

MATHS4

Primary teachers



Two pathways to number

2024

MATHS 4 PRIMARY TEACHERS PRACTICE WORKBOOK

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Maths4Primary teachers is a series which was originally funded by Zenex Foundation. The **Two pathways to number** module is designed for teachers in schools.

Its development is a collaboration between Kelello and the Nelson Mandela Institute. It is a professional development companion to **Magic Classroom Collective series of Lesson starters**. Lesson starters are week-by-week instructional materials focused on mental mathes, for teachers on Grades 1-3.

M4PT **Two pathways to number** and MCC **Lesson starters** are used by JumpStart which provides teacher training and school-based coaching.

2024 First publication



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Suggested citation:

Roberts, N. & Porteus, K. (2024) *Two Pathways to Number: Maths4Primary teachers*: Kelello Consulting and Nelson Mandela Institute

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Maths4Primary teachers: Two pathways to number

This is a learning guide companion to the **Magic Classroom Collective Lesson Starters**

Target: Foundation Phase educators in mathematics
Designed and developed by



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UNIT 1: Introduction

Two pathways to number focuses on **how young children** (from babies to 5 and 6 year olds) **learn mathematics**. Children make meaning from the world around them. They come to school (even pre-school/creche and grade R) with many mathematical experiences from their life experience. Over time they are inducted – learning from adults and their peers – into a tradition of more formal mathematics.

Key Ideas

This umthamo has the following key ideas:

1. Explain **what mathematics is**, and agree on some **ground rules (classroom norms)** for this maths course.
2. Know **how to succeed in this maths course**: what to do each week and how you are assessed.
3. Know that for mathematics to make sense, we need **a clear mental map or picture** for numbers and relationships. There are **two main pathways for learning about numbers**:
 - a. A **discrete pathway**; and
 - b. A **continuous pathway**.

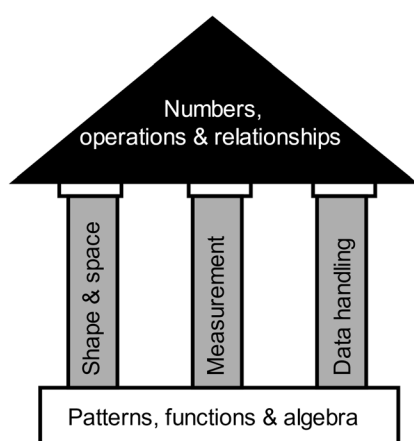
What is mathematics?

We know that many students taking this course have not liked mathematics at school. Many students find maths difficult and stressful. We believe everyone can do mathematics. Even if a person has been taught badly, and hated maths before, with effort and opportunity that person can do maths.

But, what is mathematics? Are we all talking about the same thing when we say “maths”?

Mathematics involves thinking about and working with numbers, shape and space.

The South African Curriculum and Assessment Policy (CAPS) identifies five content areas for primary school mathematics.



Numbers operations and relationships is the focus in primary school. it is given the most time

- Numbers can be whole, natural, rational or integers;
- Operations are add, subtract, multiply and divide; and
- Relationships include equal, unequal contexts, more than and less than.

There are 3 contexts in which to use numbers, operations and relationships:

- Shape and space
- Measurement and
- Data handling.

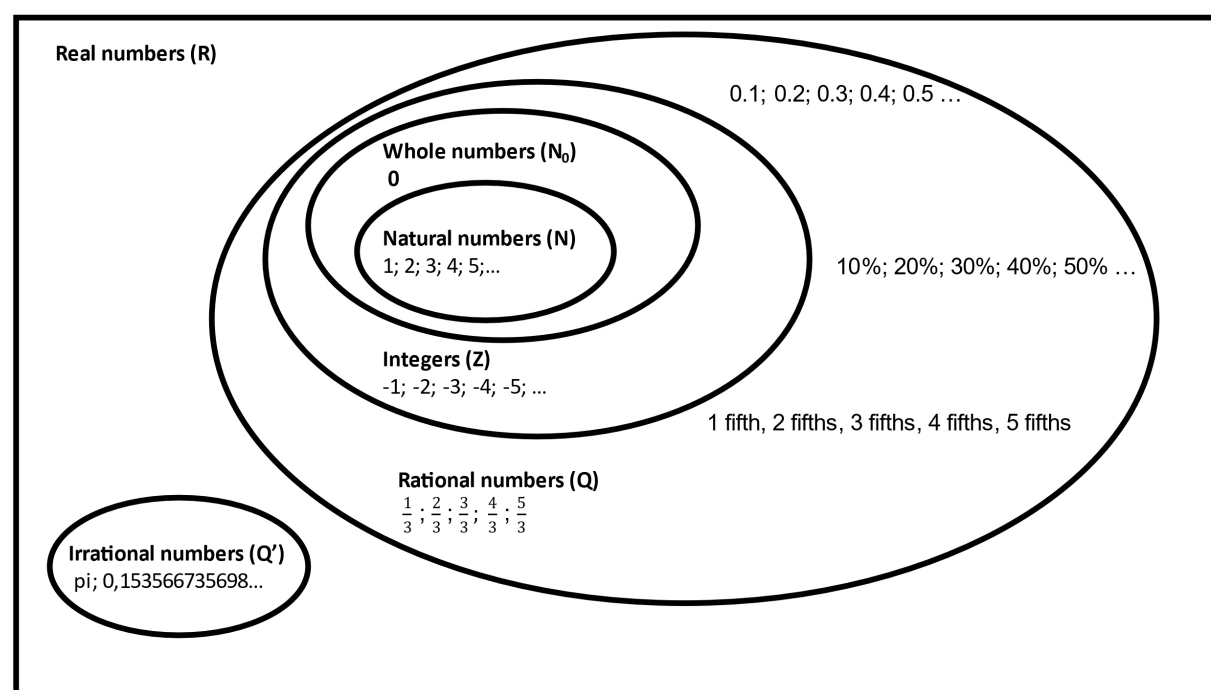
Patterns functions and algebra is the foundation. It is the way of learning about numbers, operations and relationships in the 3 different contexts.

The South African curriculum and assessment policy (CAPS) defines mathematics as follows:

Mathematics is a language that makes use of symbols and notations for describing **numerical, geometric and graphical relationships**. It is a **human activity** that involves observing, representing and investigating **patterns and quantitative relationships** in physical and social phenomena and between mathematical objects themselves. It helps to develop **mental processes** that enhance logical and critical thinking, accuracy and problem-solving that will contribute to decision-making. (DBE, 2011).

As teachers you need to know the **different types of numbers** which children encounter in primary and secondary school. This diagram shows how the different **number systems** fit together.

