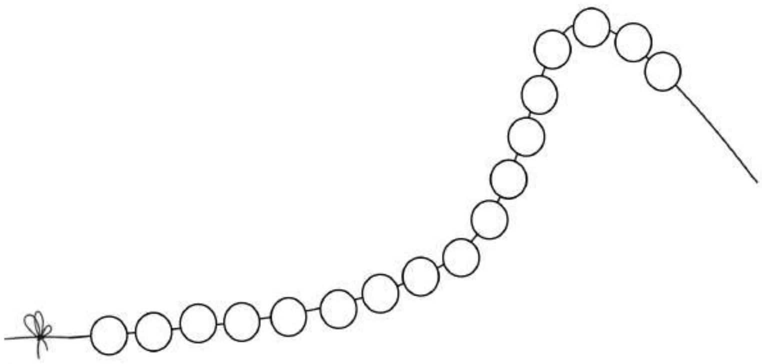
Draw the T for the table.
Write ×4 at the top. It is helpful to think of a context where there are groups of fours. Eg chairs and legs.

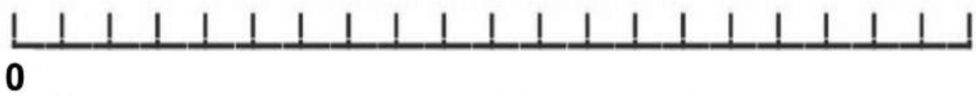
For the left hand side:
“Start with 1. **Double, double double. Stop and think**: 10”

For the right had side:
“Start with 4. **Double, double double.**

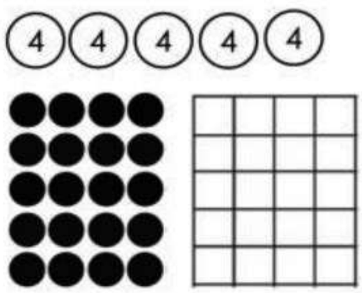
Stop and think:
4 × 10”
This is the 4 times table.

8. Colour in the necklace to show groups of 4. Make sure the 4th bead clearly shows the group of 4. Write the multiples of 4 along the number line.





9.

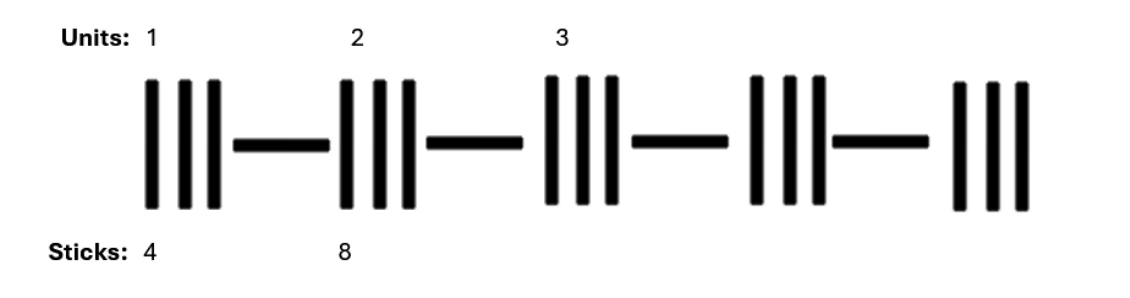


Complete the family of 8 related number sentences

4 × 5 = 20

Investigation

10. Investigate this pattern of sticks.



a. Draw a circle around the unit that repeats.

b. **Specialise.** Create a t-table to show the relationship between units and sticks. Use at least 5 cases.

c. **Generalise:** What is the relationship between units and sticks? If I know the units, how do I find out how many sticks?

d. **Generalise**: What is the relationship between sticks and units? If I know the number of sticks, how do I find out how many units?

11.

'Fill in the missing numbers.'

$20 \times 4 = 80$ so $21 \times 4 = \boxed{}$

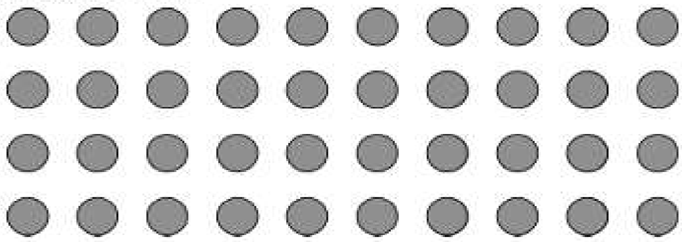
$4 \times 18 = 72$ so $4 \times 19 = \boxed{}$

$15 \times 4 = 60$ so $14 \times 4 = \boxed{}$

$4 \times 17 = 68$ so $4 \times 16 = \boxed{}$

12.

Array problem:



- *'How many groups of four are there?'*

$\boxed{} \times 4 = \boxed{}$

- *'How many groups of two are there?'*

$\boxed{} \times 2 = \boxed{}$

- *'How many groups of four are there?'*

$\boxed{} \times 4 = \boxed{}$

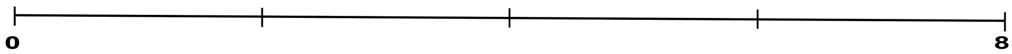
- *'How many groups of two are there?'*

$\boxed{} \times 2 = \boxed{}$

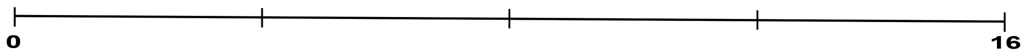
13. **Finding 1 quarter, is the same as dividing by 4**.

Draw a small circle or dot to show where 1 quarter is for each number line:

a. 1 quarter of 8 is ________



b. 1 quarter of 16 is __________



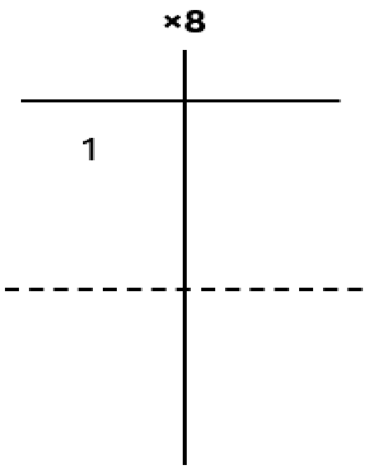
Tasks 2.2.1

Eights, clue boards and and the distributive property

1. Think of a context for ×8… ________________ and ________________

Example

Complete a ×8 clue board



You create a ×8 clue board like this.

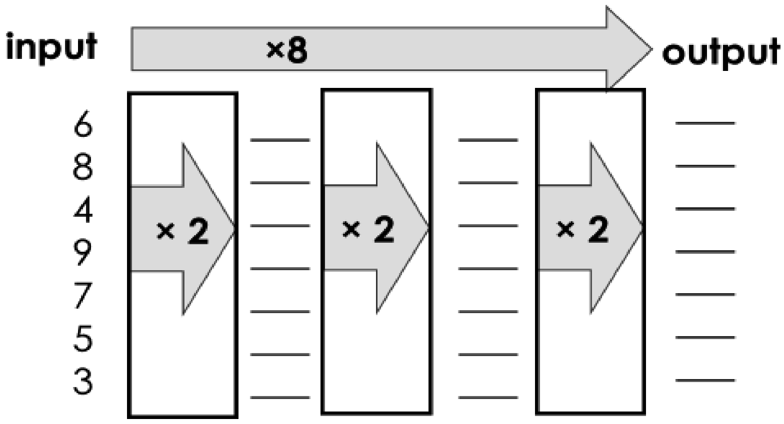
Draw the T for the table. Write ×8 at the top. It is helpful to think of a context where there are groups of eights. Eg chairs and legs.

For the left hand side:
"Start with 1. **Double, double double. Stop and think**: 10"

For the right had side:
"Start with 8. **Double, double double.**

Stop and think:
8 × 10"
This is the 8 times table.

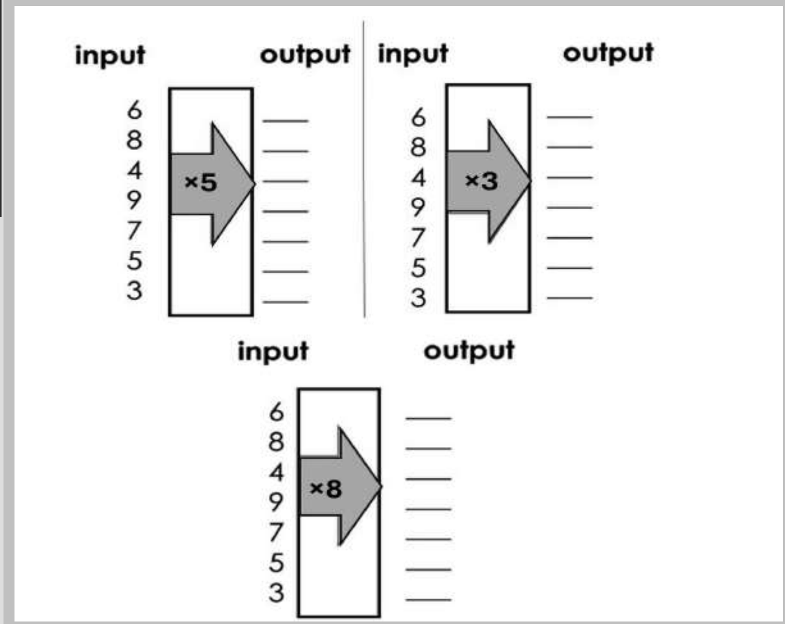
2. 'Times 8' is the same as 'times 2 times 2 times 2'.



'my number' doubled, and doubled again, and doubled again = 'my number' × 8

Investigation

3. **Explore the 8 times table, using the fact that 8 = 5 + 3**
   a. Complete the function machines.



Look for a pattern...

Choose you own input number. Call it "my number"..

my number × 5 + my number × 3

= my number × 8

Is this statement true:

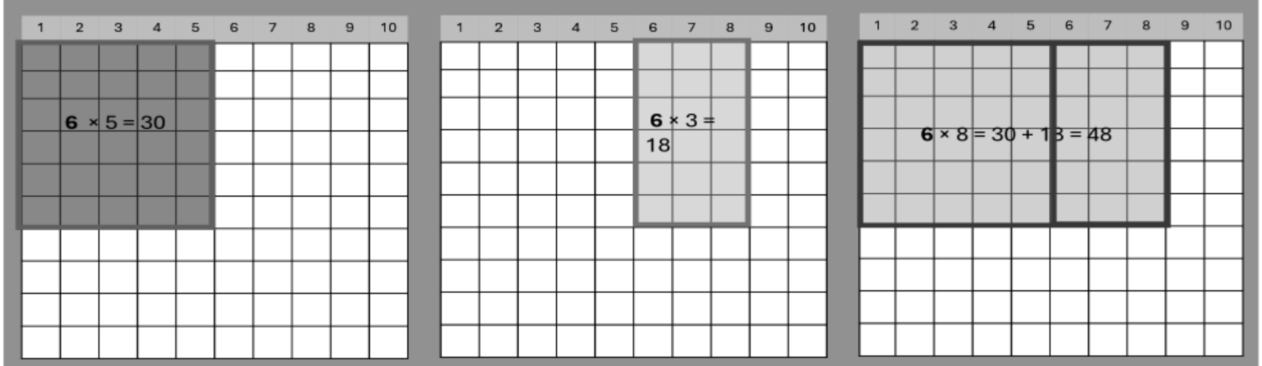
Sometimes or

Always or

Never?

Why does this happen?
Say, for example, that I choose **6** as my input number.

My number is **6.**



Look at **6** x 5 = ___

Look at **6** x 3 = ___

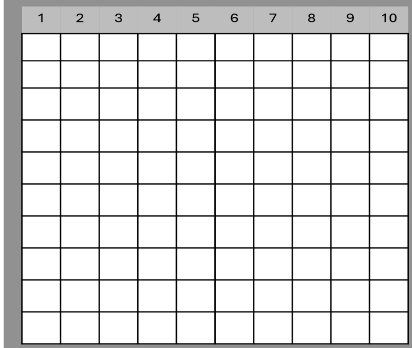
Put the two rectangles together, to get **6** x 3 = ___

   b. My number is 6. Then

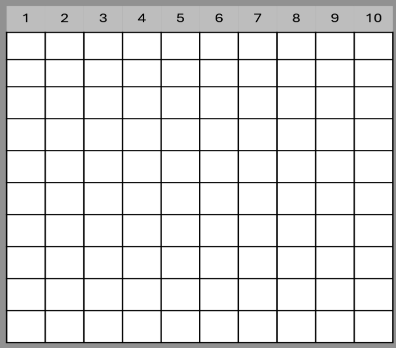
      6 × 5 = ______ and **6 × 3 =** ______. Put the two rectangles together. Then
      6 × 8 = 30 + 18 = ______.
      6 × 8 = **6** × (5 + 3) = **6** × 5 + **6** × 3

c.

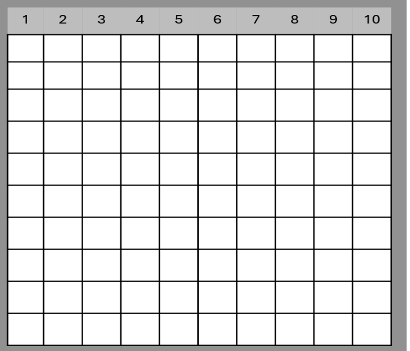
My number is: **4**



Draw **4** x 5 = ___



Draw **4** x 3 = ___



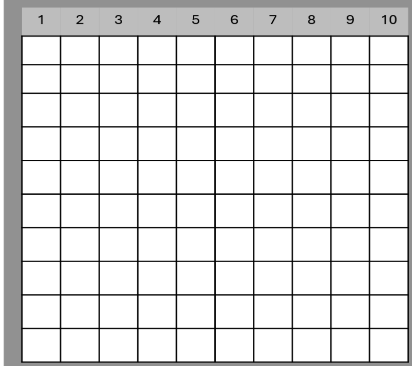
Put the rectangles together.
To get **4** x 8 = ___

My number is 4. Then

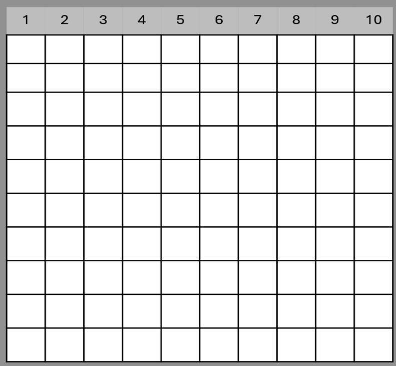
4 × 5 = _____ and **4 × 3 = _____. Put the two rectangles together. Then**
4 × 8 = ______ + ______ = 32.
4 × 8 = **4** × (5 + 3) = **4** × _____ + **4** × _______.

d. **What if, your number is 9?**

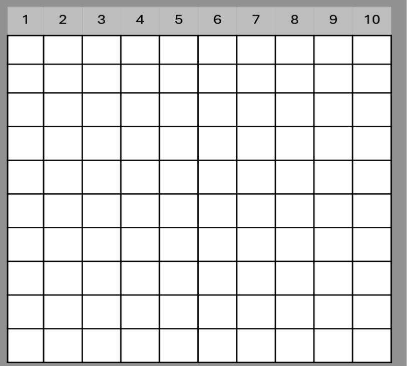
My number is: **9**



Draw **9** x 5 = ___



Draw **9** x 3 = ___



Put the rectangles together.
To get **9** x 8 = ___

9 × 5 = _____ and **9 × 3 = _____. Put the two rectangles together.**

Then

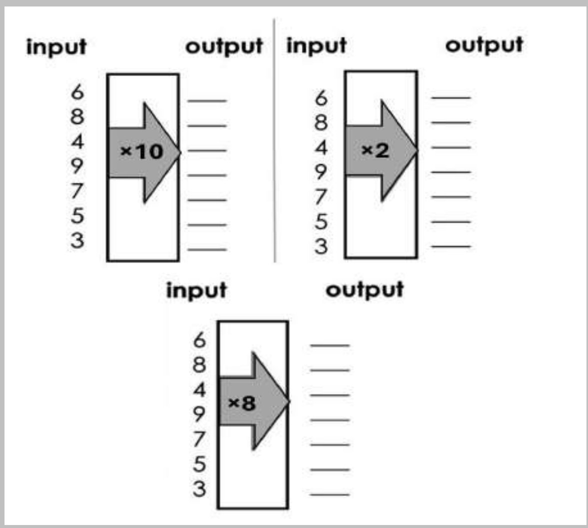
9 × 8 = ______ + ______ = ____.
9 × 8 = **9** × (5 + 3) = **9** × _____ + **9** × _______.

My number × 8 = my number × 5 + my number × 3
I can break up 8 into 5 and 3 and multiply each part by my number.

This is the distributive property.

Investigation

4. Explore the 8 times table, using the fact that 8 = 10 - 2.
   a. **Complete these function machines for × 10, × 2 and × 8**



Look for a pattern…

a. Choose you own inputnumber.
“my number” is ________.

What is the relationship between:

- my number × 10
- my number × 2; and
- my number × 8?

b. Write a statement that is always true
my number × 10

____

____

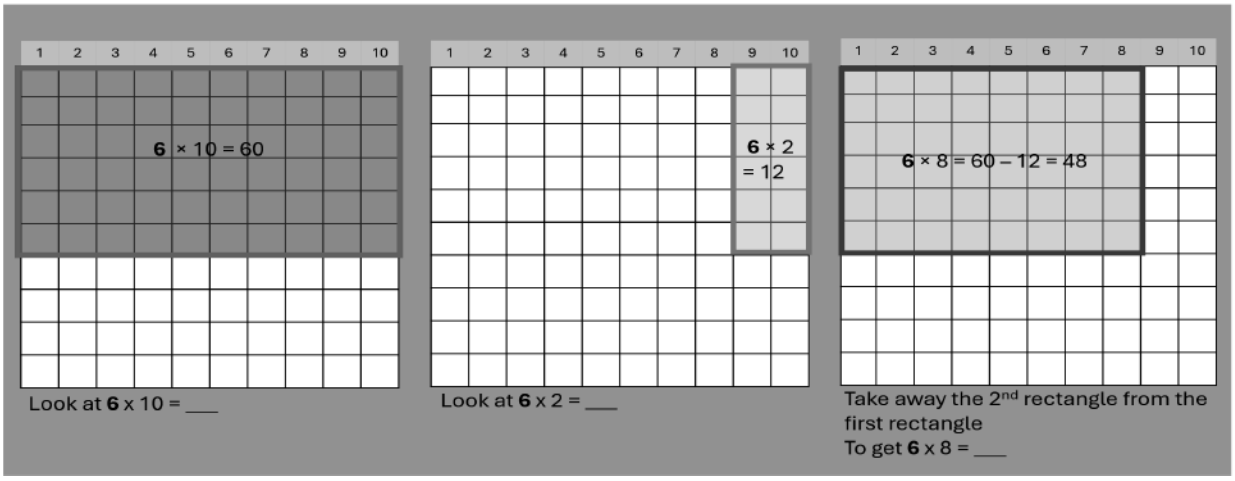
____

____

Show why this happens for different **input numbers**.

   b. What if your number is 6?

My number is: **6**



My number is 6.

6 × 10 = _____ and **6** × 2 = ________.
Therefore,
6 × 8 = _______ - _________ = 48.
This is the distributive property. To multiply by 8 i can break 8 into 10 – 2.
6 × 8 = 6 × **(10 - 2)** = _______ - _________ = 48.

c. What if, your number is 9?

My number is: **9**

1	2	3	4	5	6	7	8	9	10

Draw **9** x 10 = ___

1	2	3	4	5	6	7	8	9	10

Draw **9** x 2 = ___

1	2	3	4	5	6	7	8	9	10

Take away the 2nd rectangle from the 1st rectangle
To get **9** x 8 = ___

9 × 10 = ____ and **9 ×** 2 = _______.