Diagram: Number systems



This course is designed to help you to learn the mathematics you need to teach primary school children, well. Learning mathematics involves learning ways of thinking. Carpenter, Franke et al. (2003) write that learning mathematics

involves learning **powerful mathematical ideas** rather than a collection of disconnected procedures for carrying out calculations. But it also entails **learning to generate those ideas, how to express them using words and symbols, and how to justify to oneself and others** that those ideas are true. (Carpenter, Franke et al. 2003, p. 1)

Learning mathematics is learning **ways of thinking** about **quantities (discrete objects/events and continuous quantities which we can measure)**. Learning mathematics is **learning ways of thinking** about **shapes**, their arrangement, how they are positioned and can change (transform) in **space**. Once you have thought about your relationship with maths (how it makes you feel, right now) and you have agreed on what maths actually is, it may be worth working on some classroom norms. How do you want to be treated, and how will you agree to behave in this maths class? What kind of a maths teacher do you want to be?

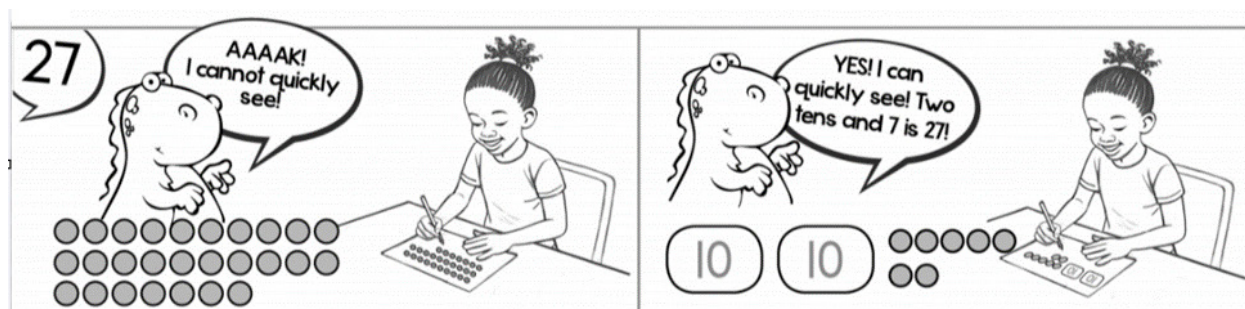
In this module we focus on how to teach children to make sense of numbers. In South Africa, many children imagine numbers as a loooooong string of 1s. They calculate in 1s, like this:



But this is very inefficient.

We **DON'T want** learners to draw number pictures as a long string of 1s!

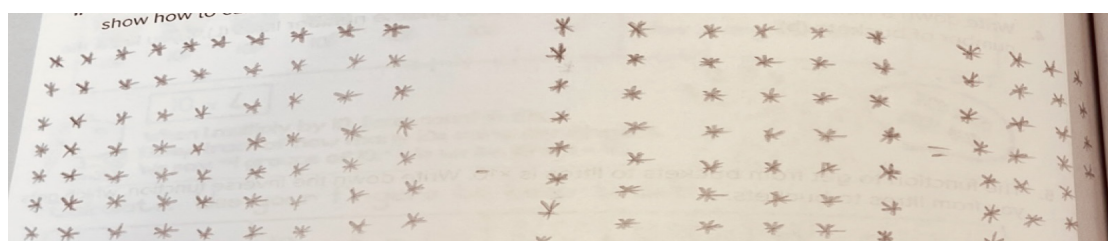
We **DO want** learners to use number symbols and structure into 100s, 10s and 1s.



As teachers, we help learners structure their number pictures into 100s, 10s and 1s.

As a teacher you should **ALWAYS structure** number pictures into 100s, 10s and 1s.

We **NEVER** want to see teachers working like this:

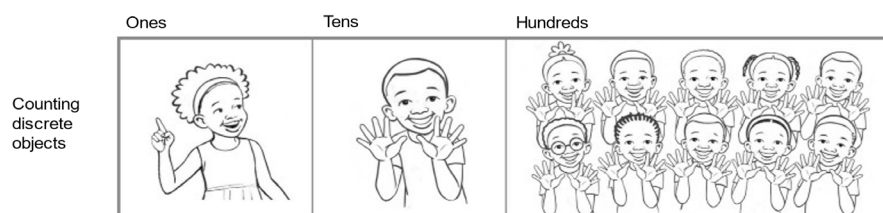


Building mental maps or pictures

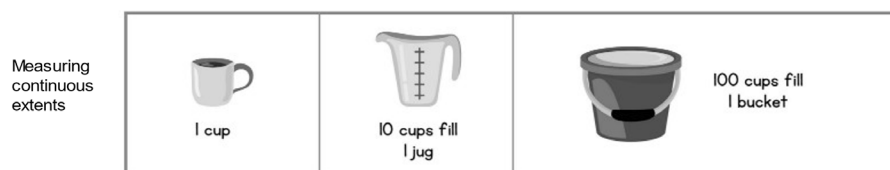
Many of us have learnt maths by memorising. Many of us don't have a mental map for maths which goes beyond counting in ones. **Our first job in this course to see maths as meaningful.** For maths to make sense we each need a coherent mental map or picture. **Our second job in this course is to help children build meaningful mental maps for learning mathematics.**

There are two main routes to developing children's number sense:

- **A counting pathway** where the starting point is discrete quantities (discrete objects or discrete events); and

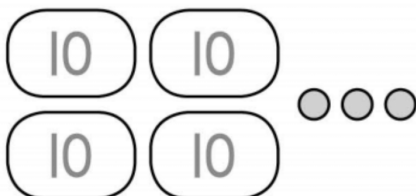


- **A measurement pathway** where the starting point is continuous quantities that can be measured using a particular attribute such as length or volume.



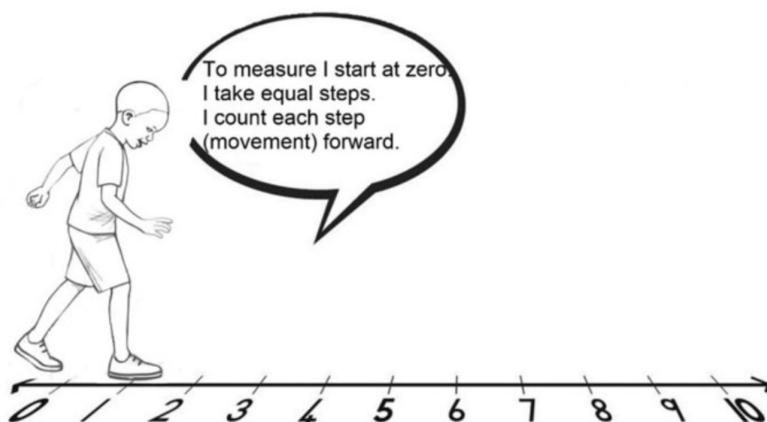
In this module we consider both pathways, and an underlying approach of structuring numbers to identify, describe and generalise the patterns of 2s, 5s and 10s.

The counting pathway uses number pictures.



Children should structure number pictures in 10s. They write the number symbol 10.

The measurement pathway uses number lines.



Children should structure number lines in 10s. Multiples of 10 are "friendly numbers".