Therefore,

This is the distributive property. To multiply by 8 I can break 8 into 10 – 2.

9 **×** 8 = 9 **× (10 - 2)** = ______ - ________ = _____.

5. **18. Fill in the missing symbols and missing numbers.**

a.

'Fill in the missing symbols (<, > or =).'

$4 \times 8 \bigcirc 5 \times 8 - 5$

$4 \times 8 \bigcirc 5 \times 8 - 8$

$4 \times 8 \bigcirc 3 \times 8 + 3$

$4 \times 8 \bigcirc 3 \times 8 + 8$

b.

'Fill in the missing numbers.'

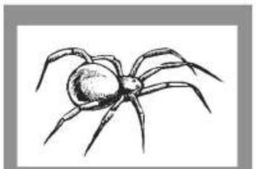
$20 \times 8 = 160$
so
$21 \times 8 = \boxed{}$

$8 \times 18 = 144$
so
$8 \times 19 = \boxed{}$

$15 \times 8 = 120$
so
$18 \times 8 = \boxed{}$

$8 \times 17 = 136$
so
$8 \times 16 = \boxed{}$

6. **Children calculate how many legs are on 6 spiders. Correct their work.**



How many legs are on 6 spiders

Mandy:

$6 \times 2 = 12$
double 12 = 10 + 10 + 2 + 2 = 20 + 4
$6 \times 4 = 24$
double 24 = 20 + 20 + 4 + 4 = 48
$6 \times 8 = 48$

Elliot:

8 16 24 32 40 48 56

$6 \times 8 = 56$

Ben:

'I know that six eights are forty-eight'

$6 \times 8 = 48$

Akisha:

'Five times eight is forty, so six times eight is forty plus eight.'

$6 \times 8 = 5 \times 8 + 8$
$= 40 + 8$
$6 \times 8 = 48$

Tasha:

$6 \times 4 = 24$
double 24 = double 20 + double 4
= 40 + 4
= 44
so $6 \times 8 = 44$

Tasks 2.2.1

Timed tango ×4 and ×8

1	10 × 4 = ______	**11**	______ × 4 = 32
2	4 × 4 = ______	**12**	______ = 28 ÷ 4
3	4 × ______ = 48	**13**	4 × 8 = ______
4	______ = 7 × 4	**24**	______ = 24 ÷ 4
5	4 × 2 = ______	**15**	______ = 4 × 5
6	______ = 4 × 5	**16**	1 quarter of 40 = ______
7	12 × 4 = ______	**17**	4 × 3 = ______
8	______ × 4 = 16	**18**	______ ÷ 4 = 10
9	Factorise: 28 = ______ × 4	**19**	Factorise: 20 = 4 × ______
10	Which is a multiple of 4? 2; 8; 15; 22; 34	**20**	Which is a factor of 4? 2; 8; 15; 22; 34

TASK

Create a clueboard for legs and chairs

I know a number is divisible by 4 when…

The factor pairs of 4 are ….

Therefore 4 is prime/composite.

8 written as a product of prime numbers is

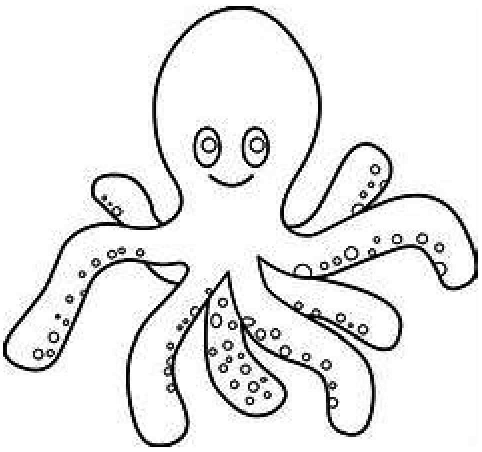
______________________________________.

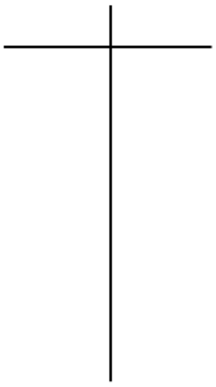
Timed tango ×8

1	6 × 8 = ______	11	______ = 7 × 8
2	______ × 8 = 40	12	8 × ______ = 80
3	8 × ______ = 80	13	______ = 96 ÷ 8
4	8 × 3 = ______	24	64 ÷ 8 = ______
5	10 × 8 = ______	15	______ ÷ 8 = 7
6	______ = 7 × 8	16	______ = 48 ÷ 8
7	8 × 8 = ______	17	11 × 8 = ______
8	______ eighth of 56 = 7	18	1 eighth of 16 = ______
9	Factorise: 40 = ______ × 8	19	Factorise: 32 = ____ × 8
10	Which is a multiple of 8? 2; 8; 32; 42; 84	20	Circle all the factors of 8: 1; 2; 4; 8; 32; 42; 84

TASK

Create a clueboard for octopuses and tentacles





I know a number is divisible by 8 when…

The factor pairs of 8 are ….

Therefore 8 is prime/composite.

8 written as a product of prime numbers is

______________________________.

______________________________.

______________________________.

SUMMARY

SUMMARY 2.2

1. Draw grids to show all the factor pairs for 2.

How do you know if a number is divisible by 2?

6 divided by 2 is the same as: Find 1 ________ of 6.

2. Draw grids to show all the factor pairs for 4.

Write 4 as a product of prime factors.

How do you know if a number is divisible by 4?

16 divided by 4 is the same as: Find 1 ________ of 16.

3. Draw grids to show all the factor pairs for 8.

Write 8 as a product of prime factors.

How do you know if a number is divisible by 8?

4. 16 divided by 8 is the same as:
   Find 1 ________ of 16.

UMTHAMO 2.3

THREES, SIXES AND NINES

In this umthamo activities for using different structures and interpretations of multiplication and division for the three, six and nine times table. It is important for you – and your learners – to notice the connections between these times tables.

TASK

10										
9	9			36	45				81	
8			24			48			72	
7										
6	6	12		24		36	42		54	
5						30				
4										
3	3	6	9	12		18	21	24	27	
2										
1										
×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

The times tables for tables for 3, 6 and 9 go together. Complete the 3, 6 and 9 times tables in the multiplication chart.

Key ideas

1. The **3, 6 and 9-times tables are closely related** to each other.
   - 6 is double 3, and 9 is triple 3.
   - Multiples of 6, are double the multiples of 3
   - Multiples of 9, are triple the multiples of 3
2. **Products in the 3, 6 and 9 times tables share the same factors**.
   - The factor pairs for 3 are 1 and 3. 3 is a prime number.
   - The factor pairs for 6 are 1 and 6, and 2 and 3. 6 is a composite number.
   - The factor pairs for 9 are 1 and 9, and 3 and 3. 9 is a composite number. 9 is a square number.
3. You can **factorise** a number to simplify the multiplication. Factorise means split into smaller factors.

For example:

- 7 × 6 = 7 × 2 × 3. Multiply by 6 is the same as double, then triple
- 7 × 9 = 7 × 3 × 3. Multiply by 9 is the same as triple, then triple

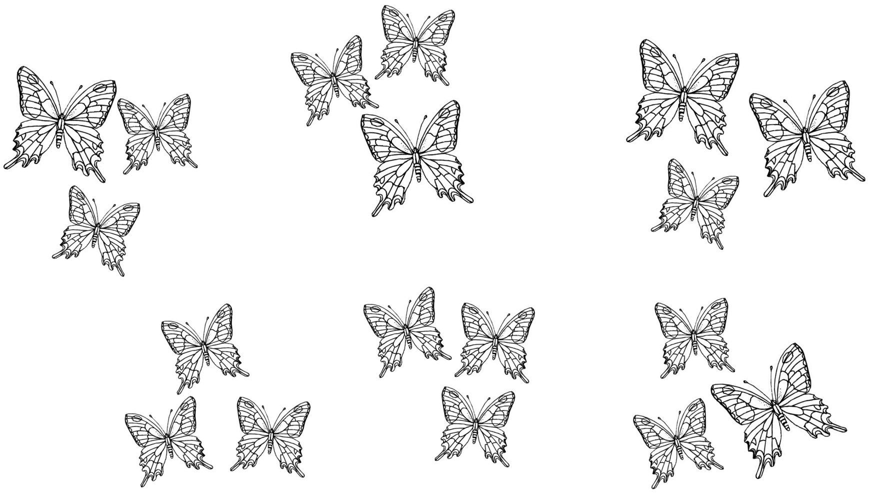
4. Skip or **group counting in 3s**, is **counting the multiples of 3**.
   - **Adjacent multiples in the 3s song**, have a difference of 3.
     - The threes song is: 3, 6, 9, 12; 15 ... 12 and 15 are **next to** each other. So the difference between 12 and 15 is 3.
   - **Adjacent multiples in the 6s song,** have a difference of 6.
     - The sixes song is: 6, 12, 18, 24,..., 18 and 24 are **next to** each other. So the difference between 18 and 24 is 6.
   - **Adjacent multiples in the 9s song,** have a difference of 9.
     - The nines song is: 9, 18, 27 ... 18 and 27 are **next to** each other. So the difference between 18 and 27 is 9.
5. **Divisibility rules:**
   - Products in the 3, 6 and 9-times table are always divisibly by 3.
     - The digit sum of the multiples of 3 is always either 3, 6 or 9.
     - Multiples of 6 are always even. The digit sum of the multiples of 6 is always either 3, 6 or 9.
     - The digit sum of the multiples of 9 is always 9.
6. You can multiply 3, 6 or 9 by 0. You always get 0. You cant divided 3, 6 or 9 by 0. It is undefined.

Threes

STEP 1:

Picture problems and number talks with threes

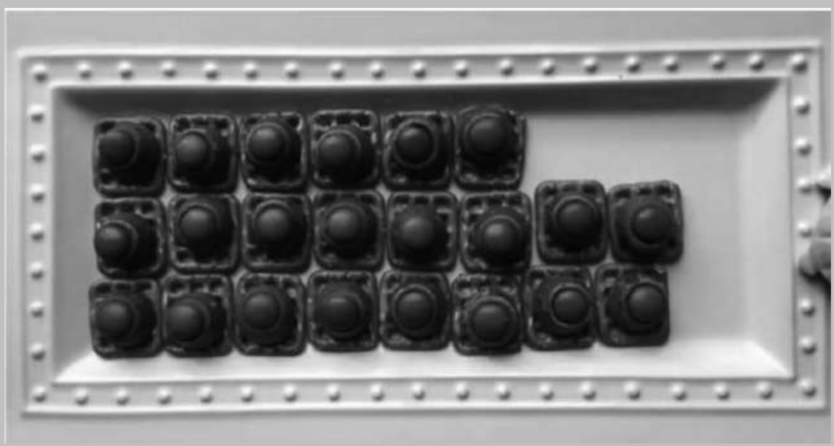
How many butterflies?



These butterflies are arranged in groups of 3. But they could be arranged in 4s, or just randomly. It is important to include picture problems and number talks where there is a built in structure of groups of 3s, like this example:

TASK

Sweet treats
How many, without counting...? _______________



Write an equation (number sentence) to show how many there are.

____

How many will fit to complete the plate? _________________
How many are missing? _________________

STEP 2:

Learning the threes song

Children need to know the 3s song…
This has a rhythm of
da,da, **dum**, da, da, **dum**, da, da
dum,…
clap clap, **stamp**, clap clap, **stamp**,
clap clap, **stamp**,…
one, two, **three,** four, five, **six,** seven
eight **nine**

Scan here for: We will, we will rock you,
by Queen
https://www.youtube.com/watch?v=-tJYN-eG1zk

Activity for learners

Complete the number pattern

3	6	9							

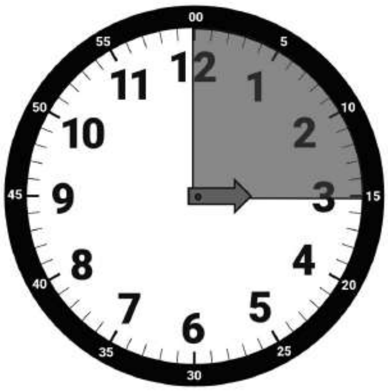
Complete the shape pattern

Time in hours uses the threes song (multiples of 3).
Time in quarter hours (15 minutes) also uses the threes song.

An analogue clock has 2 number lines in a circle.

- One number line is for **hours**. It starts at 0 and ends at 12.
- The other number line is for **minutes**. It starts at 0 and ends at 60. Many clocks do not show this second number line. You have to imagine it.

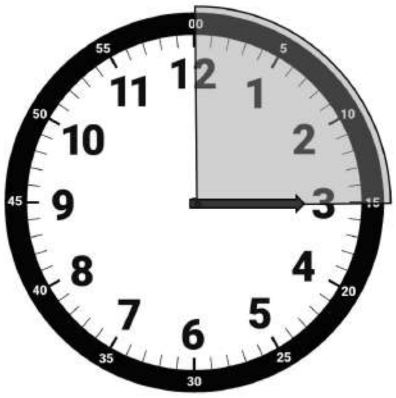
Hours



Half of 1 day has 12 hours
Read the big numbers
The hour hand is short and wide.

1 quarter of 12 hours = 3 hours.

Minutes



1 hour has 60 minutes
Read the small numbers
The minute hand is long and narrow.

1 quarter of 60 minutes = 15 minutes

The two number lines can work on the same cricle as hours and minutes have the same whole-equal part structure.

Half of a day has 12 hours

12 hours			
3	3	3	3

4 equal parts
3 hours repeated 4 times = 12 hours
12 hours shared into 4 equal parts = 3 hours
1 quarter of 12 is 3.

One hour has 60 minutes

60 minutes			
15	15	15	15

4 equal parts
15 minutes repeated 4 times = 60 minutes
60 minutes shared into 4 equal parts = 15 minutes
1 quarter of 60 is 15.

We use the 3s song when making and reading a clock.

Activity for learners

For parts of each hour we look at the minute hand. We first look for the 3, 6, 9 and 12.

- 3 parts out of 12 is 1 quarter. We say "quarter past"
- 6 parts out of 12 is 2 quarters. We say "half past"
- 9 parts out of ______ is 3 quarters. We say "quarter to" (1 quarter until the whole hour)
- 12 parts out of ______ is 4 quarters or a full hour. We say "o'clock"

For minutes we look at the minute hand. We first look for the 3, 6, 9 and 12.

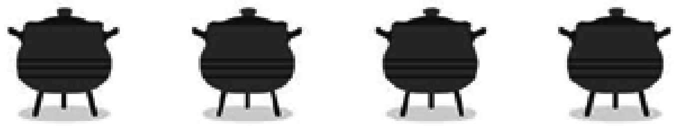
- 3 parts out of 12 is the same as 15 parts out of 60. We can say "15 minutes past"
- 6 parts out of 12 is the same as ______ parts our of 60. We can say "30 minutes past"
- 9 parts out of 12 is the same as ______ out of 60. We can say "45 minutes past"
- 12 parts out of 12 is the same as 60 out of ______ or a full hour. We say "o'clock"

STEP 3:

Use the threes song to solve problems

When thinking about contexts for exploring the 3 times table, a situation that has a structure of threes is helpful.

Activity for learners

	How many pots?	
	How many legs?	

9 pots, how many legs?		9 legs, how many pots?	
5 pots, how many legs?		15 legs, how many pots?	

3 x 4 = ___	12 ÷ 3 = ___	3 x 6 = ___	18 ÷ 3 = ___

TASK

Some other contexts for problems about threes:

1 stool has 3 legs

2 stools have _____ legs

3 _______ have ______ legs

1 box has 3 toothpaste tubes

2 boxes have _____ toothpaste tubes

______ boxes have 9 toothpaste tubes

Activity for learners

STEP 4:

Model multiplying and dividing by 3

You first **specialise** – by giving a few concrete examples. In this example there are 3 pots.

How many pots? __________

How many legs? __________

TASK

Later you **generalise**

Draw a T-table for the number of pots and number of legs.

×3

15 pots, how many legs? ____________

15 legs, how many pots? ___________

150 pots, how many legs?
____

150 legs, how many pots? ___________

What is the relationship between the number of legs, and number of pots?

Legs = ______________________

What is the relationship between the number of pots, and number of legs?

Pots = ______________________

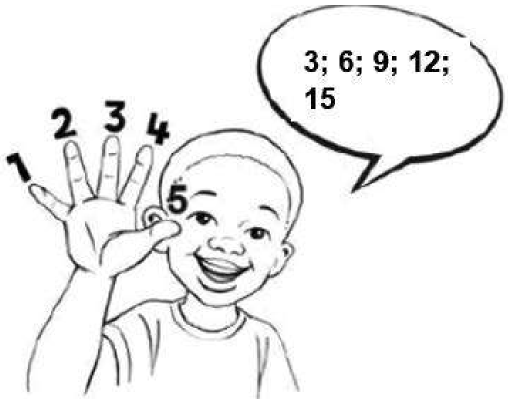
If there are 0 pots, how many legs?

If there are 0 legs, how many pots?

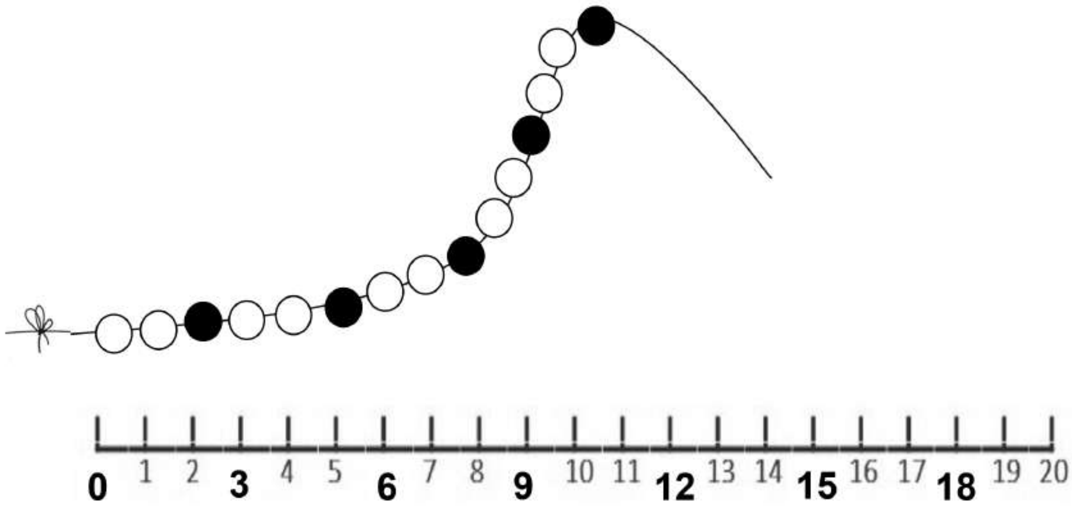
STEP 5:

Keep track of the counts

When working with 3s you keep track of the counts.



You can use your fingers. You can use a number line.



TASK

What are all the ways you can use to keep track of the counts?

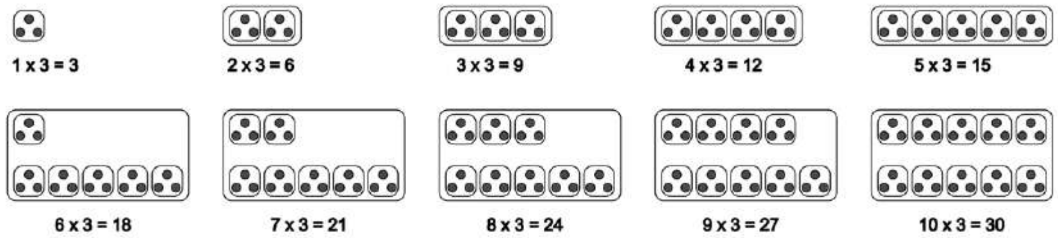
STEP 6:

Learn the 3 times table.