Reading 1: Mason & Wilder-Johnston (2004) Johnston-Wilder

John Mason has identified a set of 8 “Mathematical Powers” that all children possess and which we need to foster and develop in order to create “able mathematicians” who are able to reason about maths and problem solve. The powers, which come in pairs, are as follows:

Conjecture

Children should be encouraged to **make conjectures**, that is say what they think about what they notice or why something happens. E.g., “I think that when you multiply an odd number by an even number you are always going to end up with an even number.”

and

Convince

Children should then be encouraged to **convince**, that is to persuade people (a partner, group, class, you, an adult at home etc) that their conjectures are true. In the process of convincing, children may use some, or all, of their other maths powers.

Organise

Children should be encouraged to **organise**, putting things (numbers, facts, patterns, shapes) into groups, in an order or a pattern, e.g., sorting numbers or shapes

and

Classify

Children should then be encouraged to **classify** the objects they have organised e.g., identifying the groups odd and even numbers, irregular and regular shapes, etc.

Imagine

Children should be encouraged to **imagine**, objects, patterns, numbers and resources to help them solve problems and reason about mathematics.

and

Express

Children should be encouraged to **express their thinking**, that is show and explain their thinking and reasoning e.g., about a problem, relationship or generalisations.

Specialise

Children should be encouraged to **specialise**, that is to look at specific examples or a small set examples of something. For example, looking at the odd number 7 and the even number 8 to test their conjecture that an odd X even number = odd number. Children can also specialise in order to start to see patterns

and

Generalise

Children should be encouraged to **generalise**, that is to make connections and use these to form rules and patterns. For example, from their specific example they could generalise that any odd number multiplied by an even number gives an even number. Children should also be encouraged to use algebra to

Mason, J and Johnston-Wilder, S (2004) Learners Powers in: *Fundamental Constructs in Education*, London:Routledge Falmer pp115-142

- 1.**



Working out:



- 3.**

Mathematical disposition

Many of us have maths trauma or wounded maths stories. With support from lecturers and peers we can change our maths stories.

Task 1.2: Two minute Tango 1

1	$200 - 1 = \underline{\hspace{2cm}}$	11	$\underline{\hspace{2cm}} = 256 - 100$
2	$\underline{\hspace{2cm}} = 20 - 1$	12	100 more than 23 is $\underline{\hspace{2cm}}$
3	1 more than 300 is $\underline{\hspace{2cm}}$	13	If I decrease 134 by 100, I get $\underline{\hspace{2cm}}$
4	If I decrease 56 by 1, I get $\underline{\hspace{2cm}}$	14	207 ; 197 ; 187 ; $\underline{\hspace{2cm}}$
5	127 ; 126 ; 125 ; $\underline{\hspace{2cm}}$	15	4 plus 100 = $\underline{\hspace{2cm}}$
6	10 more than 50 is $\underline{\hspace{2cm}}$	16	If I increase 146 by 1, I get $\underline{\hspace{2cm}}$
7	$\underline{\hspace{2cm}} = 105 - 10$	17	$9 + 6 + 1 = 10 + \underline{\hspace{2cm}}$
8	If I decrease 50 by 1, I get $\underline{\hspace{2cm}}$	18	$99 + 26 + 1 = 100 + \underline{\hspace{2cm}}$
9	If I increase 25 by 10, I get $\underline{\hspace{2cm}}$	19	89 subtract 1 = $\underline{\hspace{2cm}}$
10	4 ; 14 ; 24 ; $\underline{\hspace{2cm}}$	20	$55 - 10 = \underline{\hspace{2cm}}$

Children should learn the **small, medium and large counting songs**.

- Counting forwards in 1s is the same as adding 1 each time
- Counting forwards in 10s is the same as adding 10 each time
- Counting forwards in 100s is the same as adding 100 each time

They should have a mental image of the number songs as a number picture and as a number line.



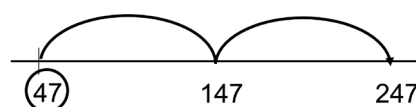
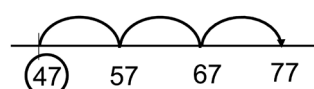
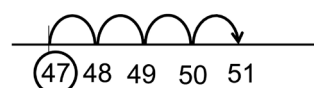
The counting pathway (number pictures)

Small song 47 $\begin{matrix} \circ & \circ & \circ & \circ \\ 48 & 49 & 50 & 51 \end{matrix}$

Medium song 47 $\begin{matrix} 10 & 10 & 10 \\ 58 & 59 & 60 \end{matrix}$

Large song 47 $\begin{matrix} 100 & 100 & 100 \\ 147 & 247 & 347 \end{matrix}$

The measurement pathway (number lines)





Summarise in your own words 4
main ideas in this introductory unit.

.....

UNIT 2: The counting pathway

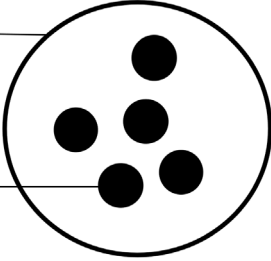
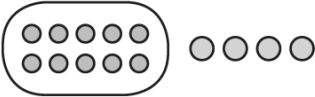
Key The counting pathway

A **counting pathway** is a route for learning about numbers, where the starting point is **discrete** quantities (discrete objects or discrete events).

The key ideas in Unit 2 are:

1. Following the **counting pathway** to learn about numbers, children count discrete objects or discrete events.
2. **Children learn to 'count with meaning'** by moving along a counting trajectory.
3. **Children first learn to count in 1s.** But to understand number names beyond ten, they need to count and structure in 1s, 10s and 100s. They also count and structure in 2s and 5s.