Try this song for learning the 3 times table:

Dice and dominoes are a familiar pattern of threes.

Groups of 3



Factor pairs and divisibility rules for 3

TASK

How many different arrays or rectangles can you make from 3 counters?

Factors of **3**

factor × factor = 3

Draw 3 dots to make arrays.

The factor pairs for 3 are:
___ and ____
___ and ____

1 2 3 4 5 6 7 8 9 10

Colour in 3 blocks to make rectangles

Here is a set of 3.
It is unstructured

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.

3 can make ___ different rectangles.
Therefore, 3 is prime / composite.
3 can make a square. True/False

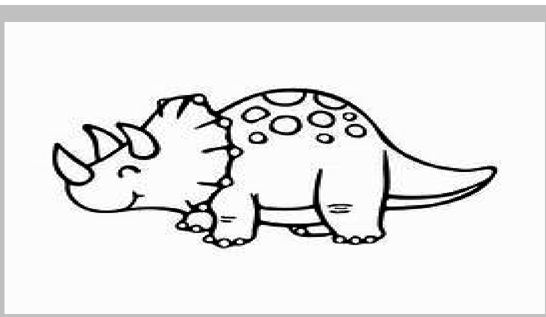
The only factor pair for 3 is 3 and 1
3 = 3 × 1 and 3 = 1 × 3

Therefore, 3 is a **prime number**.

IDEAS FOR YOUR CLASSROOM WITH 3'S

Activity for learners

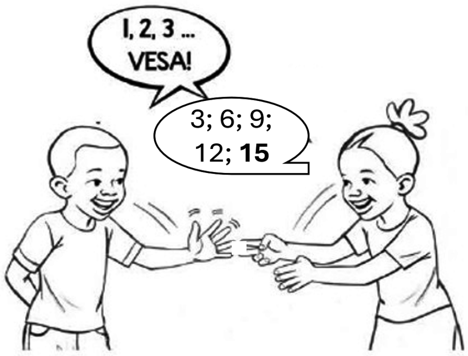
Useful contexts for 3s:
Any groups of 3 each time;
Each potjie pot has 3 legs;
3 legged stools;
Each triangle has 3 sides;
Each triangle has 3 angles;
Each tricycle has 3 wheels,
Each triceratops has 3 horns.



A triceratops dinosaur has 3 horns.
Baby bop from Barney is a triceratops.

Learn to count in 3s

Activity for learners

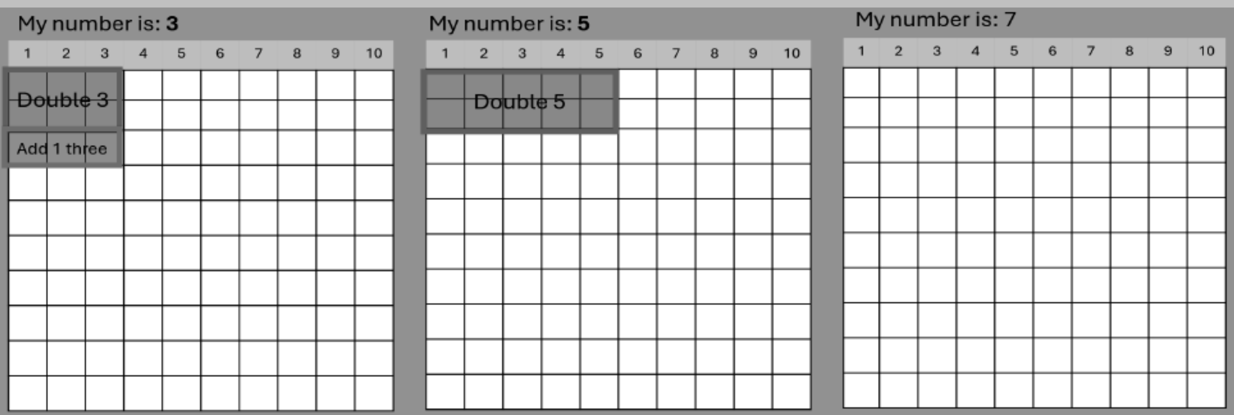


123 VESA
Count in 3s

- Play in pairs
- Put hands behanind back
- Say together, "1,2,3 VESA
- **Samp:** Show some fingers using ONE hand, e.g. 5
- **Bean**: Touches each one of Samp's fingers.
  Counts in 3s: 3, 6, 9,12,**15**,
  Says: **3 times 5 is 15**
- Do it again! Faster.
- Then use 2 hands.

TASK

Explore how the 3 times table relates to the doubles.
My number times 3 = Double my number + my number



Look at Double **3** = 6
Add 1 three, to get 9
3 × 3 = 9

Look at Double **5** = ___
Add 1 five to get ____
5 × 3 = ___

Draw Double **7** = ___
Add 1 seven to get ____
7 × 3 = ___

TASK

You can use arrays to explore how the 3 times table connects with the 2 times table.

- My number × 3 = my number × 2 + my number

Is this statement true:

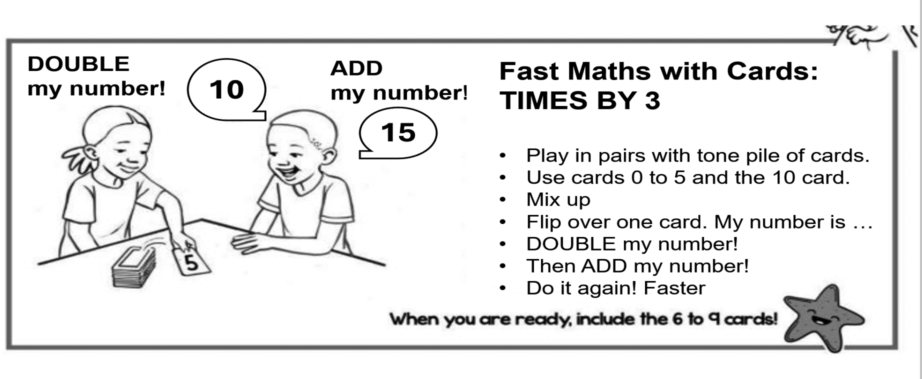
- Sometimes
- Never
- Always?

Explore this statement by choosing different numbers.

Once the arrays are understood, then you can practice this skill for ×3.

-✂-- Activity for learners

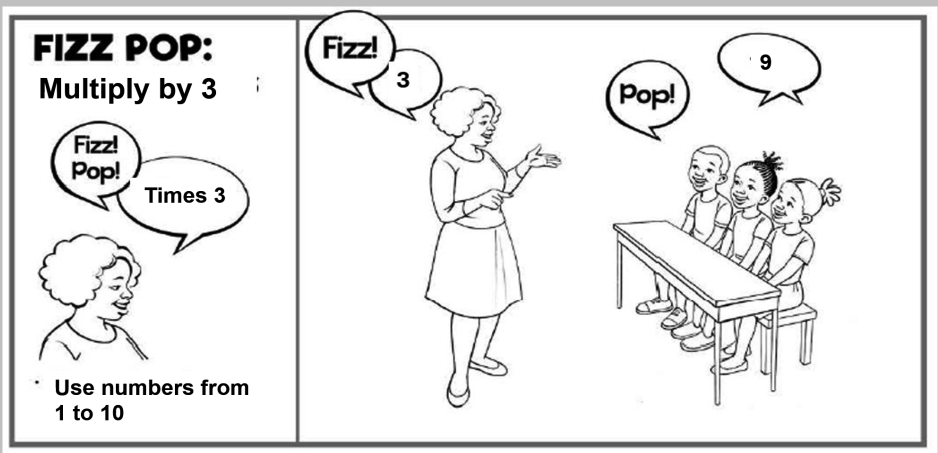
Double my number, then add my number



Children may need more help to get to know the 3 times table. Let them draw a clue board for ×3. With the clue board in front of them, play fizz pop. The function is ×3. Do this every day for at least 1 week.

-✂-- Activity for learners

Fizz pop: Times by 3



The next week play fizz pop. The function is ×3. Play a few times then they turn over their clue board so they cant see it. Play for at least another week. Revisit often.

Remember that to learn their times tables:

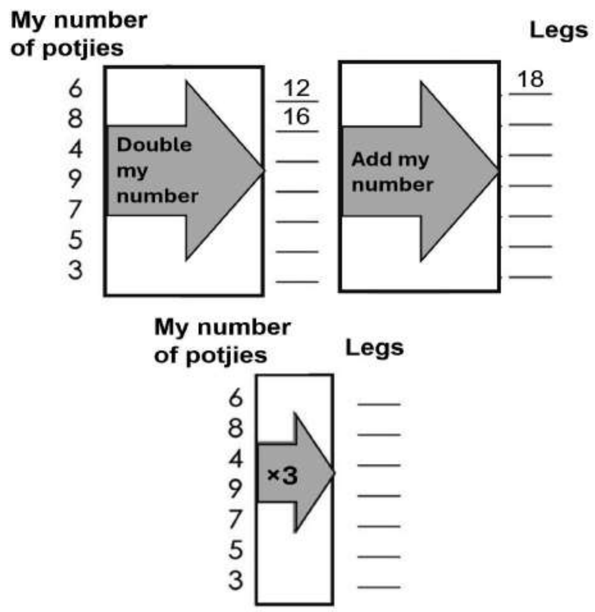
- Children need to move from **concrete to pictorial and to abstract** (CPA)...
  They also move in both directions (APC).
- Children need to connect new known facts to old known facts.
  They derive new facts from the known facts.
- Children need to learn the relationships between the different times tables.
  The times tables link together
  - » 2, 5 and 10 go together
  - » 2, 4 and 8 go together;
  - » 3, 6 and 9 go together.

Concrete	**Literal objects** are real physical objects in their surroundings: sticks, stones, forks, bowls, leaves, hats, shoes **Concrete objects** are physical objects which represent the literal objects such as bottle tops, or unifix cubes. They are also called manipulatives. They are concrete as they can be touched, joined and arranged.
Pictorial	**Iconic pictures:** Learners also use drawings or picture of collections of objects. They can draw a hand, or a few stones or a pair of shoes. **Indexical picture:** Overtime, they start to draw with less detail. Instead of drawing 5 apples, they just draw 5 circles to show how many there are.
Abstract	**Number symbols:** Only in Grade R and 1 do they start to recognise and learn to write number symbols. **Number sentences (symbolic syntactical):** A number sentence describes the relationship between collections of discrete objects or between measurement units. Learners start to write number sentences using symbols like + and – and = in Grade 1. The × and ÷ symbols are used by Grade 3.

TASK

Explore how ×6 relates to ×2 and to ×3

It is always worth starting with a real context. It is more **concrete** to work with a problem about 1 potjie pot that has 3 legs.



You can be helpful to work with a **picture** of arrays, once the real context of potjies and legs is secure.

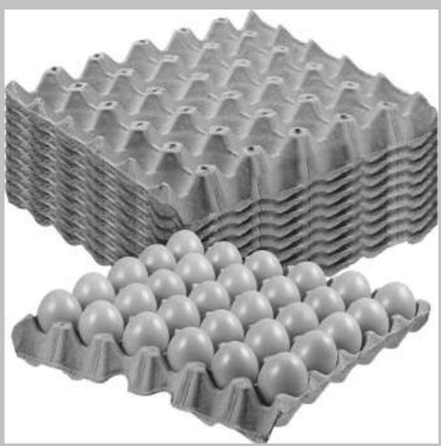
Sixes

By now you should know the 6 steps in building up children's knowledge of any times table. So we only focus on a few of those steps for sixes.

Picture problems and number talks

TASK

Put on your six glasses. How many sixes?



TASK

How many dough nuts?

Write an equation (number sentence) to show how you worked out how many doughnuts:

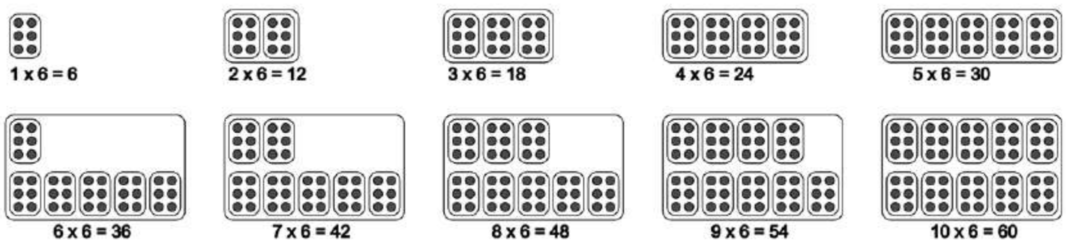
Children are often familiar with 6 dots on a dice or 6 dots on a domino.

Double six on a domino.

Six on a dice

Many dice showing 6, gives the 6 times table. 1 group of 6, 2 groups of 6, 3 groups of 6 etc...

Groups of 6



The 6s song

Try this song for learning the 6 times table:

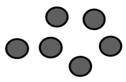
SCAN ME

Factor pairs and divisibility rules with 6s

TASK

How many different arrays or rectangles can you make from 6 counters?

Factors of **6**



Here is a set of 6.
It is unstructured

factor × factor = 6

Draw 6 dots to make arrays.

The factor pairs for 6 are:
___ and ____
___ and ____

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.

1	2	3	4	5	6	7	8	9	10

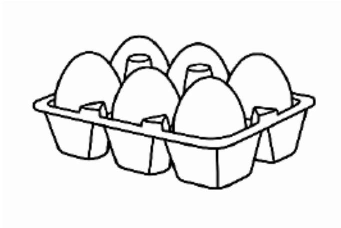
Colour in 6 blocks to make rectangles

6 can make ___ different rectangles.
Therefore, 6 is prime / composite.
6 can make a square. True/false

Pick a number on the 1-100 number chart. Is it divisible by 6?
The divisibility rule for 6:

IDEAS FOR YOUR CLASSROOM WITH 6s

Useful contexts for 6s:
Any groups of 6 each time;
Each insect (bug, beetle, butterfly) has 6 legs;
Each hexagon has 6 sides;
6 on a dice
6 on a domino
Plants in rows of 6



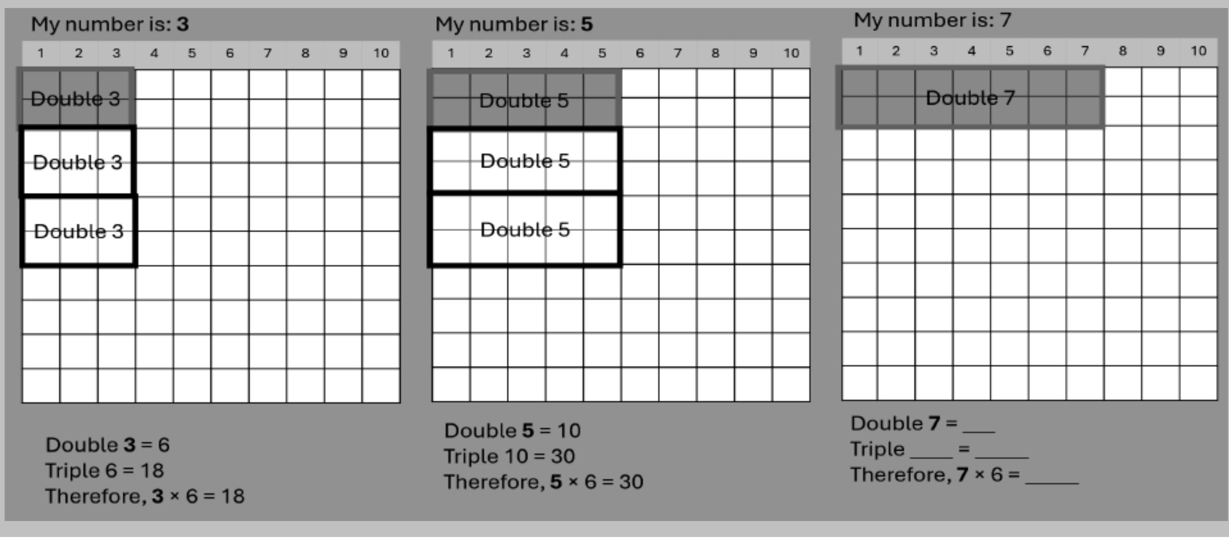
A box holds 6 eggs

It is important to link the **times 6** facts with facts the children already know.

- Link × 6 to × (3 + 3)… Link × 6 to double × 3
- Link × 6 to × (5 + 1)

Activity for learners

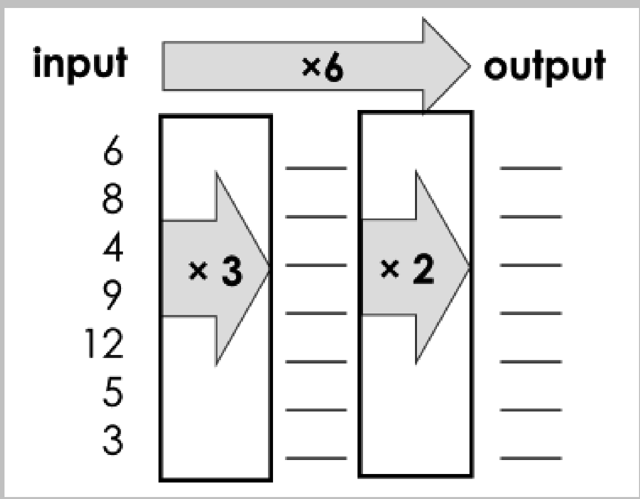
Explore arrays to visualise how ×2×3 = ×6



Using function machines to break up ×6 into double and triple is more **abstract,** but it still gives a picture or diagram**.**

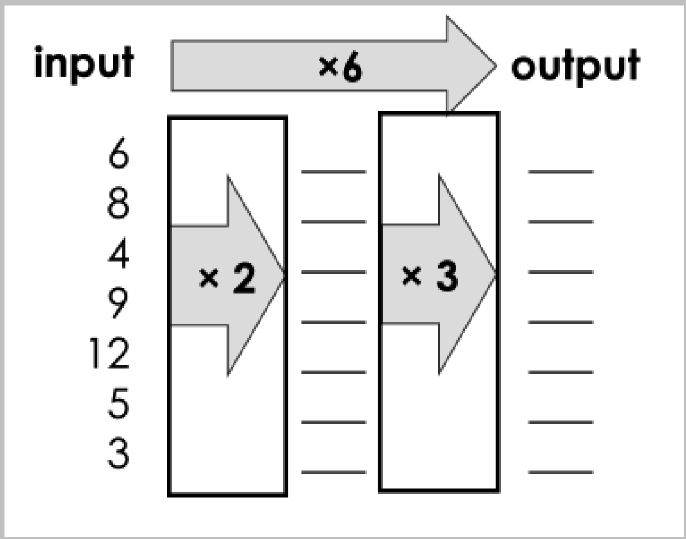
Activity for learners

Is ‘triple then double’ the same as or different to ‘double then triple’?