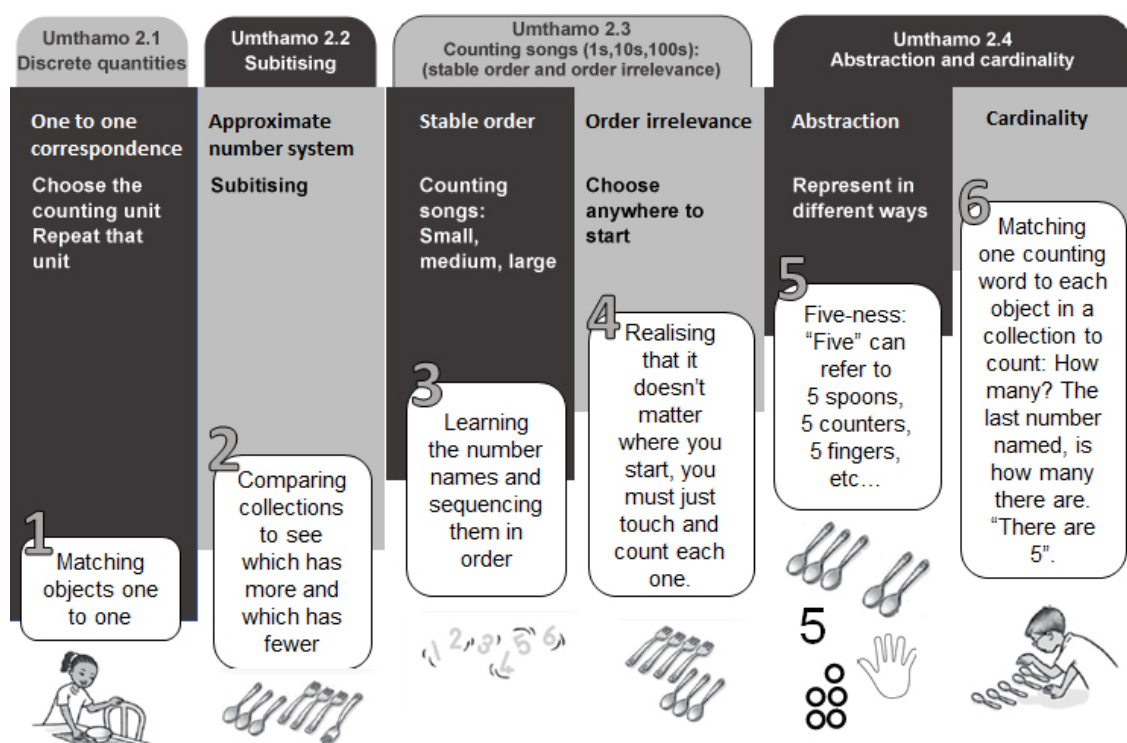
These are the main features of the counting pathway.

Quantities	Discrete quantities that can be counted
Pathway into number sense	Counting pathway
Unit	Counting unit (often in 1s, 2s, 5s or 10s)
Metaphor	A container The container or the set of 5 objects A discrete object or An element 
Representations	Number pictures 
Number meaning	Cardinality Number is the size of a set/collection of discrete quantities.

Children learn to 'count with meaning' by moving along a counting trajectory

Children learn to count, moving along a **counting learning trajectory**:

- Stage 1:** Choose what to count, and **match** objects one to one
- Stage 2:** **Compare** collections (by subitising) to identify which has more or fewer
- Stage 3:** Learn the **number song** (say the small number sequence to count in ones)
- Stage 4:** Realise that **it does not matter where you start**, you must touch and count each one. So they notice where they start and end to count a set of objects (so they don't count the same object twice) (order irrelevance)
- Stage 5:** **Abstract** the number-ness of a number. Know that five can refer to five objects or five pictures or five events or 5 steps etc.
- Stage 6:** Know how many there are. Match one counting word to each object in the collection and realise that the last number named is how many there are in the set (**cardinality**).



Source: Adapted from Kuhne, et al. (2017) *R-Maths Concept Guide*, p. 62

UMTHAMO 2.1

Discrete quantities

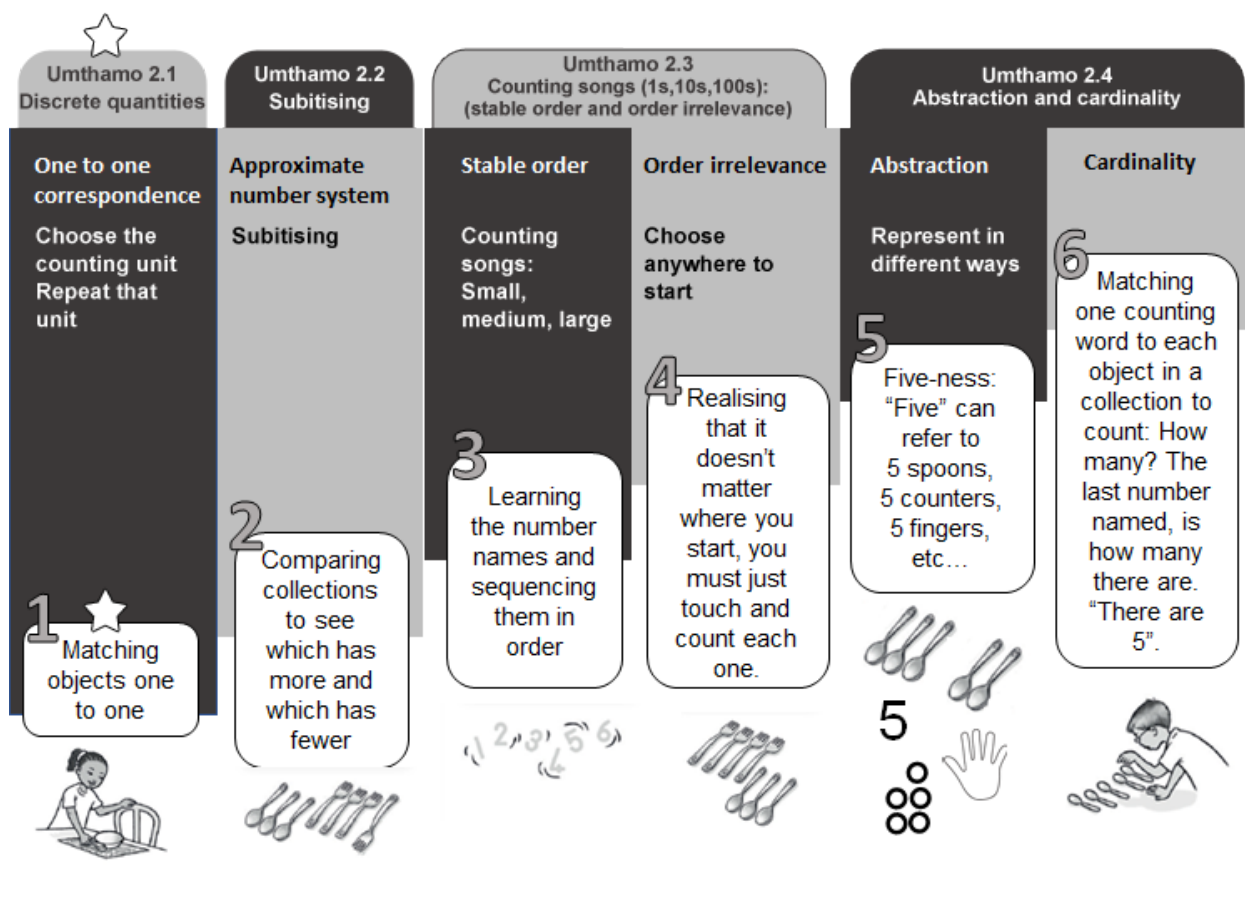
Introduction

Key Ideas

The key ideas in umthamo 2.1 are:

1. **Stage 1 of the counting trajectory is choosing a counting unit.** What are we counting?
2. **In Stage 1 the chosen counting unit is repeated, and discrete objects are matched one-to-one** (a finger is matched to an object, forks are matched to knives, a counting word is matched to 1 discrete object)
3. **A mental picture for a number of discrete objects is a container** (tomatoes on plates)
 - We can express and imagine whole numbers as discrete objects and events, using **many different representations**
 - We use whole numbers in many **different contexts**: measurement, shape & space, and data handling
4. **Maths does not make sense if your only mental picture is counting and structuring in ones.** To understand the base-ten number system of place value, you need to:
 - **Count and structure in 1s, 10s and 100s.**
 - You use the **small, medium and large counting songs**. Count in 1s, 10s and 100s from any number
 - **Add and subtract 1s, 10s and 100s** from any number

The counting trajectory



Stage 1a: Choose what to count (the counting unit)

When children count how many discrete objects they are in a set, they must first decide what should be in the set. They use their mathematical powers to **organise and classify**. They choose which objects to count. They choose an object, and count how many times that object is repeated.

Activity for learners

Number talk

There are many “**things**” or **units** to count. Some children may count the **number of plant tubs**. Some children may count the **number of plants** or the **number of leaves**. There is no right answer. What you count, depends on what you decide is the unit.

In every collection, there is a **unit** which we count to see how many of “**somethings**” is in that collection or set or group. In any collection you should define the unit: What are you counting or structuring?



How many?

How did you work it out?

Stage 1b: Repeat the unit and match discrete objects one-to-one

Once children have decided what they are focusing on (the counting unit), then they can match discrete objects one-to-one. Even before they can say the number song for counting, children can match objects one-to-one.

Diagram: Matching one finger to one dot

The child can put one finger on each sticker. A child could put one sticker on each finger. To do this, they don't need to know that there are 'eight' stickers, or 'eight' fingers. They can just match the discrete objects one-to-one. Children can also match discrete events one-to-one.

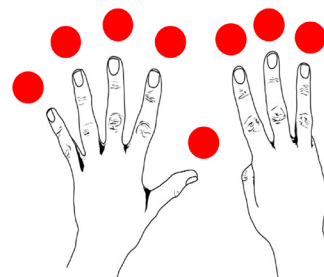


Diagram: Matching on action/event to one sound



Jump/skip each time I clap.

Clap	Skip
Clap	Skip
Clap	Skip
Clap	Skip
Clap	Skip

The child can jump each time they hear a clap. The don't need to be able to count. They match discrete events one-to-one.

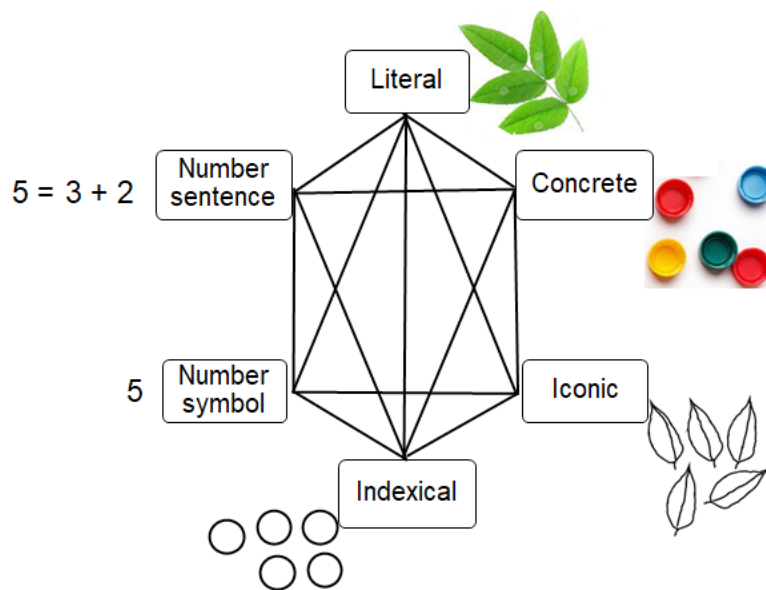
Children use one-to-one correspondence, even before they can talk.

A mental picture for discrete objects:

We **imagine and express** discrete objects in many different ways.

Children move between **representations** of numbers: literal, concrete, iconic, indexical, and number symbols.

Diagram: Types of representations



Note:

As this is a book/ presented online there are no “literal” objects. We use a photograph to depict the “literal” objects.

Concrete	Literal objects are real physical objects in their surroundings: sticks, stones, forks, bowls, leaves, hats, shoes Concrete objects are physical objects which represent the literal objects such as bottle tops, or unifix cubes. They are also called manipulatives. They are concrete as they can be touched, joined and arranged.
Pictorial	Iconic pictures: Children also use drawings or picture of collections objects. They can draw a lock or key or hat, or a few stones or a pair of shoes. Indexical picture: Overtime, they start to draw with less detail. Instead of drawing 4 leaves, they just draw 4 circles to show how many there are.
Abstract	Number symbols: Only in Grade R and 1 do they start to recognise and learn to write number symbols. Number sentences (symbolic syntactical): Later they start to write number sentences (using symbols like + and – and = in Grade 1, and only using × and ÷ by Grade 3). A number sentence describes the relationship between collections of discrete objects.

Researchers used to think that children **must always start** with the concrete, move to the pictorial and then move to the abstract (Concrete→Pictorial→Abstract). We now know this is not true. Children need to work in both directions: Concrete→Pictorial→Abstract and Abstract→Pictorial→Concrete. Children need to have a **strong mental picture** of what the number symbols mean and of how different numbers fit together.

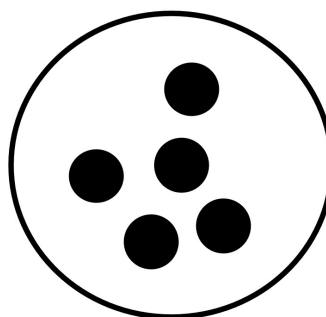
A key representation for the counting pathway are **sets of discrete objects** (like apples or marbles, or people) which can be counted in ones. The discrete objects are contained or encircled into a **set**. We use a '**container**' metaphor for the number of discrete objects contained in a set (Lakoff & Núnes 2001).

Diagram: A number picture for discrete objects



One way that we think about a number is using the idea of **a container**.

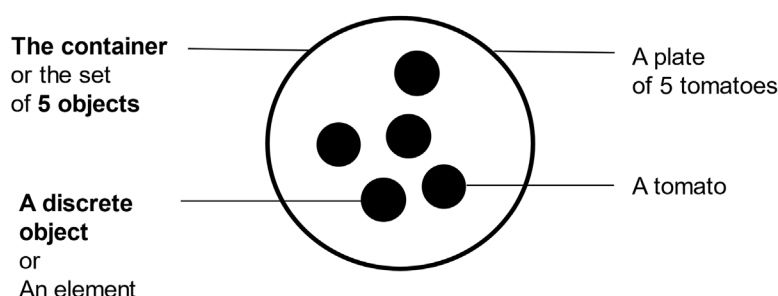
5 is a collection of 5 discrete objects,



The discrete objects are the elements of the set. The elements are the units we use to count how many there are.