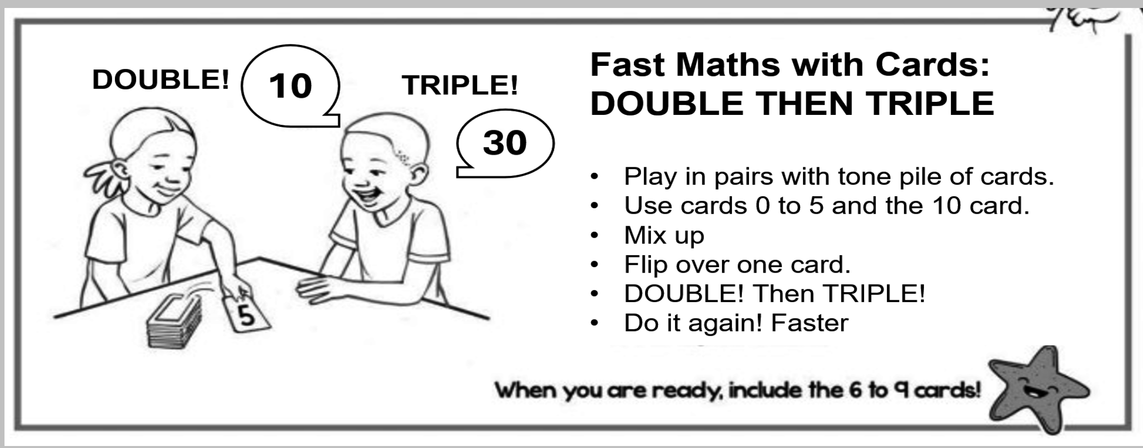
When the **concrete, pictorial and abstract** are secure for ×6, you can expect learners to work mentally for ×6.

Activity for learners

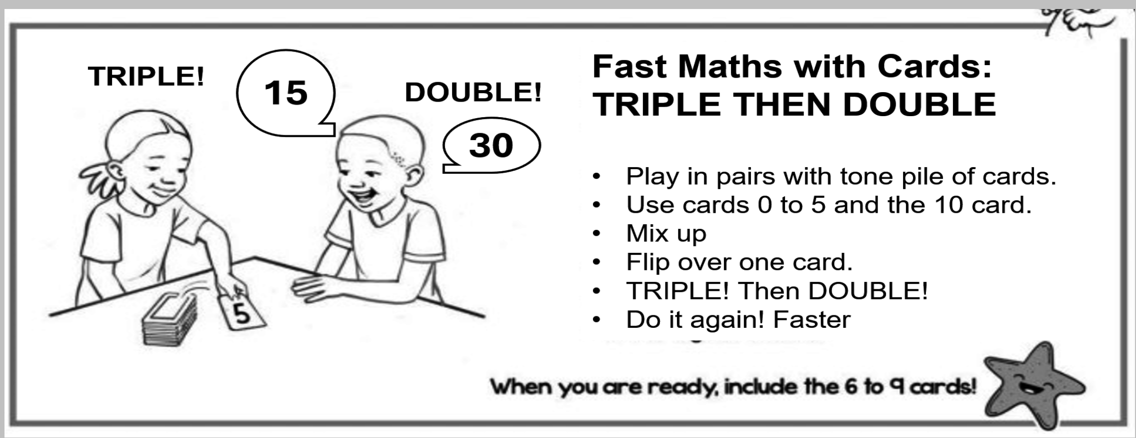


Double then triple with cards

Activity for learners



Explore: Do you get the same result if you 'Triple then double' with cards



Nines

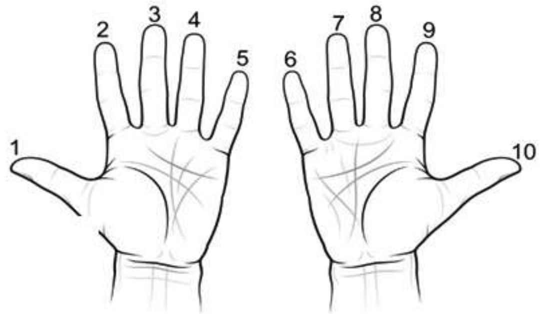
SCAN ME

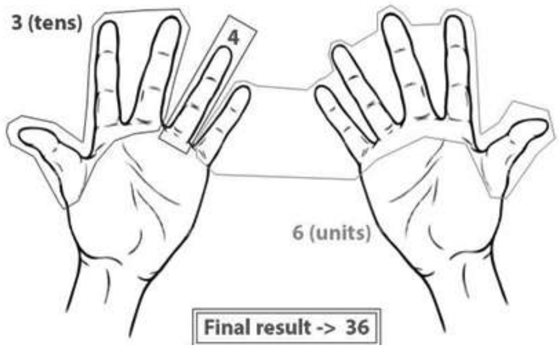
Picture problems and number talks with 9s

TASK

How many cupcakes?

Learning the 9 times table





SCAN ME

https://www.youtube.com/watch?v=_qmYcvChQwo Skip to 1:28

Factor pairs and divisibility rule for 9

TASK

How many different arrays or rectangles can you make from 9 counters?

Factors of **9**

factor × factor = 9

Draw 9 dots to make an array.

The factor pairs for 9 are:

Colour nine squares to make rectangles

1	2	3	4	5	6	7	8	9	10

Here is a set of 9.
It is unstructured

An array is structured.
It has equal rows and equal columns.
You can always have just 1 row.

9 can make ____ different sized rectangles. The rectangles could be vertical or horizontal.
Therefore, 9 is not prime.
9 is a composite number

The factor pairs for 9 are 9 and 1, and 3 and 3
9 = 9 × 1 and 9 = 1 × 9 and 9 = 3 × 3
Therefore, 9 is a **composite number**.
9 is a square number.

TASK

Number	Sum of digits	**Number**	**Sum of digits**
9		72	7 + 2 = 9
18		81	
27		90	
36	3 + 6 = 9	99	
45		108	
54	5 + 4 = 9	117	
63		126	

Divisibility rule for 9

The 9 times table goes with the 3 and 6 times tables.

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	65	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

A number is divisible by 3, if its digit sum is 3, 6 or 9
A number is divisible by 6, if it is even, and its digit sum is 3,6 or 9
A number is divisible by 9 if it's digit sum is 9.

Activity for learners

Eight for the price of one

You need: 2 players; 2 dice

Roll the two dice

Write down a number sentence for multiplying the two numbers

Write down the seven other related number sentences

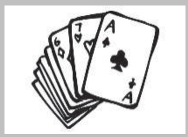
The first player who correctly writes down all eight number sentences gets a point

The winner is the player with the most points after 5 rounds

-✂-- Activity for learners

Salute (multiply)

You need: A pack of cards



- The captain chooses two sailors to come to the front
- Each sailor is given a card
- The sailors may not look at the cards
- When the captain says 'Salute!' the sailors put the cards on their forehead so that the other sailor can see the card but they can't
- The captain tells the sailors the product of the two numbers
- The first sailor to say what the number on their forehead is, stays at the front, the other sailor sits down
- A new sailor is chosen to challenge the winner

IDEAS FOR YOUR CLASSROOM WITH 9s

Useful context for 9s:
Any groups of 9 each time;
Plants can grow in rows of 9

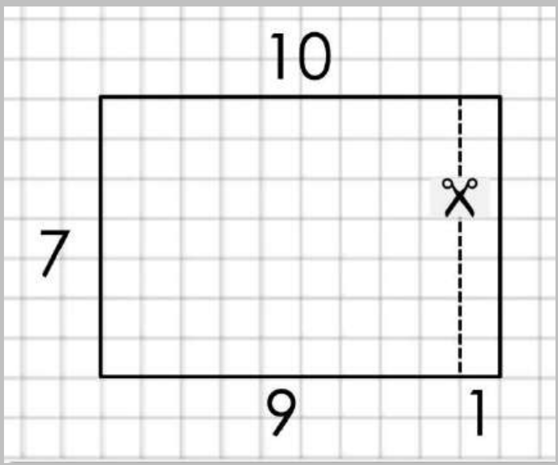
Cupcakes are packed with 9 in a box.

All of these activities are helpful for children to learn their 9 times table

- **Explore real contexts, like boxes and cupcakes...**1 box has 9 cupcakes
- **Count forwards in 9s**. Start at 9. Keep track on your fingers.
- **1,2,3 Vesa**, **Count forwards in 9s**: Samp shows some fingers on one hand.
  Bean touches each of Bean's fingers and counts in 9s.
- **Fizz Pop.** The function is " my number times 9". The function is "my number repeated 9 times". The function "my number groups of 9".
- **Fast maths with dice or with cards:**
  - » Multiply by 9
  - » Triple then triple again,
  - » Multiply my number by 10 then subtract my number.

Investigation

Explore arrays to visualise my number × 9 = my number × 10 – my number.



2 x 9 = (2 x 10) – (2 x 1) = 20 – 2 = 18
3 x 9 = (3 x 10) – (3 x 1) = 30 – 3 = 27
4 x 9 = (4 x 10) – (4 x 1) = – =
5 x 9 = (5 x 10) – (5 x 1) = – =
6 x 9 = (x 10) – (x 1) = – =
7 x 9 = (x) – (x) = – =
8 x 9 = (x) – (x) = – =
9 x 9 = (x) – (x) = – =

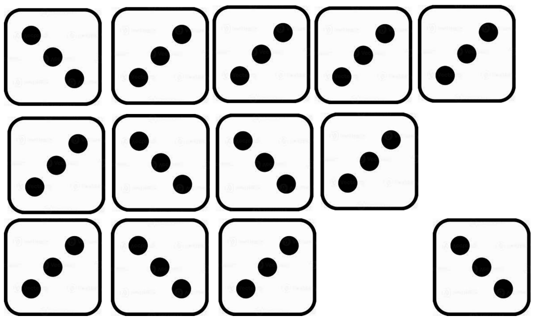
Draw how to imagine 18 x 9 as a grid of 18 x 10 then subtract 1 eighteen.

Draw how to imagine 24 x 9 as a rectangle of 24 x 10 then subtract 1 twenty four.

Tasks 2.2.2

Threes

1. **Design a number talk** using with this picture:

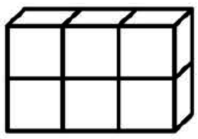


Ask 3 questions to prompt some discussion:

Finding 1 third is the same as dividing by 3.

2.

Build. Break into 3 equal pieces.



How many cubes altogether?	
How many cubes in one third?	

6 ÷ 3 = ______

3. **Digit sums**

   a. Calculate the digit sums for the multiples of 3 (the threes song or the multiples of 3).

Number	Sum of digits
3	3
6	6
9	9
12	1 + 2 = 3
15	
18	1 + 8 = 9
21	

Number	Sum of digits
24	2 + 4 = 6
27	
30	
33	
36	
39	2 + 9 = 12 and 1 + 2 = 3
42	

   b. Explain a divisibility rule for 3. How do you know if a number is divisible by 3 or not?

4. Divisible by 3? Multiple of 3?

Close your eyes, pick a number from the 41-100 number chart.
Is it divisible by 3?

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50

Circle all the multiples of 3 in the grid.

Write your number here: ___________________

5. Consider these tricycles.

a. **Specialise.** How many tricycles in the picture? ____________.

How many wheels? ____________

b. **Generalise.** Draw a function machine which shows the number of tricycles as the input, and the number of wheels as the output. Choose 5 examples.

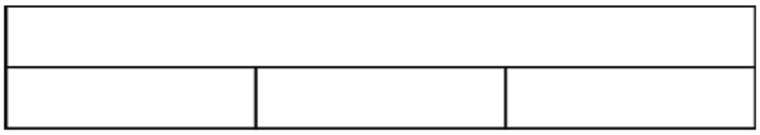
c. Complete a number sentence (equation) that shows the number of tricycles in relation to the number of wheels.

Number of tricycles = ____

d. Complete a number sentence (equations) that shows the number of wheels in relation to the number of tricycles.

Number of wheels = ____

6. **True of false?**



This whole equal part diagram can ONLY be used to represent six divided into 3 groups of two.

a. Lily says that the whole equal part diagram can also be used to represent 15 divided into 3 groups of five. Is she right? Explain

b. Mandla says that is represents 25 divided into 3 groups of ten, ten and five. Is he right?

7.a Draw an array for 6 x 9 = 54

7.b Draw a rectangle for 6 x 9 = 54. Show that 6 x 9 = (2 x 9) + (4 x 9), by partitioning the rectangle.

Tasks 2.3.2

Timed tango ×3 and ×6

1	10 × 3 = ______	**11**	______ × 3 = 24
2	3 × 3 = ______	**12**	______ = 27 ÷ 3
3	3 × ______ = 33	**13**	3 × 8 = ______
4	______ = 7 × 3	**24**	______ = 24 ÷ 3
5	3 × 2 = ______	**15**	______ = 3 × 5
6	______ = 3 × 5	**16**	30 ÷ 3 = ______
7	12 × 3 = ______	**17**	4 × 3 = ______
8	1 third of 15 = _____	**18**	______ ÷ 3 = 10
9	Factorise: 27 = ______ × 3	**19**	Factorise: 18 = 3 × ______
10	Circle the multiples of 3: 27; 43; 72; 104; 114; 289	**20**	Circle the factors of 3 1; 3; 27; 72; 104; 114; 289

TASK

Create a clue board for legs and potjies.

I know a number is divisible by 3 when...

____

____

____

____

____

The factor pairs of 3 are ....

Therefore 3 is prime/composite.

3 written as a product of prime numbers is _____________.

3 groups of zero =

3 repeated 0 times =

3 shared equally between 0 =

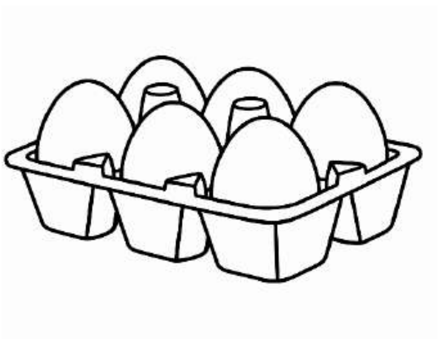
0 shared equally between 3 =

3 shared equally between 0 =

1	6 × 8 = ______	**1**	______ = 7 × 6
2	______ × 6 = 48	**2**	6 × ______ = 60
3	6 × ______ = 60	**3**	______ = 42 ÷ 6
4	1 sixth of 18 = ______	**4**	60 ÷ 6 = ______
5	10 × 6 = ______	**5**	______ ÷ 6 = 7
6	______ = 7 × 6	**6**	______ = 18 ÷ 6
7	8 × 6 = ______	**7**	11 × 6 = ______
8	______ × 6 = 48	**8**	5 × 6 = ______
9	Factorise: 30 = ______ × 6	**9**	Factorise: 54 = ______ × 6
10	Circle all the multiples of 6 6; 24; 32; 41; 60; 66	**10**	List all the factors of 6 ______________________________

TASK

Create a clue board for boxes and eggs.



I know a number is divisible by 6 when…

____

____

____

____

____

The factor pairs of 6 are ….

Therefore 6 is prime/composite.

6 written as a product of prime numbers is ____________.

0 multiplied by 6 =

6 multiplied by 0 =

0 divided by 6 =

6 divided by 0 =

SUMMARY 2.3

Consolidating the times tables
Which times tables go together?

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	65	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
X	1	2	3	4	5	6	7	8	9	10

× ______
and
× ______
and
× ______
go together.

They are always even.
× ______ is the same as ×2×2
× ______ is the same as ×2×2×2.

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	65	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
X	1	2	3	4	5	6	7	8	9	10

× ______
and
× ______
and
× ______
go together.

For ×_____ the last digit is 0
For ×____ the last digit is 0 or 5
my number × _____ = my number

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	65	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
X	1	2	3	4	5	6	7	8	9	10

× ______
and
× ______
and
× ______
go together.

For ×_____ the digit sums are 3, 6 or 9.
For ×_____ it is even, and the digit sums are 3, 6 or 9.
For ×_____ the digit sums are 9.

UNIT 3: WORD PROBLEMS AND USING ARRAYS, T-TABLES AND CLUE BOARDS

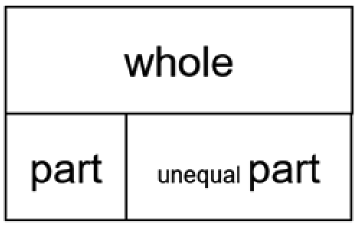
Key ideas

1. Multiplicative thinking it about **multiplication and division.** Teachers should **link division to multiplication.**
2. **Multiplication is commutative.** The order does not matter. 5 × 2 = 2 × 5
3. **Division is not commutative.** The order does matter. 10 ÷ 2 ≠ 2 ÷ 10.
4. There are important **representations** for multiplicative thinking
   a. Whole – equal part diagrams
   b. Arrays
   c. T-tables
   d. Clue boards

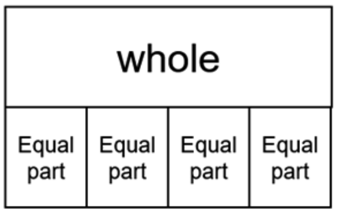
Foundation Phase multiplicative word problem types

Recall that for **additive word problems**, there were two different equations that were important:
start +/- change = result
part + part = whole

The additive word problem could be solved using either addition or subtraction. Because there is a family of equivalent number sentences, any subtraction problem can be re-written as an addition problem.
9 – 7 = ... is the same as 7 + ... = 9.

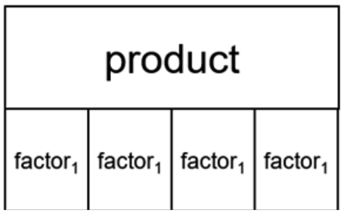


For **multiplicative word problems**, there is a whole which is made of equal parts.



In this diagram the whole is made up of 4 equal parts. Each equal part is 1 quarter of the whole

Up to this point, we have mostly worked with the general number sentence (formula) for multiplication and division as:



Factor$_1$ × Factor$_2$ = Product

In this diagram,

Factor$_2$ is 4.

Factor$_1$ is repeated 4 times to make the product.

UMTHAMO 3.1

WORD PROBLEMS AND USING ARRAYS, T-TABLES AND CLUE BOARDS

Key ideas

1. Multiplicative thinking it about **multiplication and division.** Teachers should **link division to multiplication.**
2. For multiplication word problems, you should use **both meanings of ×**
   - ______ 'groups of' _______; and
   - ______ repeated _______ times
3. For division you should use **both meanings of ÷**
   - The equal sharing situations; and
   - The grouping/packing situations
4. For division you should include problems:
   - that result in **left overs or remainders**
   - that **result in a fraction**; and
   - about '**finding a fraction**' (of a discrete object, or a measurement, or of a group of discrete objects)
5. For **Intermediate Phase** should also include problems about:
   - **Scaling** (to make bigger or smaller)
   - Repeating **patterns** (investigate number patterns with a common difference)
   - Combinations (**Cartesian product**)

Types of multiplicative word problems

There are many different types of multiplicative word problems.

Your word problems examples should always include:

- **discrete objects** (from the **counting pathway**)
- **continuous** extents (from the **measurement pathway**). All measurement contexts are multiplicative. There is a standard or non-standard unit of measurement that is repeated (time, mass, capacity, volume, length).

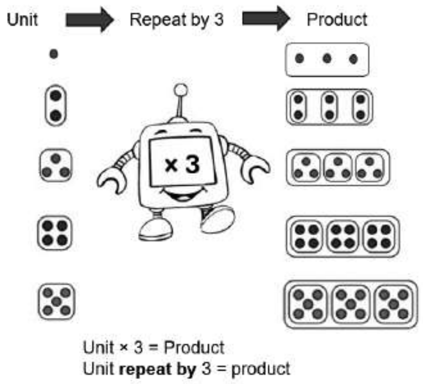
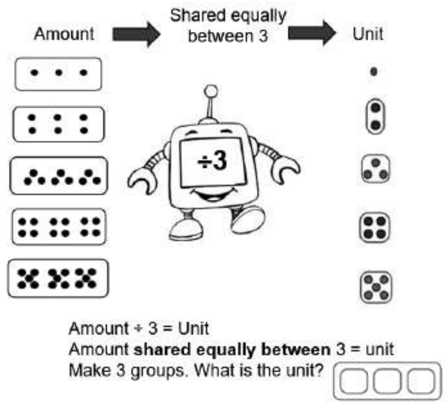
You should encourage the use of all the different representations for multiplication and division.