Diagram: A number picture – the container metaphor

The number 5 is a collection of 5 discrete objects



Thinking about the number 5, we imagine 5 objects in a set or collection. The 5 discrete objects are grouped together into a container. This is the **cardinal** use of number. There are five discrete objects in the set – so the set has a cardinality (size) of 5.



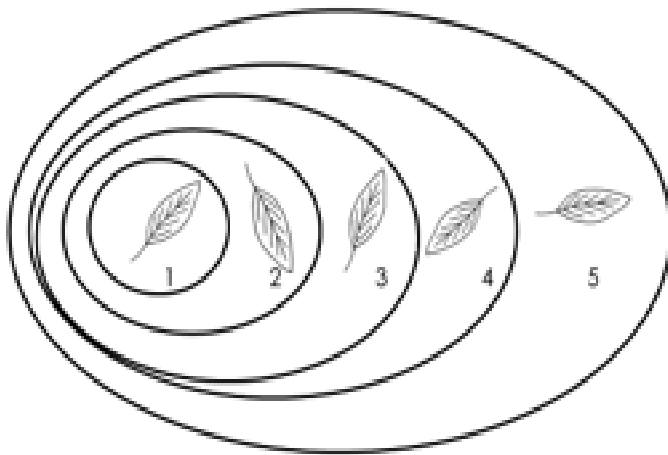
Tasks: Fluencies and structuring (1s, 10s, 100s)

Task 2.1.1 Number pictures – A container metaphor for discrete objects

Sets of leaves

In the counting song for ones (small number song), each number is within the next number.

They are a **nested set**.



Mathematical acting and thinking

Express and imagine discrete objects and events in many different ways
A metaphor for discrete objects is a container (tomatoes on plates)
Smaller numbers are contained within bigger numbers – they form nested sets

1. Continue the nesting pattern:

1 is inside the set of 2

2 is inside the set of 3

_____ is inside the set of _____

_____ is inside the set of _____

_____ is inside the set of _____

The big idea here is that 1; 2; 3; and 4 are all within the set of 5.

2. Choose a discrete object to draw (eg. an apple, a pencil, a hand, a brick...). Draw 12 of these discrete objects. Draw a circle around the ten objects to show the set of 10.

3. Now draw the 12 discrete objects using an **index/symbol** (e.g. a circle or a triangle, or line) for each object. Draw a circle around the ten objects to show the set of 10.

Structuring representations

Number pictures: A container metaphor for a collection of discrete objects

4.

Small song	How many?		There are ____ ones.
		1 2 3 4 5 6 7	
Medium song	How many?		There are ____ tens. There are ____ ones.
		1 2 3 4 5	
Large song	How many?		There are ____ hundreds There are ____ tens There are ____ ones
		1 2 3	

As teachers we ALWAYS draw by structuring in 100s, 10s and 1s
 Children will start drawing with 1s.
 But, as teachers, we help children to structure in 100s, 10s and 1s

Fluencies

Bonds of 5
Bonds of 10

 Task 2.1.2 Two minute tango 2

1	$2 + \underline{\hspace{2cm}} = 5$	11	$5 - 5 = \underline{\hspace{2cm}}$
2	$20 + \underline{\hspace{2cm}} = 50$	12	$75 - 5 = \underline{\hspace{2cm}}$
3	$200 + \underline{\hspace{2cm}} = 500$	13	$375 - 5 = \underline{\hspace{2cm}}$
4	$\underline{\hspace{2cm}} + 1 = 5$	14	$\underline{\hspace{2cm}} + 7 = 10$
5	$\underline{\hspace{2cm}} + 10 = 50$	15	$70 + \underline{\hspace{2cm}} = 100$
6	$\underline{\hspace{2cm}} + 100 = 500$	16	$\underline{\hspace{2cm}} + 700 = 1000$
7	$4 + \underline{\hspace{2cm}} = 25$	17	$4 + \underline{\hspace{2cm}} = 10$
8	$\underline{\hspace{2cm}} + 4 = 25$	18	$4 + \underline{\hspace{2cm}} = 20$
9	$250 - \underline{\hspace{2cm}} = 40$	19	$14 + \underline{\hspace{2cm}} = 20$
10	$250 - 40 = \underline{\hspace{2cm}}$	20	$\underline{\hspace{2cm}} + 4 = 20$

Equals means **is the same as**.

Equals DOES NOT mean “makes” or “find the answer”.

My number plus 2 **is the same as** 5. What’s my number? or
Something add 2 **is the same as** 5. What’s my number?

$$\square + 2 = 5$$

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Two Minute Tango 2

Practice bonds of 5, 10 and 20.
Add together to make 5, 10 or 20.
Subtract from 5, 10 or 20.

I can add ones. So I can add tens!



I know that $5 + 3 = 8$

Therefore: I know that $50 + 30 = 80$

$$\begin{array}{ccccccc} 5 & + & 3 & = & 8 \\ \begin{array}{c} \text{5 blocks of 10} \\ \text{50} \end{array} & + & \begin{array}{c} \text{3 blocks of 10} \\ \text{30} \end{array} & = & \begin{array}{c} \text{8 blocks of 10} \\ \text{80} \end{array} \end{array}$$

Number facts connect in **1s, 10s and 100s**.

- If I know that $5 + 3 = 8$, then I know that $500 + 300 = 800$
- If I know that $5 + 3 = 8$, then I know that $5\,000 + 3\,000 = 8\,000$

Make connections

If you know one number fact, then use the **small, medium & large numer songs** to work out new facts



Name and draw the different **types of representations** children use.
Draw a **number picture** (tomatoes on plates) for **discrete objects**.
Is your drawing **indexical or iconic**?

.....

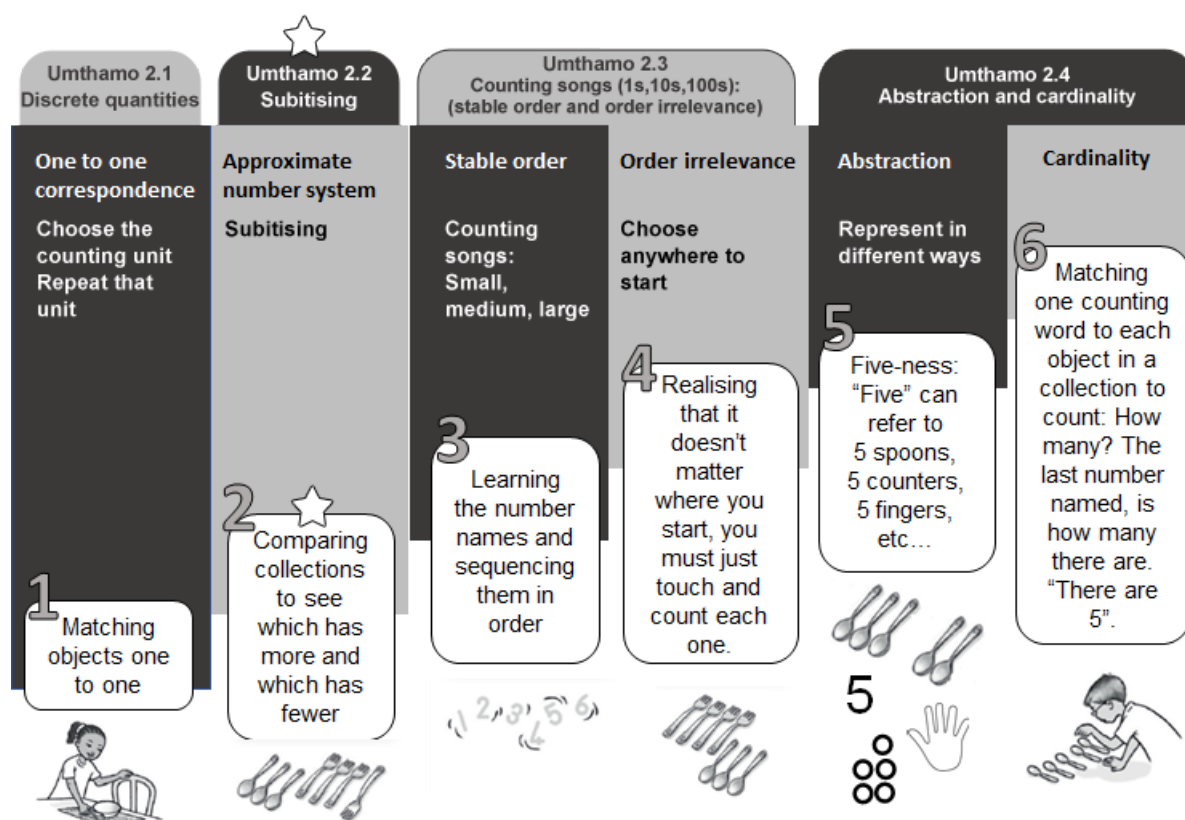
Subitising

Introduction

Key Ideas

The key ideas in umthamo 2.2 are:

1. **There are 6 principles which children must apply to count with meaning:**
one-to-one correspondence, comparison, stable order, order irrelevance, abstraction and cardinality.
2. **Stage 2 of the counting trajectory uses the approximate number system to compare** which has more or fewer.
3. **In Stage 2 children subitise** (quickly see without counting)
 - First they use **perceptual subitising** (for small sets less than 5 or 6)
 - Then they use **conceptual subitising** (bigger sets are split up)
 - Next they use **conceptual subitising with place value and skip counting** (using the small, medium and large counting songs)
 - Finally they use **conceptual subitising with place value and multiplication.**
1. Maths does not make sense if a child's only mental picture is counting and structuring in ones. Children can:
 - Use **number pictures** that are structured into 1s, 10s and 100s.
 - Structure number in **2s and in 5s**
 - Be fluent with **bonds of 10, and counting in 5s.**



With plenty of hands-on activities and guidance from the teacher, children begin to understand and apply the following counting principles:

One-to-one correspondence principle: Matching discrete objects and discrete events one-to-one. Children match discrete objects by touching and arranging them. Children match discrete events with an action. They can track events on their fingers or by moving stones or by drawing a mark.

Compare using the approximate number system: Children use the Approximate Number System (ANS) to compare small sets of objects and identify which set has more and which has fewer. They use perceptual subitising to compare two small sets.

Stable order principle: Children know the small number song. Number names are always arranged in the same fixed order, e.g. one is followed by two, two is followed by three, three is followed by four, and so on.

Order-irrelevance principle: The order of counting the objects in a collection does not matter. Children need to understand that however we arrange the objects, the total number of objects in the collection remains the same.

Abstraction principle: Children understand that even if groups with the same number of objects look very different (e.g. five grapes, five people, five houses) they have the same numerosity, i.e. 'fiveness'. They realise that counting can be applied to objects, pictures, colours, shapes, or even actions or sounds.



Source: Kuhne, et al. (2017) R-Maths Concept Guide

Cardinality principle: The last number word said when counting a collection represents the total number in the collection. When a child is able to answer: 'How many are there?', they have reached an important counting milestone. They can now count with meaning. They realise that the last number that they say in the number song name, is also how many there are in the collection. They have found the cardinality of the set.

In this umthamo, we focus on comparing discrete quantities using the ANS. We learn about subitising

Humans are born with an OBJECT TRACKING SYSTEM (OTS). This allows a baby to track the movement of up to 4 objects (or people).

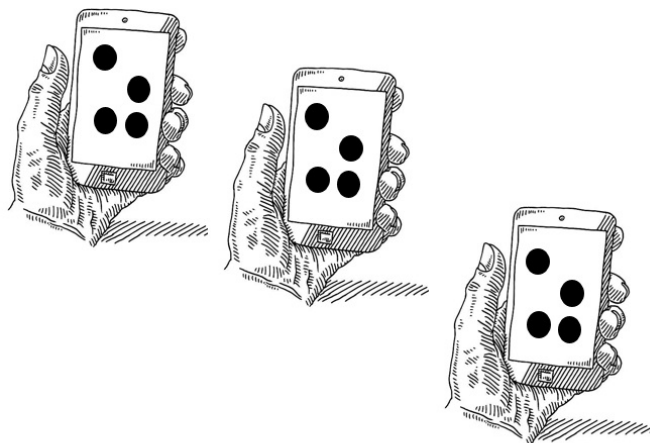
Humans can reason about quantities BEFORE THEY CAN TALK

Humans are born with an APPROXIMATE NUMBER SYSTEM (ANS). This allows a baby notice which has more or less, or when an object is missing (up to 5 or 6)

The approximate number system

At a very early stage children use their **approximate number system**. Babies as young as three months notice when a discrete object is missing.

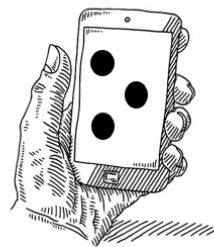
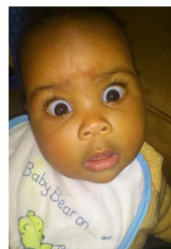
If you show a baby a screen with 4 dots,
and then
show them the same screen with 4 dots,
and then
show them the same screen with 4 dots,
they will get bored.



But then if you show them a screen with 3 dots, the baby will look puzzled,

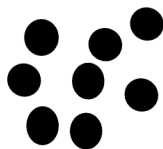
They will look longer and wait to see what you do next.

They look for the missing dot.



Children can compare two small collections and identify which collection has more. They can show you which set has more and which set has less.

Activity for learners



Black dots



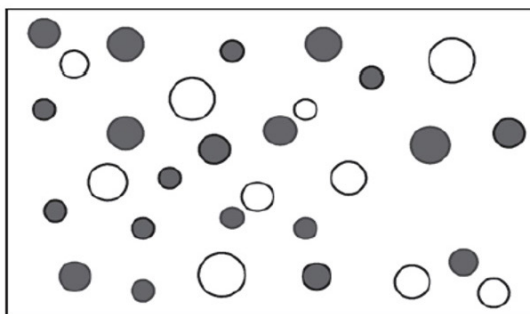
Grey dots

Are there more black dots or grey dots?