In Foundation Phase

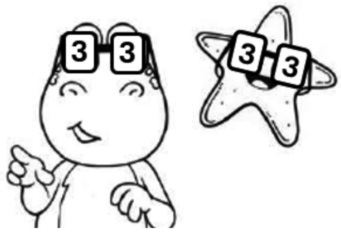
- For multiplication word problems, you should use both meanings of ×
  - » ______ '**groups of**' _______; and
  - » ______ **repeated** _______ times
- For division you should use both meanings of ÷
  - » The **equal sharing** situations; and
  - » The **grouping/packing** situations
- For division you should include problems:
  - » that result in **left overs or remainders**
  - » that result in a **fraction**; and
  - » about '**finding a fraction**' (of a discrete object, or a measurement, or of a group of discrete objects)

You should ask **what if?** Questions, and expect children to explore and investigate multiplicative situations.

Times as 'repeat by' and divide as 'sharing equally'

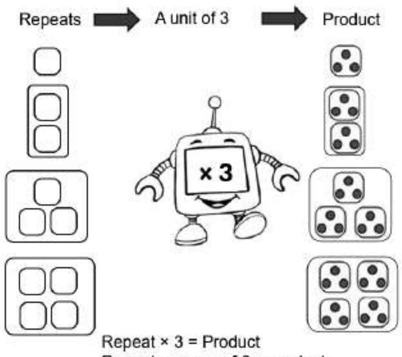
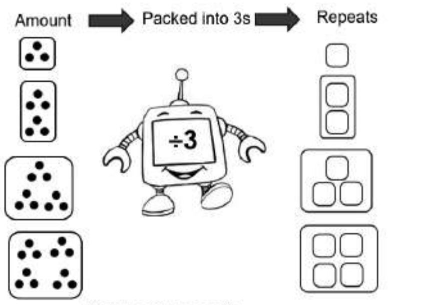
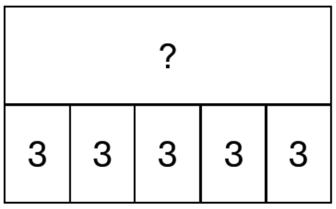
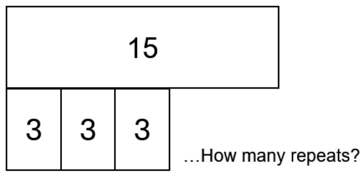
	Multiplication	Division
Equation /Formula /Rule	$\text{Factor}_1 \times \text{Factor}_2 = \text{Product}$ Unit × Repeats = Product	$\text{Product} \div \text{Factor}_1 = \text{Factor}_2$ Product ÷ Repeats = Unit
Meaning	5 × 3 = 15 means 5 **repeated** 3 times gives 15.  This is the '**repeated by'** concept for multiplication.	15 ÷ 3 = 5 means **share** 15 **equally** between 3. Make 3 equal groups from 15. How many in each group? Given 15, find the unit that is repeated 3 times. This is the '**equal sharing'** concept for division
Function	An input (unit) is **repeated by 3** to produce an output (the product).  	An input (total amount) is **shared equally between 3** to find the output (the unit) 
Whole-equal part	**5 'repeated by' 3 = product** ? 5 5 5 The unit is 5. There are 3 repeats	**15 shared equally between 3** 15 ? ? ? The whole is 15. Share into 3 equal parts
Typical situation	1 teaspoon holds 5 mℓ. Josh must take 3 teaspoons of medicine. How much medicine does he take?	15 apples are shared equally between 3 people. How may apples does each person get?

When we multiply and divide we think about equal groups!



Let's think in 3s
Let's put on our 3 glasses!

Times as 'groups of' and divide as 'grouping or packing'

	Multiplication	Division
Equation /Formula /Rule	$\text{Factor}_1 \times \text{Factor}_2 = \text{Product}$ Repeats × Unit = Product	$\text{Product} \div \text{Factor}_2 = \text{Factor}_1$ Product ÷ Unit = Repeats
Meaning	5 × 3 = 15 means 5 **groups of** 3 gives 15. 	15 ÷ 3 = 5 means Make or pack into groups of 3. How many groups? Given 15, how many 3s? This is the **grouping/packing** concept for division.
Function	An input (the repeats) of **a group of 3** (unit) produces an output (the product). 	An input (the product) is packed into 3s to find the output (repeats) 
Whole-equal part	5 repeats of the unit 3 = product. There are 5 repeats. The unit is 3.	**15 packed out into 3s** The unit is 3. How many repeats to reach 15?
Typical situation	There are 5 bags. There are 3 apples in each bag. How many apples all together?	15 apples are packs into bags. There are 3 apples in each bag. How many bags?

Intermediate Phase multiplicative word problems

It is helpful for Foundation phase teachers to know what is expected in **Intermediate Phase**. It is helpful for intermediate Phase teachers to know what is expected in Foundation Phase, so they can remediate if necessary.

In **Intermediate Phase** you extend the multiplicative situations given in Foundation Phase, you also start to include a wider range of multiplicative word problems

- For multiplication you should also include problems about:
  - **Scaling** (to make bigger)
  - Repeating **patterns** (investigate number patterns with a common difference)
  - Combinations (**cartesian product**)
- For division you should also include problems:
  - that result in **left overs or remainders**
  - that result in a fraction; and
  - about '**finding a fraction**' (of a discrete object, or a measurement, or of a group of discrete objects)
  - about **repeating patterns** (investigate number patterns with a common difference)
  - about **scaling / reducing** (making smaller).

Example

A scaling word problem

Recipe for scones

2 cups flour
2 tsps. baking power
200 g butter
1 egg
Half a cup of milk

Instructions:
1. Mix the dry ingredients
2. Mix in butter until flour has consistency of mielie meal
3. Stir in egg, and slowly add milk until mixture comes away from the side of the bowl.
4. Do not knead or handle dough
5. Cut into 8 scones.

Ingredients	16 scones	24 scones

Here the learner has to first work out the scale factor. How do you get from 8 scones to 16 scones? And from 8 scones to 24 scones? Then they apply this scale factor to increase the recipe proportionally.

A simpler scaling problem would involve doubling the size of a simple image, using a grid that is twice as big.

Activity for learners

Repeating pattern word problem

Squares	1	2	3		10	100	1000			
Matches								10	100	1000

What is the rule to relate matches to squares?

Matches = ________________________________

What is the rule to relate squares to matches?

Squares = ________________________________

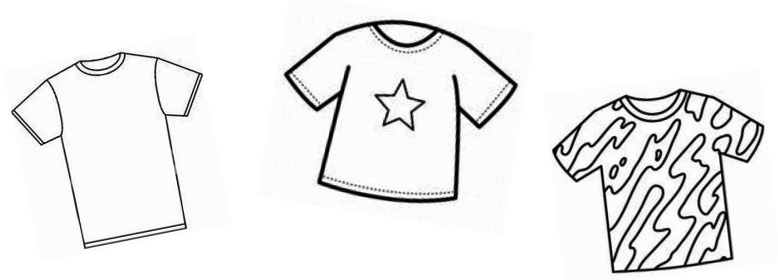
PrimTEd Std. NA 7
Early algebra. These pattern exercises are very helpful for introducing rules or functions. They scaffold into algebraic equations in Senior Phase.

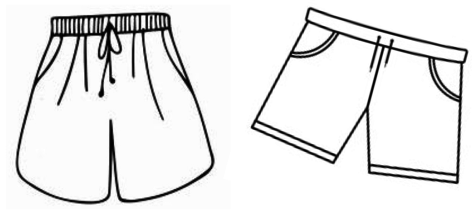
We have included **many investigations** in this manual. These kinds of problems form part of **patterns, functions and algebra** in the South African CAPS. It is very helpful to find the part of the pattern that stays the same, and the part that is being added on each time (the unit that repeats). This is a very important scaffold to algebra in high school.

All Foundation Phase word problems can be turned into investigations by asking **what if** questions. This is a good pedagogic strategy to extend learners, and keep those who can solve a single problem quickly, to extend and explore their thinking.

Activity for learners

Example: Cartesian product





Okuhle has 3 T-shirts and
2 pairs of shorts.

How many outfits has she got?

This is an example of what is called a Cartesian product problem. The problem related two sets: the t-shirt set, to the shorts set. There are then combinations which can be made my taking each element in one set, and combining it with one element in the other set. It is important to allow learners to explore these kinds of problems for themselves. They should not be given too much structure. Over time, they will start to realise that arranging the sets spatially can be helpful. Using rows and columns is also helpful.

We do not discuss all the multiplicative problem types in the CAPS. **It is most important to get student confident with sharing and grouping problem using both discrete and continuous situations.**

Hackmack (2025) provides the following framework for thinking about types of multiplicative word problems.

Framework for multiplicative word problems		
Equivalent Groups	Number of groups unknown	"*Karen puts 24 pencils in cases. If she puts 8 pencils in each case, how many cases does Karen need?*"
	Size of group unknown	"*Karen shares 24 pencils equally between 3 cases. How many pencils are in each case?*"
	Product unknown	"*Karen has 3 pencil cases. There are 8 pencils in each case. How many pencils are there altogether?*"
Unit Rate	Number of units unknown	"*Sam wants to buy chips for R250 for his shop. Each bag of chips costs R5. How many bags of chips can he buy?*"
	Unit ratio unknown	"*Sonti travels at a constant speed for 300 km. If she travels for 3 hours, what is her speed per hour?*"
	Product unknown	"*James travelled at 80km per hour for 13 hours. How far did he travel?*"
Ratio	Number of units unknown	"*Hlumi is baking. Her recipe states that every 5 grams of sugar is mixed with 35 grams of flour. What is the ratio of sugar to flour in the recipe?*"
	Group size unknown	*"In my class I have 4 boys for every 3 girls. If I have 40 children in the class, how many boys and girls do I have?"*
	Product unknown	*"Sonti earns twice as much as Feroza. Sonti earns R2 500 per month. How much does Feroza earn?"*
Proportional Sharing	Group size unknown	"*Feroza works for 3 hours, and Daniel works for 1 hour cleaning homes. Together they are paid R520. How should the money be shared fairly between the two?*"

Rectangular arrays/ Rectangular area	No of groups unknown	"*Thabo has 20 cookies he needs to put them into 5 cookies in each row. How many rows of cookies can he make?*"
	Size of group unknown	"*Thabo has 20 cookies he needs to put them into 4 rows how many cookies will he put in each row?*"
	Product unknown	"*Thabo packs 4 rows of 5 cookies in each row. How many cookies did he pack?*"
Cartesian Product		"*Thabo is making pizzas. He has 6 different toppings. He wants to put 2 toppings on each pizza. How many different kinds of pizza can he make?*"

PrimTEd Std. NA 4.6
Describe and use multiplication and division problem types and the range of useful associated representations.

Choose one of the example problems. Keep most of the problem as it is. Change just one or two aspects that make the problem harder.

The Hackmack (2025) framework offers two different generalised equations for multiplicative word problems types

In Foundation Phase the word problems are usually:

- Equivalent groups (vary the position of the unknown);
- Unit rate (number of units unknown);
- Rectangular arrays (vary the position of the unknown)

Word problem type	General formula or equation
Equivalent groups: Sharing or grouping	Size of the group × Number of groups = Product
Unit rate	Unit ratio × Number of units = Product
Ratio	Size of the group × Number of units = Product

Then she **varies the position of the unknown.** She changes which part of the equation is missing. When setting tests, and choosing examples for work in class, it is helpful to think about which problem types you have included, using the CAPS and Hackmack frameworks.

IDEAS FOR YOUR CLASSROOM

Foundation phase children need lots of practice working on multiplication and division problems in context.

Sometimes children should work on a problem and make their own arrangement with cubes. They make their own picture or drawings. The teacher reads the problem to them.

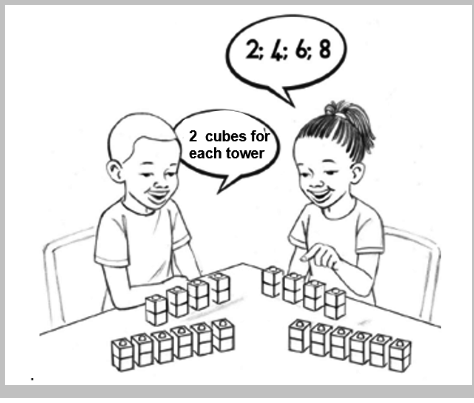
-✂-- Activity for learners

The teacher says:

Here are some cubes.

Arrange the cubes in towers with 2 in each tower.

How many towers?
How many cubes?



-✂-- Activity for learners

The teacher says:

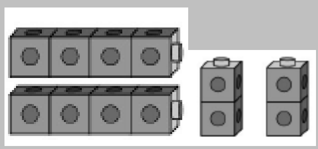
We have 7 amagwinya.

Can you share them equally between 2 children?

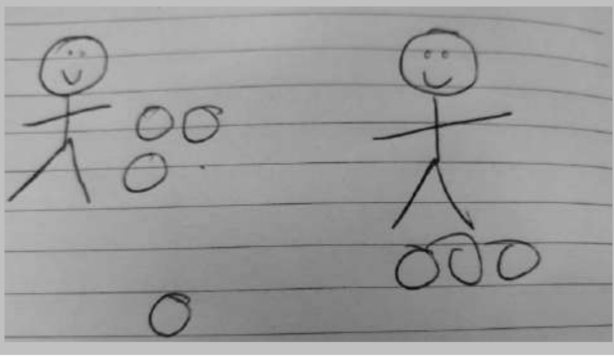
How may does each child get?

Can they share the left over one?

Use these cubes to help you.
(Give each pair at least 10 cubes)



Or draw a picture to help you.



-✂-- Activity for learners

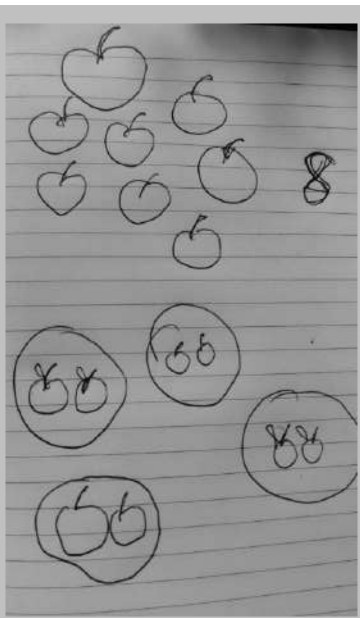
The teacher says:

We have 8 apples.

We are going to put 2 apples on each plate.

How many plates to we need?

Draw a picture to help you.



PrimTEd Std. GPM 1
Plan effective learning experiences

Activity for learners

	How many bottles?	
	How many litres?	

Children should work on problems that have structured pictures.

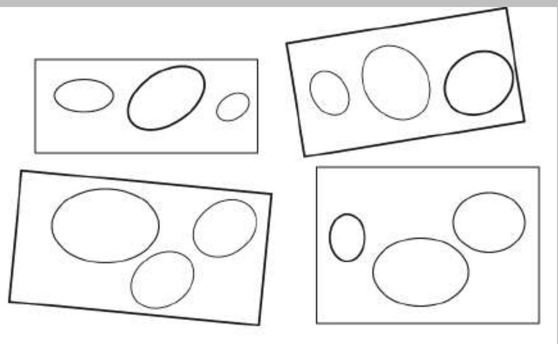
Slowly the children's drawings and working will become more structured. **A teacher can prompt them to get more structured drawings**. Can you line up the apples neatly? Can you make your drawing in rows so we can clear see how boxes repeat?

Activity for learners

The teacher says:

We are packing mangoes in boxes.
You must put 3 mangoes in each box.
Draw 1 box and show the 3 mangoes inside the box.
There are 4 boxes.
How many mangoes are there?

Draw a picture to help you



Activity for learners

One bag has 5 apples..

	How many bags?	
	How many apples?	

Children should work on problems where they have to circle the unit (eg 5 apples in 1 bag) and repeat the unit.

Pack 5 apples in 1 bag.

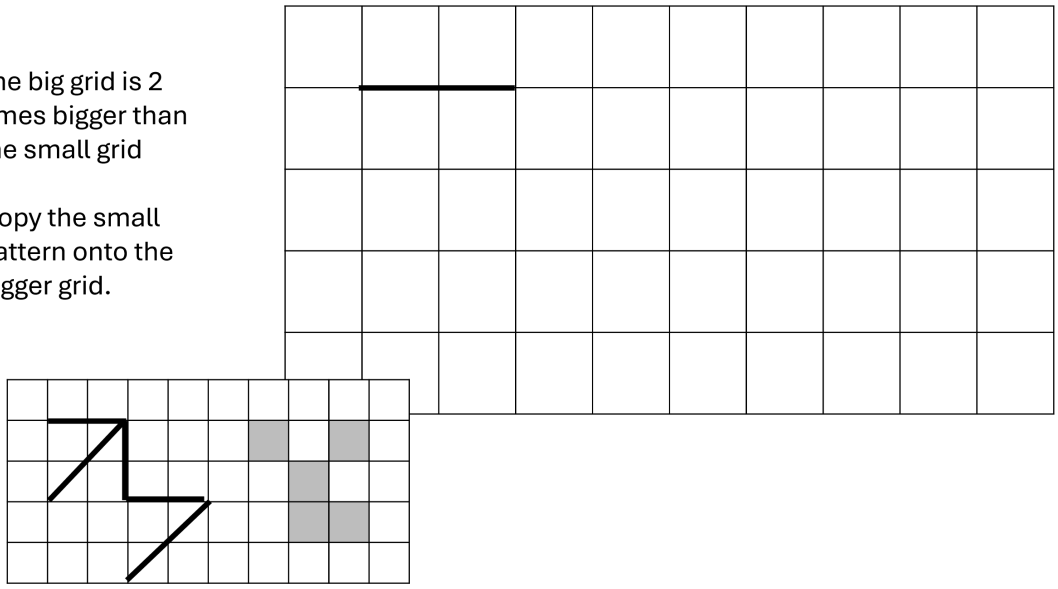
	How many apples?	
	How many bags?	
	How many left over?	

	How many apples?	
	How many bags?	
	How many left over?	

Activity for learners

The big grid is 2 times bigger than the small grid

Copy the small pattern onto the bigger grid.

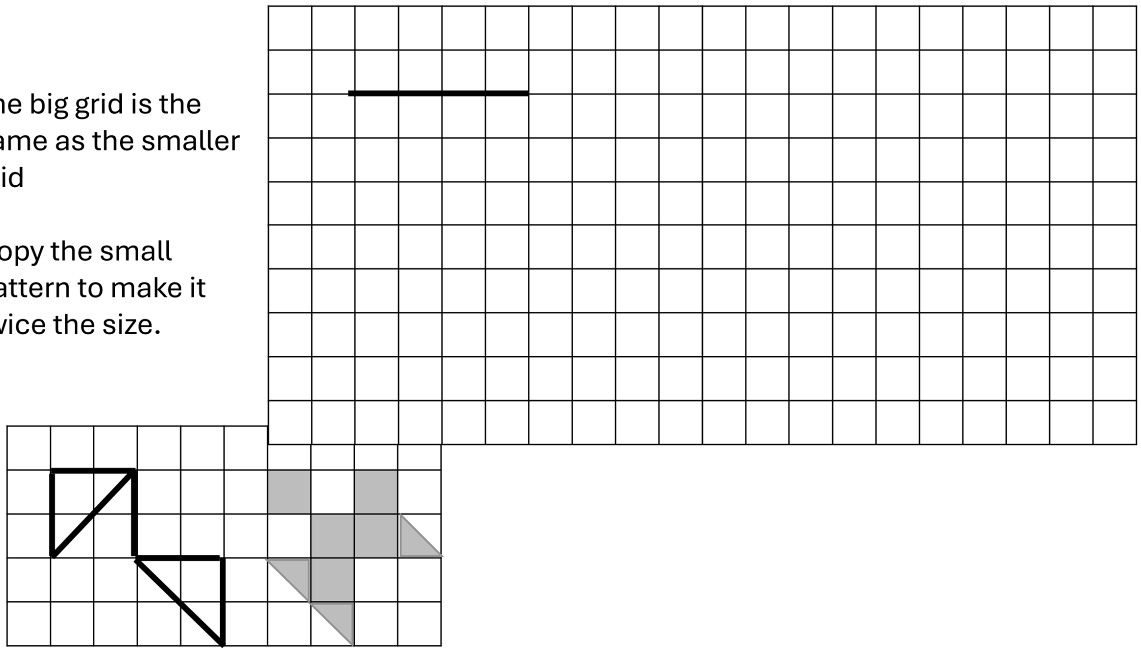


This is an example of **scaling**. **A scale factor** is used for geography with maps. A scale factor is used on photo copy machines to enlarge and reduce images.

Activity for learners

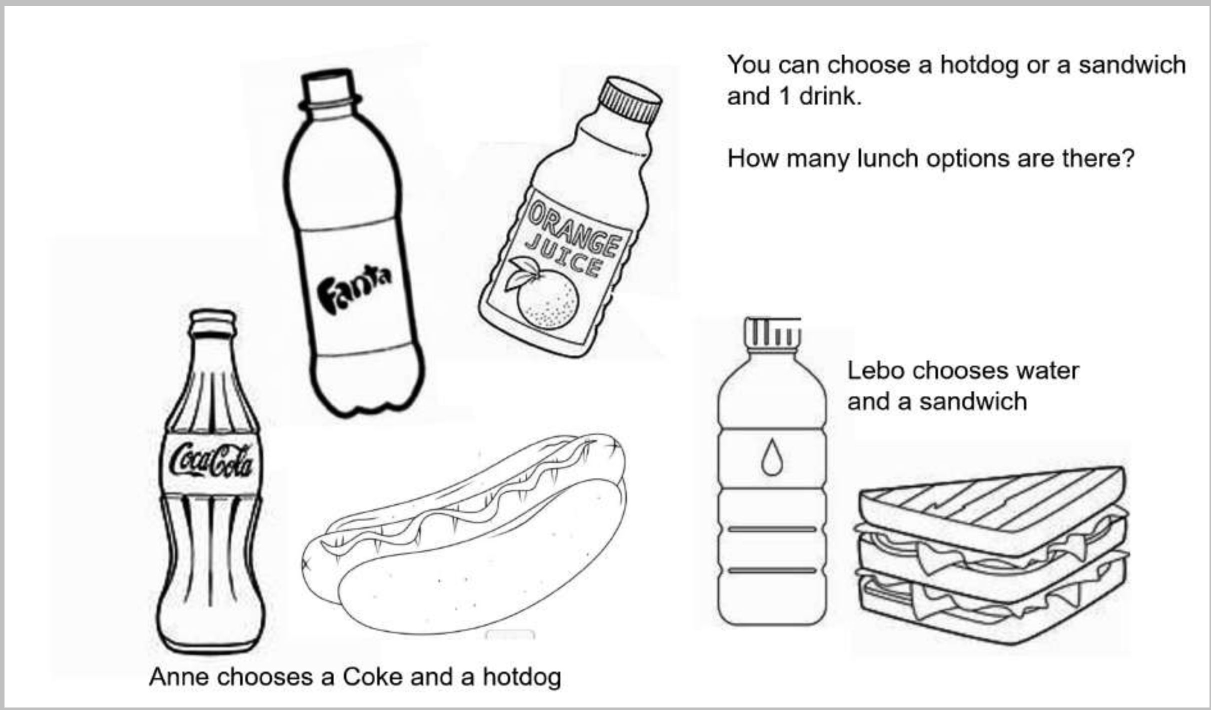
The big grid is the same as the smaller grid

Copy the small pattern to make it twice the size.



This is another example of **scaling.** This is harder, for two reasons. First, the grids are the same, so the learners have to work out how to increase the small image. Also the images to copy are more difficult

-✂-- Activity for learners



This is an example of **Cartesian product** problem. This problem is unstructured. This problem would become easier if the drinks were arranged in a row and the food in a row. It is also helpful to arrange the two sets like an array – for example the drinks are arranged across in a horizontal row; and the food is arranged vertically in a column.

You should give learners both structured and unstructured versions of **Cartesian products** problems. By later in Intermediate phase you should be giving the learners problems without diagrams. They then have to create the image or representation to help them solve it.

Representations to build multiplicative thinking

Arrays

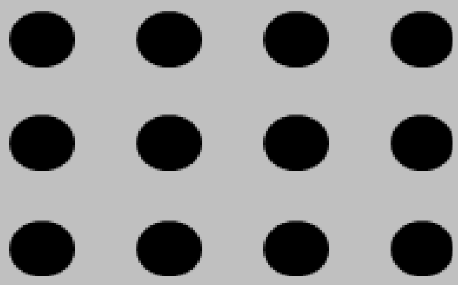
An array shows dots going across (horizontally), and dots going down (vertically). We can think of an array as neat planting of cabbages, or carrots, or mielies in rows and columns. Multiplying and dividing are related. So each array has a family of equivalent number sentences.

TASK

Draw an array to show 5 x 3. Draw 5 dots across and 3 dots down. Write the family of 8 equivalent number sentences, which all represent the same relationship: 5 x 3 = 15.

Example

Family of number sentences



Multiply	**Factorise**	**Find a fraction**
4 × 3 =12 3 × 4 =12	12 = 4 x 3 12 = 3 x 4	1 third of 12 = 4 1 quarter of 12 = 3
Divide 4 = 12 ÷ 3 3 = 12 ÷ 4	**Divide** 12 ÷ 3 = 4 12 ÷ 4 = 3	

With each array, there are two ways to find a unit and repeat it, to count quickly.

Activity for learners

Circle to show that the **unit you repeat is 4.**

Complete these number sentences:

4 repeated _____ times = _________

4 × _____ = _____

______ groups of 4 = _______

Circle to show that the **unit you repeat is 3.**

Complete these number sentences:

3 repeated _____ times = _________

3 × _____ = _____

______ groups of 3 = _______

Arrays are helpful for children to start structuring their drawings for multiplicative thinking.

Draw an array that has 10 dots across and 3 dots down

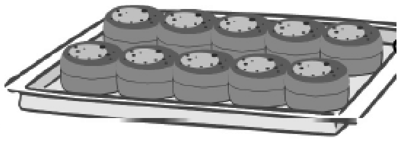
Activity for learners

Write all the related number sentences for this array.

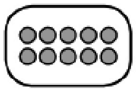
Multiply	Factorise	Find a fraction
Divide	Divide	

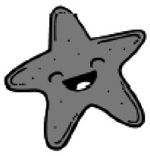
When Zibalo bakes she puts 10 scones in a tray. The trays help Zibalo count in tens. She can quickly see how may scones there are.

Zibalo always bakes 10 scones on one pan.



Every day she records how many scones she baked.
In the past, she drew a dot for EVERY scone.
Infinity showed her a quicker way.





Look at this! When I draw numbers, I draw a 10 like this:

10

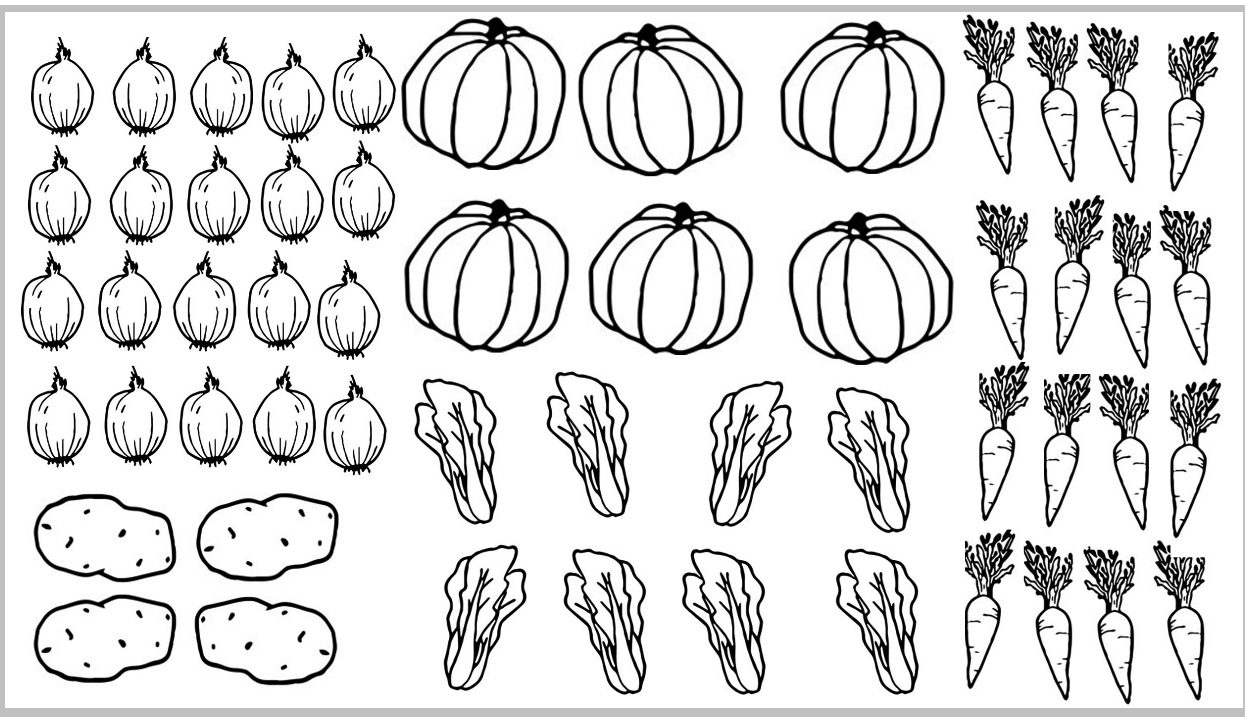
So I draw 30:

10 10 10

When Zibalo gardens, she plants the vegetables neatly in rows. She repeats to make neat equal rows. Even when Zibalo picks her vegetables she arranges them neatly into rows and columns. Then she can **group count** and see how many she has.

She counts quickly in 2s and 5s. She also counts in 3s and 4s.

Activity for learners



Each type of vegetable is arranged in its own array.

She puts each vegetable neatly into rows and columns

TASK

	How many in the first row?	How many altogether?
 Onions		
 Potatoes		
 Spinach		
 Carrots		
 Pumpkin		

Arrays are helpful representations for children to realise that **multiplication is commutative.** It does not matter the order in which you write a multiplication calculation.

Example

If **4 is the unit** we are repeating 3 times, then

- 4 repeated 3 times can be written 4 × 3.
- This is the same as 3 groups of 4 which can be written 3 × 4.

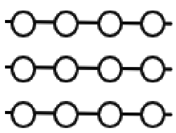
We get the same result if **3 is the unit** we are repeating 4 times, then

- 3 repeated 4 times can be written 3 × 4.
- This is the same as 4 groups of 3 which can be written 4 × 3.

So 4 × 3 = 3 × 4.

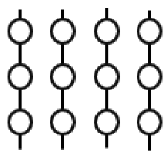
For multiplication the order does not matter.

4 repeated 3 times



3 groups of 4

3 repeated 4 times



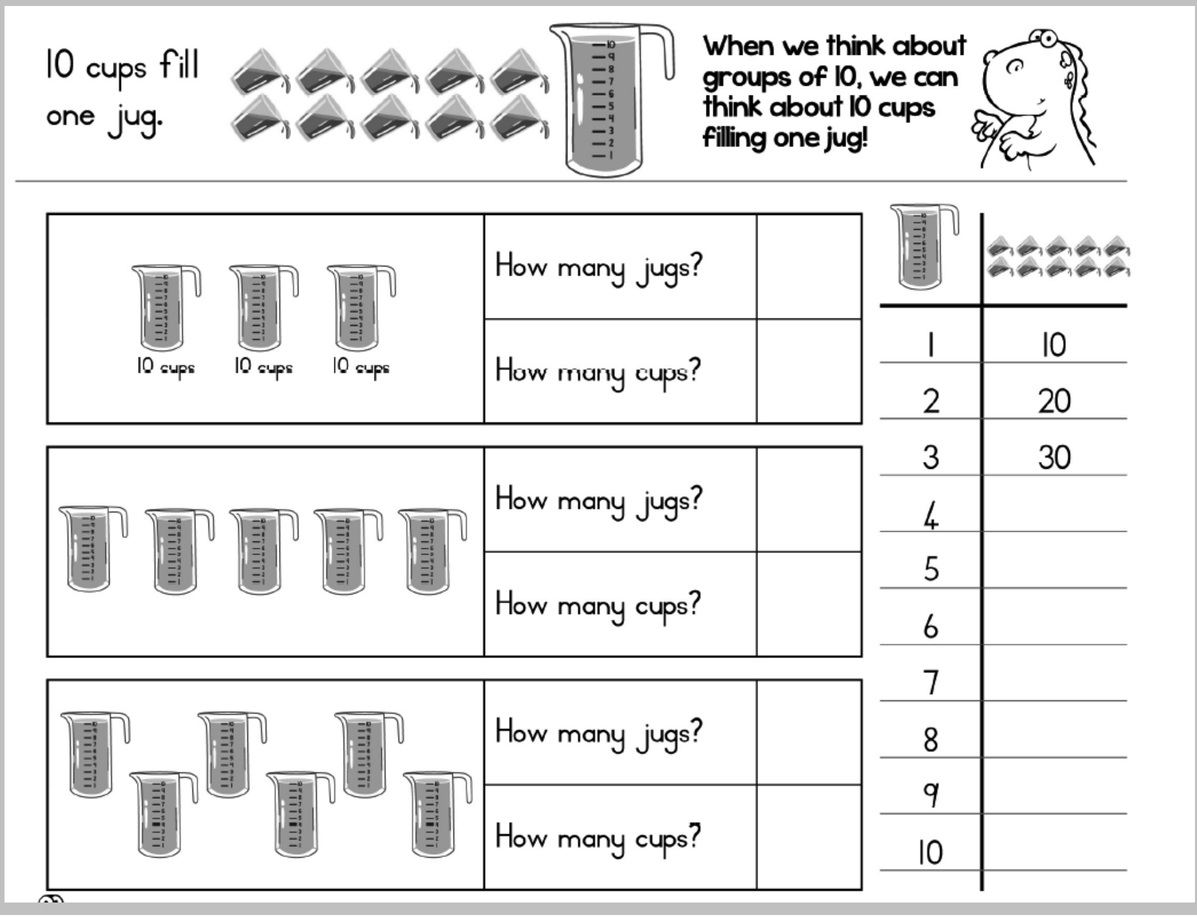
4 groups of 3

T-tables

A **t-table** is a good way to help children record the multiplication situation or multiplicative structure. It helps to shift children from concrete manipulatives, to picture problems and then to a more abstract table or numbers.

This multiplication problem is about 10 cups filling one jug.

Activity for learners



The **t-table** is given to help children record their thinking. This problem is suitable for **Grade 2.**

In **Grade 3,** the same context could be asked. The teacher could prompt the children to use a t-table.

10 cups fill one jug.

- Draw a t-table to record how may cups are needed to fill 2, 3, 4 and 5 jugs.
- Continue the t-table.
- How many jugs are needed to fill 60 cups?

By **Grade 4,** the children would be expected to draw their own t-table or use their own method or know facts, when solving multiplication and division problems.

10 cups fill one jug.

- How may cups can be filled from 7 jugs?
- How many jugs are needed if there are 80 people and you want everyone to have 1 cup?

Clue boards

By the end **Grade 3**, children should know the 2s, 5s and 10s fluently know facts for rapid recall. They should already be building up their multiplication facts for 3s and 4s too.

Grade 4 is really the year of learning their times tables and knowing them well.

Clue boards are very helpful to scaffold learning the times tables for Grade 4s (and even 5s and 6s). **Clue boards are a condensed t-table**. They only use the numbers 1, 2, 4, 8 and 10 on the left hand side of the table. To build a clue board you only need to know the following number facts:

- Doubles and halves
- ×10

And those are easy! They should **have** been learnt in Grades 2 and 3.

Example

This is a x4 clue board

×4	
1	4
2	8
4	16
8	32
10	40

You create a x4 clue board like this.

Draw the T for the table.
Write x4 at the top.

For the left hand side:

Start with 1

Double 1 to get 2. **Double** 2 to get 4.
Double 4 to get 8
Those are the doubling facts.

Stop and think.

The other easy number fact is x10
So write 10.

For the right had side:

This is the 4 times table.
1 repeated 4 times is 4.
Double 4 to get 8. **Double** 8 to get 16.
Double 16 to get 32

Stop and think.

The other easy number fact is x10
So 4 x10 = 40.

Now you can use the x4 clue board to work out all the x4 multiplication facts!

×4	
1	4
2	8
4	16
8	32
10	40

1 + 2 = **3**

4 + 8 = **12**

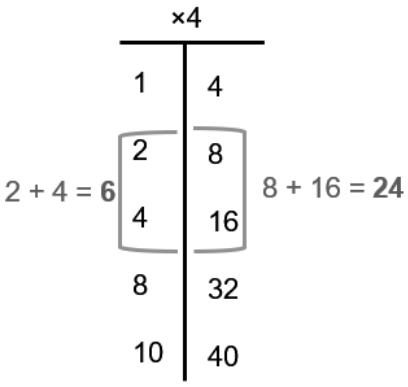
Therefore, 3 × 4 = 12

×4	
1	4
2	8
4	16
8	32
10	40

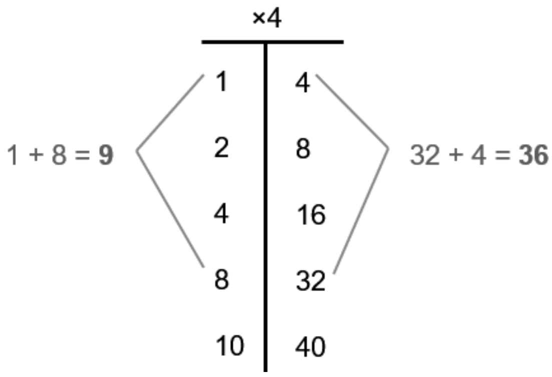
1 + 4 = **5**

4 + 16 = **20**

Therefore, 5 × 4 = 20
5 is also half of 10.
10 × 4 = 40, so 5 × 4 = half of 40 = 20



Therefore, 6 × 4 = 24
6 is also 5 + 1
5 × 4 = 20, therefore 6 × 4 = 20 + 4



Therefore, 9 × 4 = 36
9 is also 10 -1
10 × 4 = 4, therefore 9 × 4 = 40 – 4 = 36

The **clue board** is a very useful scaffold for learning the times tables and working flexibly with multiplication and division. **It does not replace knowing the multiplication number facts fluently**. Children need to know multiples ad factors up to 10 × 10. This is very important for their maths in high school. The clue-boards helps to learn the number facts.

Activity for learners

To learn the number facts for ×4 in Lesson Starters.

Practice counting in 4s on the multiples:

- **Group counting**. We are counting forwards in 4s. Start at 4. Stop after 8 counts. Keep track on your fingers. 8 × 4 = ...?
- We are counting forwards in 4s. We want to reach a target of 24. Keep track of the 4s on your fingers. ... × 4 = 24? Therefore 24 ÷ 4 = ...?

Now let children make ×4 a clue board.

They can look at their clue board for all these tasks:

Fizz-pop

» The rule is times by 4. The rule is repeat 4 times. The rule is groups of 4 etc
» The rule is divide by 4 (use multiples of 4). The rule is share between 4. The rule is pack into boxes with 4 in each box. etc.

- **I am thinking of a number.** Four times my number is 16. What is my number? Four times my number is ... etc.
- **Games in pairs:** Use a pack of 1-10 cards. Bean shuffles the cards. Bean turns over one card at a time. Samp multiplies the card by 4. How fast can you go?
- **Swop roles:** 123 Veza! Bean shows fingers on one hand. Samp multiplies by 4. Repeat 10 times then swop over. Now do with two hands.

Do these × 4 lesson starter tasks for a whole week (or more).
When most of your class are fluent, repeat it but this time they can't use a clue board.

Link it all together

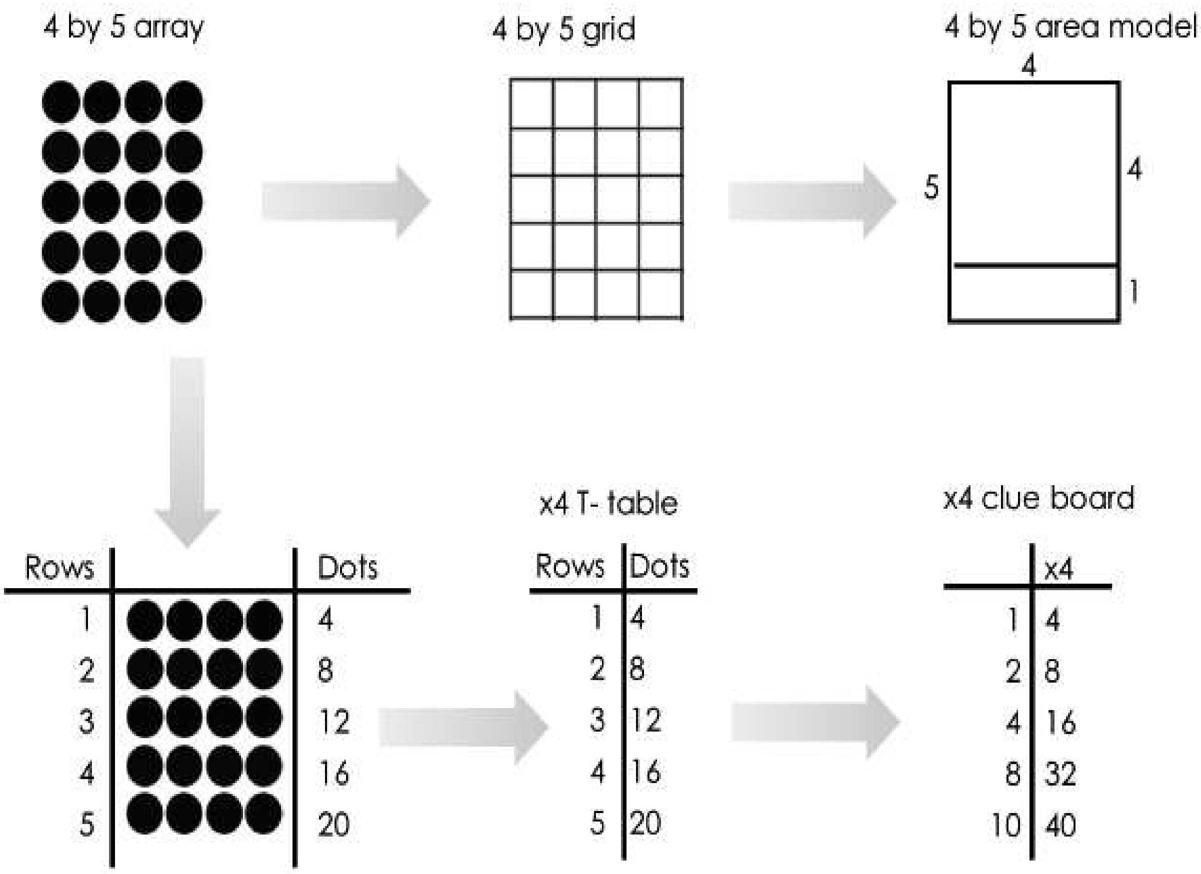
A first important connection is that **multiplying and dividing are inverses**.

Multiplicative thinking uses **multiplying and dividing:**

- **Multiplication** brings equal groups together. Group count. Count forwards in groups. Keep track of the groups on your fingers.
- **Division** separates into equal groups. Group count.
  Count forwards in groups to reach a target. Keep track of the groups on your fingers.

Children can **group count** to solve multiplication calculations. They count forwards on the multiples. They keep track of the number of groups on their fingers.

A second important connection is that **all the representations for multiplying and dividing fit together.** We build them up slowly from Grade R to Grade 6.



In **Foundation Phase** children are encouraged to **make their own drawings**. They arrange cubes. They are exposed to multiplicative representations by their teachers. They work on problems about planting (in arrays). They work on problems about sharing and packing. They work with arrays, and grids and then areas.

In **Intermediate Phase** children work with **arrays, and t-tables and clue boards**. Over time the representations become familiar. They are not always presented to children, rather, the children think about them as a tool to use for problem solving and multiplicative thinking.

PRACTICE

Tasks 3.1.1

Recording multiplicative thinking

Answer these word problems by showing your working. You should show at least 1 way of answering. Create a memorandum for the strategy and representation that you use.

1.a **Learners sell bags of mangoes during market day.**

They put 6 mangoes in each bag.
How many mangoes will they need to fill 9 bags?

They need ______mangoes.

1.b. **How you can change this word problem about mangoes to be an investigation.** Adapt this question by asking What if? questions.

You can help children by drawing a horizontal table or a t-table for them to complete…

2. A class of 56 children go out to play games.

They get into teams of 8.
How many teams do they make?

They make ______teams.

3. The total weight of 7 same-sized bags of potatoes is 63 kg.

What is the weight of one bag of potatoes?

The weight of one bag is ______ kg.

4. Corin puts out 81 pies on one tray and 19 pies on another tray.

How many pies does Corin put out all together?

Corin puts out ______ pies.

5. Hamsa is counting how many tiles cover the bathroom floor.

She counts 12 rows of tiles.
There are 11 tiles in each row.
How many tiles cover the floor?

There are ______ tiles.

6. Zwi collected 8 bottles for recycling.

Her friend collected 6 times as many bottles as Zwi.
How many bottles did the friend collect?

Her friend collected ______ bottles.

7. 105 people are going on a car trip.

Each car can carry 7 people.
How many cars will the people need?

They ______ need cars.

8. One bar of chocolate costs R12.
How much will 19 bars of this chocolate cost?

The cost is R ______ .

9. A baker puts pies in rows on a tray.
He puts the same number of pies in each of 8 rows.
The baker has 128 pies altogether.
How many pies must he put in each row?

There are ______ pies in each row

10. Joey bought 108 metres of wire to put up a fence.
This is 6 times more wire than Peter bought.
How much wire did Peter buy?

Peter bought ______ metres of wire.

11. Sam cycles 112 km.
Sameera cycles 99 km.
How much further does Sam cycle than Sameera?

Sam cycles ______ km further.

12. A company packs pencils into boxes.
Each box contains 15 pencils.
How many boxes are needed to pack 195 pencils?

______ boxes are needed.

13. Viren has some pencils.
His friend gave him 12 pencils.
Now Viren has 144 pencils.
How many pencils did Viren start with?

Viren started with ______ pencils.

14. Denozo cycles at 14 km per hour.

How many hours will it take for Denozo to travel 84 km?

It takes ______ hours.

15. A farmer is planting rows of apple trees.

He has 144 apple trees.
He plants 12 trees in each row.
How many rows will there be?

There are _____ rows of apple trees.

16. Now look back at all the word problem you have worked on in questions 1-15. Classify the word problems by showing:

- Whether they are additive or multiplicative
- Use discrete objects or continuous quantities that cant be measured.
- Whether a factor or a product is the unknown:
  - » factor × factor = product
  - » product ÷ factor = factor
- Whether there is a remainder or a fraction in the answer.

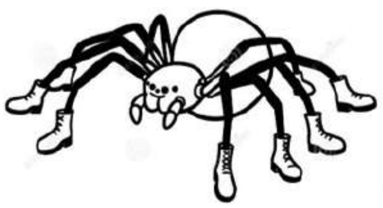
	Context or situation	Multiplicative or Additive?	Discrete or Continuous	Unknown is the product or a factor?	Reminder or fractional part
1	Pack mangoes	M	D	9 × 6 = ____ Unknown Product	n/a
2	Teams of children				
3	Bags of potatoes				
4	Corin puts out pies				
5	Hamsa counts tiles				
6	Zwi collects bottles				
7	People on a car trip				
8	Bars of chocolate				
9	A baker and pies				
10	Joey, Peter and the wire				
11	Sam cycles				
12	Pencils in boxes				
13	Viren's pencils				
14	Denozo cycles				
15	Farmer plants apple trees				

Tasks 3.1.3

Timed tango ×9

1	6 × 9 = ______	**11**	______ = 7 × 9
2	______ × 6 = 48	**12**	9 × ______ = 90
3	9 × ______ = 90	**13**	______ = 82 ÷ 9
4	9 × 3 = ______	**14**	63 ÷ 9 = ______
5	10 × 9 = ______	**15**	______ ÷ 9 = 7
6	______ = 7 × 9	**16**	______ = 18 ÷ 9
7	8 × 9 = ______	**17**	1 ninth of 36 = ______
8	______ × 9 = 45	**18**	5 × 9 = ______
9	Factorise: 72 = 9 × ______	**19**	Factorise: 27 = _____× 9
10	Circle all the multiples of 9: 9; 27; 45; 53; 207; 208	**20**	The factor pairs for 9 are: _____& _____ and _____&_____

Create a clue board for spiders and spider shoes.



I know a number is divisible by 9 when...

The factor pairs of 9 are ....

Therefore 9 is prime/composite.

9 written as a product of prime numbers is _____________.

9 multiplied by 0 =

0 multiplied by 9 =

9 divided by 0 =

0 divided by 9 =

PRACTICE

Tasks 3.1.2

Arrays, t-tables and clue boards

1. Draw an array with 3 dots across in the first row. Repeat to make 5 rows.

Circle to show that the **unit you repeat is 3**

Complete these number sentences:

3 repeated _____ times = ________

_____ × _____ = _____

______ groups of 3 = _______

2. Draw an array with 5 dots going down in the first column. Repeat to make 3 columns.

Circle to show that the **unit you repeat is 5**

Complete these number sentences:

5 repeated _____ times = ________

_____ × _____ = _____

______ groups of 5 = ______

3. Zibalo is arranging the potatoes she collected from her garden. She picked 24 potatoes. Draw a neat array of 24 potatoes. How many different arrays for 24 potatoes can you draw?

4. Don't give the answer. Just re-write the ÷ calculation as a '×' calculation.

Working out 12 ÷ 3 =... is the same as working out 3 x... = 12

Working out 20 ÷ 4 =...... is the same as working out ______________________

Working out 9 ÷ 3 =...... is the same as working out ______________________

Working out 8 ÷ 2 =...... is the same as working out ______________________

5. Eggs are packed with 18 eggs in each box.

Make a clue board for ×18.

Then complete the other table to show
- how many boxes for 'my number' of eggs.
- how many eggs fill 'my number' of boxes.

boxes	eggs
1	18

boxes	eggs
5	
20	
	72
12	
18	
	720

2 × 18 =

72 ÷ 18 =

8 × 18 =

900 ÷ 18 =

Word problems and using arrays, t-tables and clue boards

Draw all the representations that you couild use to help children learn about x 7.
Make sure you include a real or imaginable context

Explain times as 'groups of' for 5 × 2, and divide as 'grouping or packing' for 10 ÷ 2

For Intermediate Phase, we add in:

- Scaling (to make bigger or smaller)
- Repeating patterns (investigate number patterns with a common difference)
- Combinations (cartesian product)

Choose one of these problem types, and create an example word problem.

UMTHAMO 3.2

Distributive property, area model and division by chunking

Key ideas

1. **Distributive property**:

   a. You can **break up a number and multiply each part.** For example:

   $23 \times 5 = 20 \times 5 + 3 \times 5$ or
   $23 \times 5 = 10 \times 5 + 10 \times 5 + 3 \times 5$
   $23 \times 5 = 18 \times 5 + 5 \times 5$

   b. In other words, **when you multiply two factors, you can partition one of the factors** (break up the factor using addition or subtraction).

   $\text{factor}_1 \times \text{factor}_2 = \text{multiple}$
   $\text{factor}_1 \times \text{factor}_2 = \text{product}$
   $\text{factor}_1 \times (a + b) = \text{factor}_1 \times a + \text{factor}_1 \times b = \text{product}$

   c. You can **break up a number and divide each part**.

   $27 \div 5 = 20 \div 5 + 7 \div 5$ or
   $27 \times 5 = 10 \div 5 + 10 \div 5 + 7 \div 5$
   $27 \times 5 = 15 \div 5 + 10 \div 5 + 2 \div 5$

2. **Factorising** (finding the factor pairs, to find all factors)

- You can **factorise** a number to simplify the multiplication. For example

  $7 \times 4 = 7 \times 2 \times 2$. Multiply by 4 is the same as double, then double.

  $7 \times 6 = 7 \times 2 \times 3$. Multiply by 6 is the same as double, then triple

  $7 \times 8 = 7 \times 2 \times 2 \times 2$. Multiply by 8 is the same as double, then double, and double again.

- When you multiply two factors, you can **factorise** one or both factors to simplify the problem. Factorise means split into smaller factors.

  $\text{Product} = \text{factor}_1 \times \text{factor}_2$

The distributive property

Watch this maths-is-visual silent video, which introduces the distributive property.

SCAN ME

TASK

Can you work out the number of windows in:

- The 10 by 10 building? ____________________
- The 10 by 3 apartment building? ____________________
- The 6 by 3 building? ____________________
- The 6 by 10 building? ____________________

How do you use that information to work out the windows in the 16 by 13 building?

Windows in tall buildings are good visual context for arrays. **Arrays grids and areas** are helpful for visualising multiplication with big numbers.

Mix and match: Jackets and pants

A shop is selling jackets and trouser sets to mix and match.

A set (of 1 pair of trousers and 1 jacket) costs: R900

One pair of trousers cost R350, and one jacket costs: R450

TASK

Calculate:

1. How much do you pay for 4 sets.?

2. How much do you pay for 4 jackets?

3. How much do you pay for 4 pairs of trousers?

4. What do you notice?. Can you write an equation which states the relationship between the cost of the set, and the cost of the jacket and cost of the pair of trousers?

You can break up a set, and pay for each part separately. It makes no difference to the price. This is how the **distributive property** for multiplication works too. When you are multiplying, you can break up the number, and multiply each part and it makes no difference to the total.

PrimTEd Std. NA 4.2
Demonstrate knowledge of equivalent relations, in general, and equality in particular, relating to multiplication and division problems.

Correct or incorrect?

Double 3 + Double 7 = Double 10

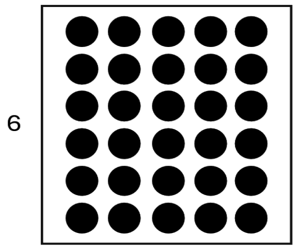
Double 4 + Double 7 = Double 21

Double 12 - Double 4 = Double 8

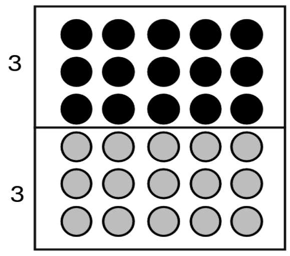
Double 12 - Double 4 = Double 3

Children can explore the distributive property and look for patterns of multiplication facts using arrays.

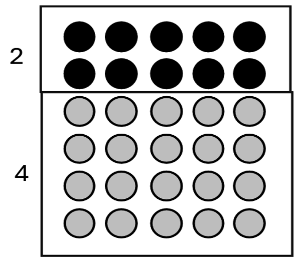
Example



Rows	Tomatoes
1	5
6	30



Rows	Tomatoes
1	5
3	15
3	15



Rows	Tomatoes
1	5
2	20
4	40

In this example the number fact 6 × 30 can be explored by breaking up the 6 in different ways. 6 = 3 + 3 or 6 = 2 + 4. We can break up the 6 in different ways. We multiply each part by 5 and add them. We always get the same result.

T-tables help to keep track of the relationships. You first define the unit that is repeating:

- 1 row has 5 tomatoes

You can break up the number of rows in many ways.

- 6 rows = 3 rows + 3 rows, or
- 6 rows = 2 rows + 4 rows, or
- 6 rows = 1 rows + 5 rows.

Break up flexibly and multiply each part. Do you always get the same result?

- 6 rows has 30 tomatoes

PrimTEd Std. NA 4.6
Describe and use multiplciation and division problem types and the range of useful associated representations.

Using the distributive property to multiply

You can break up a number and multiply each part, then add together again.

Example

24 × 6 =

$$\begin{array}{l} 20 \times 6 = 120 \\ \underline{+\ 4 \times 6 \ \ = 24} \\ \qquad\quad 144 \end{array}$$

break up 24 = 20 + 4

24 × 6 =

$$\begin{array}{l} 10 \times 6 = 60 \\ +\ 10 \times 6 = 60 \\ \underline{+\ 4 \times 6 \ \ = 24} \\ \qquad\quad 144 \end{array}$$

break up 24 = 10 + 10 + 4

15 × 7 =

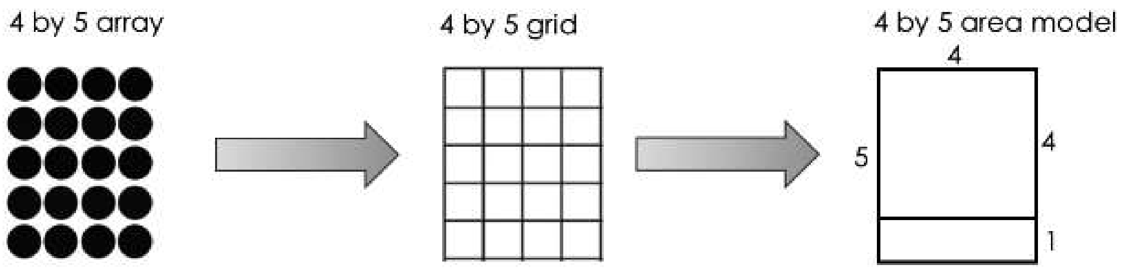
$$\begin{array}{l} 10 \times 7 \ = 70 \\ \underline{+\ \ 5 \times 7 = 35} \\ \qquad\quad 105 \end{array}$$

break up 15 = 10 + 5

Area model for multiplication

For medium and large numbers it is harder to keep track of the longer calculations. As a teacher it is important that you can work flexibly with different representations. You must support children to be flexible. In this section you will work on multiplication using an **area model**.

You have already worked with arrays, and with grids, and with the area (or a rectangle) model:

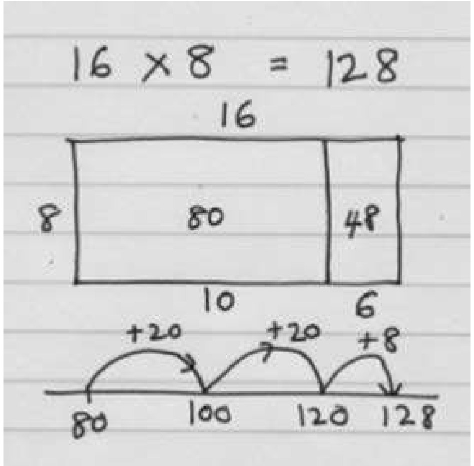


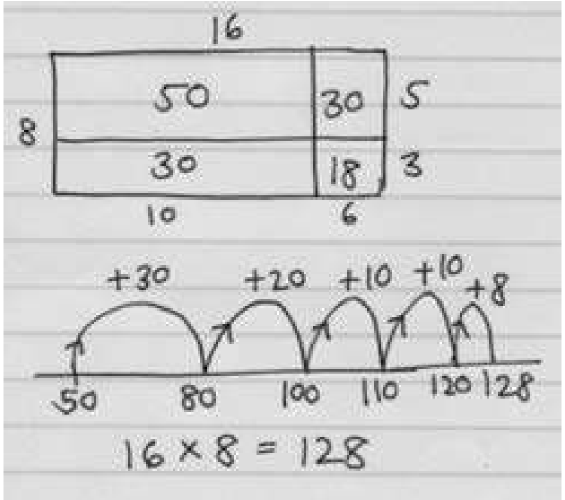
Now we will work with using the area model for medium and large numbers.

Example

Example: 16 x 8

The area of a rectangle model can be used to do multiplication with bigger numbers.
The two rectangles below show two different ways in which the area model can be used to calculate 16 x 8.



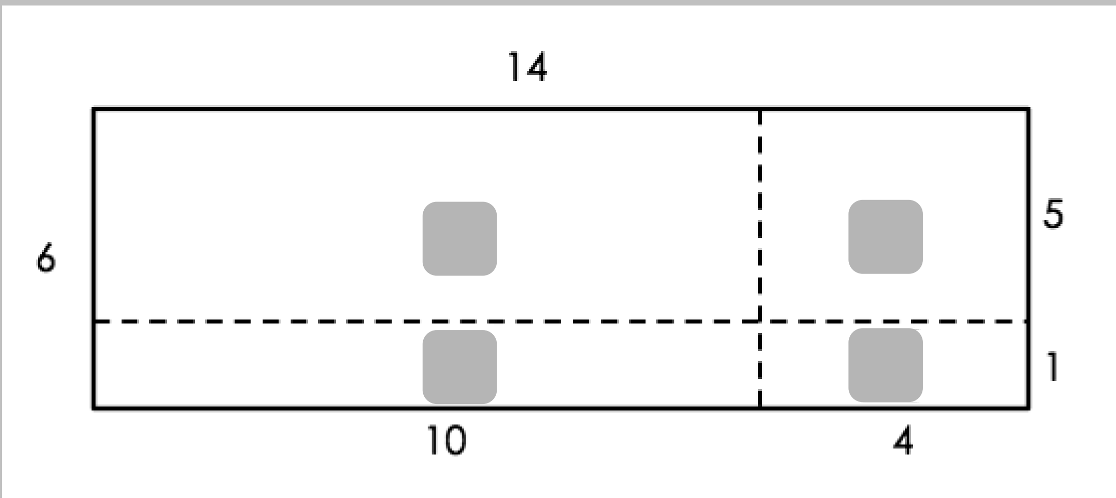


Example

Example 14 x 6

This rectangle showing 14 x 6, is an example of the area of a rectangle model.
The area or a rectangle model can be used to do multiplication with bigger numbers.

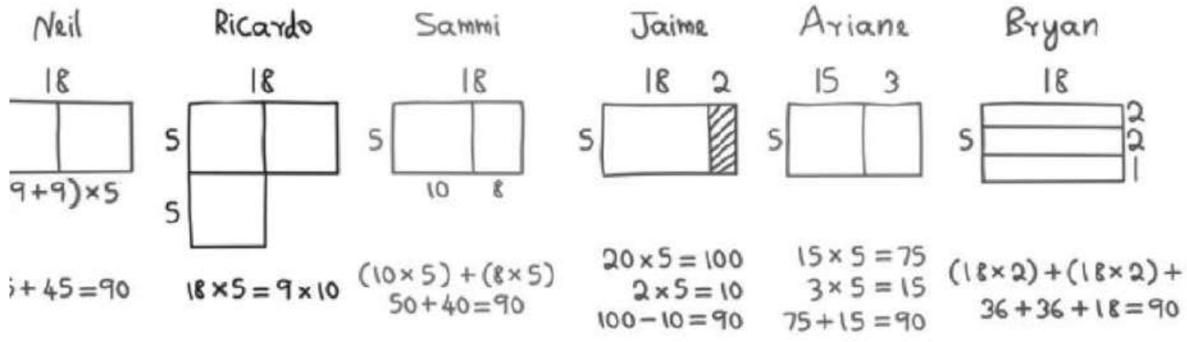
This is an example for 14 across by 6 down (14 x 6).
Work with each smaller array first: 10 by 5 and 4 by 5, 10 by 1 and 4 by 1. Add together all the arrays to get the big array.



14 x 6 =

Look at all the different ways that learners worked on 5 x 18 using area models

How do you work out 5 x 18 without a calculator?



from Jo Boaler. Mathematical Mindsets

SCAN ME

SCAN ME

Watch these short videos about the number talk for 5 x 18

Using the distributive property to divide

You can break up a number and divide each part, then add together again.

Example

24 ÷ 6 =

$$\begin{array}{r} 12 \div 6 = 2 \\ \underline{+\ 12 \div 6 = 2} \\ 4 \end{array}$$

break up 24 = 12 + 12

35 ÷ 5 =

$$\begin{array}{r} 30 \div 5 = 6 \\ \underline{+\ 5 \div 5 = 1} \\ 7 \end{array}$$

break up 35 = 30 + 5

Example

35 ÷ 5 =

  10 ÷ 5 = 2

\+ 10 ÷ 5 = 2

\+ 10 ÷ 5 = 2

\+ 5 ÷ 5 = 1

  7

break up 35 = 10 + 10 + 10 + 5

Dividing each part and adding together again, is the same as dividing the whole

It is easier to break up a number using known facts.
There is then only one remainder at the end.

So being able to quickly find known facts/ multiples is an important skill. In the next section you will learn how to use T-tables and clue boards help with this.

Division by chunking

Example

75 ÷ 3 =

Use expanded notation

70 ÷ 3 =

5 ÷ 3 =

75 ÷ 3 =

75 ÷ 3 =

Use known facts about ×3

30 ÷ 3 =

30 ÷ 3 =

15 ÷ 3 =

75 ÷ 3 =

89 ÷ 6 =

Use expanded notation

80 ÷ 6 =

9 ÷ 6 =

89 ÷ 6 =

89 ÷ 6 =

Use known facts about ×6

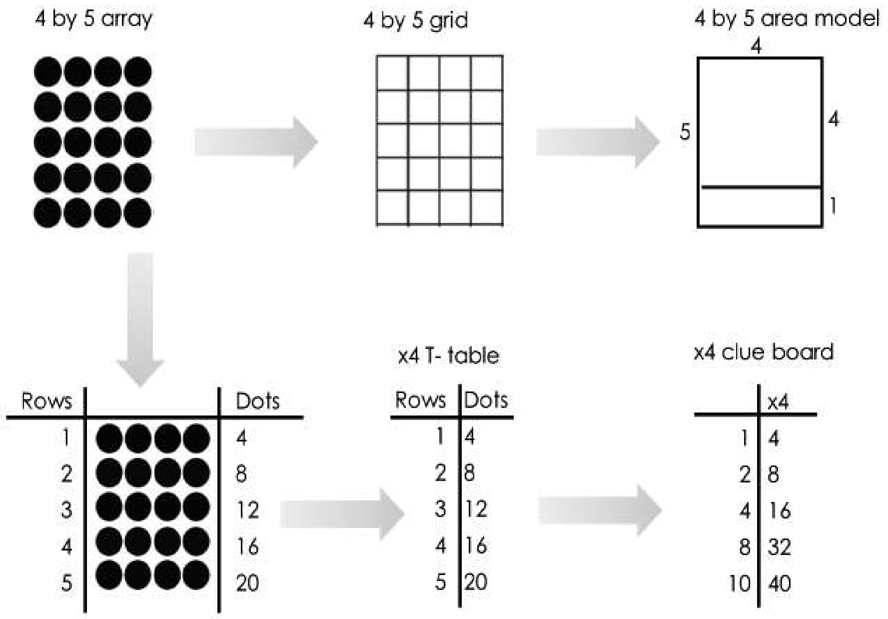
60 ÷ 6 =

24 ÷ 6 =

5 ÷ 6 =

89 ÷ 6 =

Bring the representations together: Specialize with arrays, grids, areas and generalise with rows and dots, t-tables and clue board



Multiplication and division can be represented in many different ways.
Here we discuss 6 of these representations.

When we **specialise** we think about a particular or a special case. The top of this diagram shows the special case of 5 multiplied by 4. This same example (5×4) is shown in three different ways: as 4 by 5 **array**, as a 4 by 5 **grid** and as a 4 by 5 **area** model. You can use an array, a grid or a rectangle with its area to picture any multiplication fact.

When we **generalise**, we start to explore the patterns involved in a whole timestable, and beyond. So instead of just looking at the example of 5×4, we think about "any number × 4". To explore the 4 times table in this more general way, we need ways to record the groups of 4. We can label the array counting the rows and total number of dots in an array. Each time, we are thinking about many examples of × 4:

- 1 row has 4 dots
- 2 rows as 8 dots
- 3 rows has 12 dots, etc.

We are looking for patterns. We can record this pattern in a t-table – which lists each number of rows in order. The clue board is then a summary version of the t-table. The clueboard can be used to work out all the facts in the t-table.

What's the difference between a T-table and a clue board?

- The **t-table** lists all the multiples of the times-table.
  The rows in the array are counted in ones (1,2,3,4...).
- The **clue board** only uses doubling and x10 facts.
- The rows in the array are doubled three times: (1, 2, 4, 8), then the known fact of x10 is included.

Long division

Grade 5 learners use different strategies to solve a long division problem: 489 ÷ 13 = ... They all use the distributive property. They break down the big number into parts, and divide each part.

Maria

489 ÷ 13 =

20 × 13 = 260
10 × 13 = 130
5 × 13 = 65
2 × 13 = 26
37 × 13 = 481
rem 8

×13

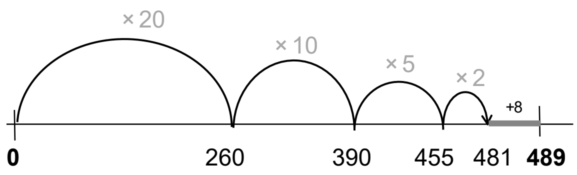
Jumps	Products
1	13
2	26
4	52
8	104
10	130
5	65

Qetelo

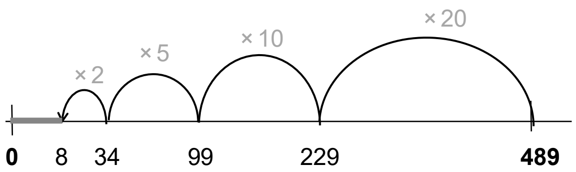
489 ÷ 13 = 20 + 10 + 5 + 2 **rem 8**

489
- 260 ... 20 × 13
229
- 130 ... 10 × 13
99
- 65 ... 5 × 13
34
- 26 ... 2 × 13
8

Kenya



Solly



Jo

489 ÷ 13 = 30 + 6 + 1 rem 8

400 ÷ 13 = 30 rem 10

80 + 10 = 90
90 ÷ 13 = 6 rem 12

9 + 12 = 21
21 ÷ 13 = 1 rem 8

Jamie

13) 489
390 ... 30 × 13
99
91 ... 7 × 13
8

489 ÷ 13 = 37 rem 8

Connie

3 7 rem 8
13) 489
39
99
91
8

13 × 3 = 39

13 × 7 = 21 + 70 = 91

TASK

1. Which learner(s) added up groups of 13, to reach a target of 489?
2. Which learner(s) subtracted groups of 13 from 489?
3. Which learner(s) broke up 489 using place value. 489 = 400 + 80 + 9
4. Which learner(s) broke up 489 using multiples of 13?

PRACTICE

Tasks 3.21

Flexible multiplication and division

1. Show how you can break down the first number in different ways to more easily do the calculation.

   $16 \times 5 =$

   $32 \div 2 =$

   $24 \times 3 =$

   $45 \div 5 =$

2. Show the difference between breaking down a number in expanded notation, and breaking down a number using known multiples.

85 ÷ 3 =

Use expanded notation

80 ÷ 3 =

5 ÷ 3 =

85 ÷ 3 =

85 ÷ 3 =

Use known facts about ×3

60 ÷ 3 =

24 ÷ 3 =

1 ÷ 3 =

85 ÷ 3 =

99 ÷ 6 =

Use expanded notation

90 ÷ 6 =

9 ÷ 6 =

99 ÷ 6 =

99 ÷ 6 =

Use known facts about ×6

60 ÷ 6 =

30 ÷ 6 =

9 ÷ 6 =

9 ÷ 6 =

83 ÷ 6 =

Use expanded notation

83 ÷ 6 =

Use known facts about ×6

3. Make a clue board for x 7

2019 FEBRUARY

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28		

www.calendar-to-print.com

Use your clue boards to work out:

Weeks	days
1	7
2	
4	
8	
10	
5	

weeks	**days**
10	
	28
3	
30	
18	
	350

2 × 7 =

56 ÷ 7 =

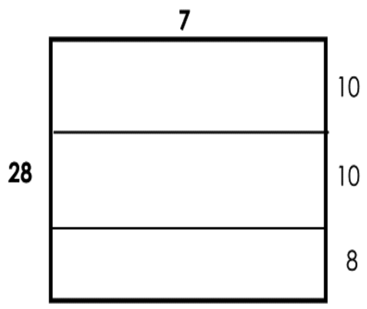
8 × 7 =

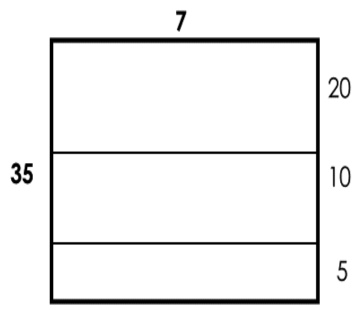
560 ÷ 7 =

800 × 7 =

5 600 ÷ 7=

4. Calculate 28 x 7 and 35 x 7 using these rectangles





5. Practice doing multiplication with clue boards and/or areas

23×6
41×7
73×4

6. Work out 18 x 35 in different ways

7. Calculate 35 × 17

35 x 17 =

17

35	10	7	
	200	140	20
			10
			5

Show a different way to work out 35 × 17

8. Stickers come in sheets of five. How many stickers could I have altogether?
   a. a. Circle the correct answers.

   40 105 52 5 75 90

   b. What is the divisibility rule for 5?

PRACTICE

Tasks 3.2.2

Division with remainders or with fractions

1	1 quarter means 1 part out of _____ equal parts.	11	You fold a strip of paper into 8 equal parts. Each part is 1 _____.
2	30 divided by 10 = ______	12	1 _____ of 30 = 3
3	36 = _____ tens and _____ ones	13	31 = _____ tens and _____ tenths
4	1 half of 9 = _____ and ____ half	14	1 tenth of 40 is ....
5	1 half of 31 = _____ and 1	15	1 tenth of 41 is 4 and 1 _____.
6	15 = _____ twos and _____ ones	16	1 half of 15 = _____.
7	Avos are packed with 5 in each box. 15 avos, need _____ boxes	17	Avos are packed with 5 in each box. 16 avos, need _____ boxes. The last box is only _____ full
8	1 fifth of 30 = _______	18	16 divided by 5 = _____ rem ______.
9	 10 cups fill 1 jug. 22 cups = ______ jugs and _____ cups	19	 10 cups fill 1 jug. 5 cups = ______ jug(s)
10	36 divided by 5 = _____ rem ______.	20	36 divided by 5 = ______ and ____ fifth.

Notice how **division links to fractions**:

- - '12 divided by 2' is the same as 'find 1 half of 12'
- - 13 divided by 2 = 6 and 1 left over OR 6 and 1 half

AND

- - '30 divided by 10' is the same as 'find 1 tenth of 30'
- - 31 divided by 10 = 3 and 1 left over OR 3 and 1 tenth.

1. Work on different ways to learn long division, with meaning, to calculate: 678 ÷ 15

a. Make a clue board for ×15.

b. Calculate 678 ÷ 15 by using the clue board. **Add up multiples of 15 to reach a target** of 678. You could use Maria's or Kenya's representation.

c. Calculate 678 ÷ 15 by using the clue board. **Subtract multiples of 15 from 678**. You can use Qetelo's or Solly's or Jamie's representation.

d. Calculate 678 ÷ 15 by **breaking up 678 using place value**. You can use Jo's or Connie's representation.

Distributive property, area model and division by chunking

First choose one example from your times table (**specialise**).

I choose the ________________ time table.

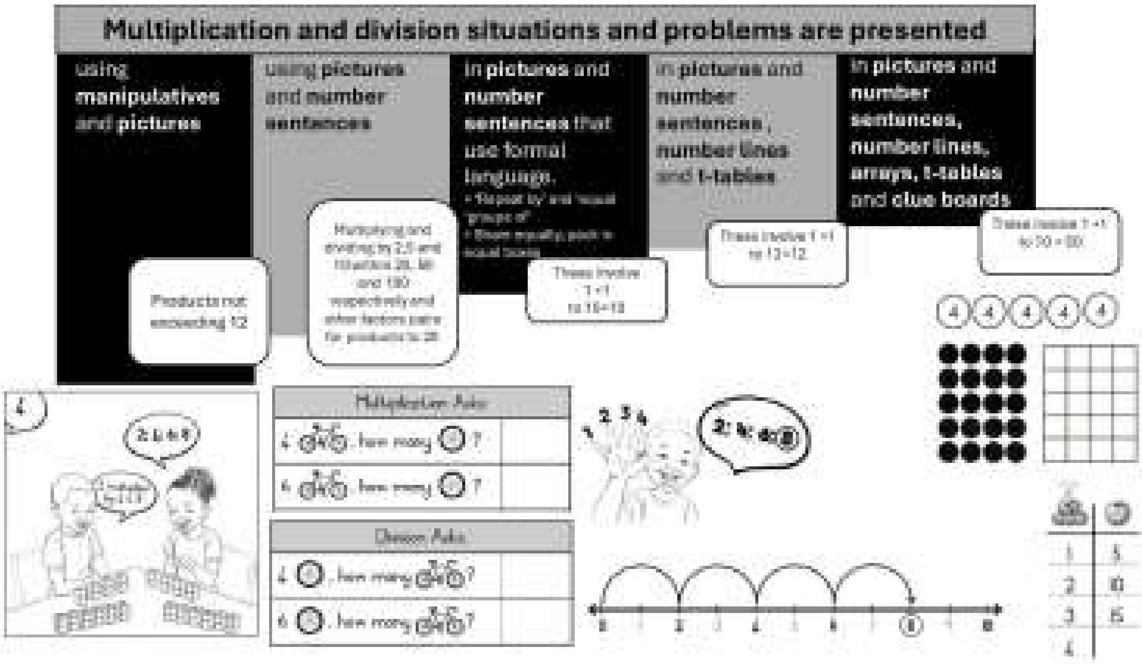
Show all the ways to represent one fact from your times table

Array	Grid	Area model

Now **generalise**. Show many examples from your times table, and show how to record the different number facts from that times table.

Rows and dots	T-table	Clueboard

Maths4Primary Teachers is a learning series for Bachelor of Education students in Foundation and Intermediate Phase specialisations. Each module is aligned to the **Primary Teacher Education (PrimTEd)** standards) which have been agreed as minimum core content for teachers of mathematics in primary schools in South Africa.



Multiplying and dividing answers questions like:

- You can skip count forwards for multiplication. Can you also skip count forwards for division?
- Is division about sharing equally, or packing into equal boxes?
- How do you help children learn their times tables?