If this activity is too difficult try the same task with a small sets of objects. The bigger the difference in size between the two groups, the easier it is to compare.

You use your approximate number system to compare sets.

Are there more white dots or grey dots?



Early on (before and during Grade R) children use perceptual subitising. Soon there after (Grade R and Grade 1) they use conceptual subitising (in 1s). As they move beyond a group of ten, they use conceptual subitising to link this with place value. First they use skip counting (of the small and medium counting songs), later (Grade 2 and 3) they link place value to multiplication.

SOME IDEAS FOR YOUR CLASSROOM

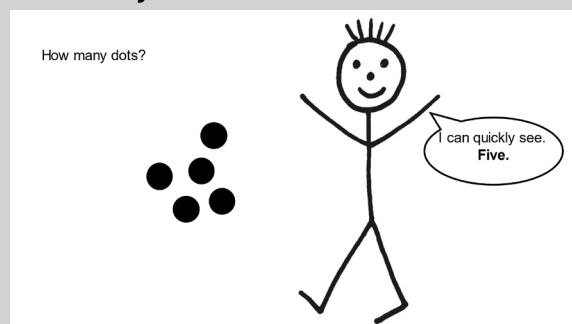
Teachers can design tasks that are the right level for their children. Here are examples of activities which move from perceptual subitising (Grade R and before) to conceptual subitising with place value and multiplication (Grade 3 and after).

Perceptual subitising

Perceptual subitising is for small sets of objects (up to 5 or 6). It is instantly recognizing and naming quantities without counting.

Activity for learners

How many dots?



This child can subitise up to 5 or 6. They use perceptual subitising.

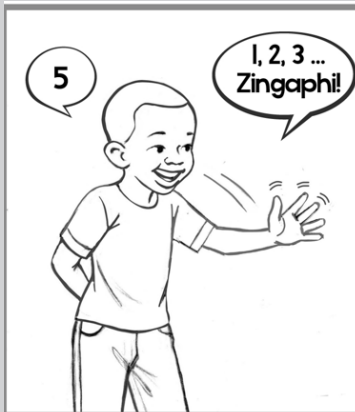
Children use their approximate number system to compare two groups. They can see who has more and who has less (without counting).

Children subitise to play **1,2,3 Show!** or **1,2,3 Veza!**

1. Start with children using only one hand. They use **conceptual subitising** to play
 - Who has more?
 - Who has less?
 - How many? Zingaphi?
 - How many more? How many less?
2. Later, children can use **both hands**. They use **perceptual subitising** to play
 - Who has more?
 - Who has less?
 - How may? Zingaphi?
 - How many more? How many less?

Use perceptual subitising to play 1,2,3 Show! with only one hand.

Game



GAME OF THE WEEK:

123! How many? Zingaphi?

- Play in pairs.
- Put hands behind back.
- Say together, "1, 2, 3! Zingaphi?"
- One child shows fingers on their one hand.
- The other child says how many they see, quickly.
- Do again! Faster!

To find **How many?**, children can subitise - they don't need to count. They can use their approximate number system to find **Who has more?** and **Who has less?**

Game



GAME OF THE WEEK:

123! Who has less?

- Play in pairs.
- Put hands behind back.
- Say together, "1, 2, 3! Who has less?"
- Both children show fingers on both hands.
- Who has less? Ngubani onokuncinci?
- Do again! Faster!

Parents or siblings can play 1,2,3 Veza at home. The child uses perceptual subitising to quickly say **How many?**

Game



1-2-3 VEZA: ONE HAND

- You and your child put one hand behind your back.
- Say together, "1, 2, 3, Veza!"
- Both of you show fingers on one hand.
- Your child says how many they see, as quickly as possible.

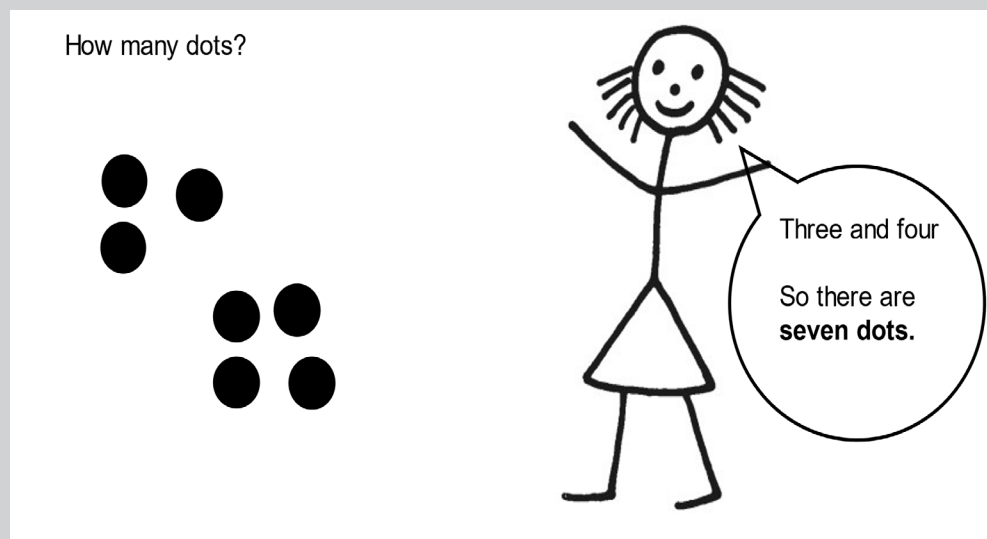
Conceptual subitising

Conceptual subitising is for bigger sets of objects (more than 7). Subsets are created and then combined.

Children **subitise** and use their fingers to **structure numbers** by breaking up bigger numbers into smaller parts. When children break up a bigger collection into smaller sub-groups, this is called **conceptual subitising**.

Activity for learners

How many dots?



This child can subitise beyond 5 or 6. They use conceptual subitising.

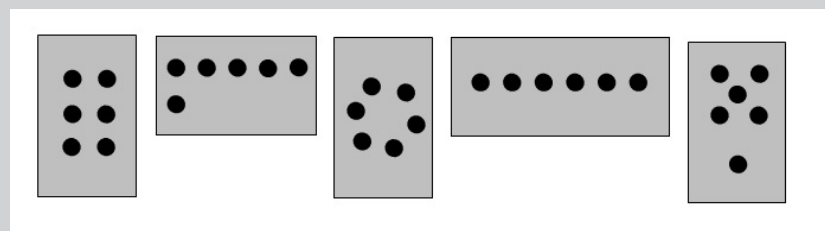
Conceptual subitising by structuring in 5s

Because we have 5 fingers on each hand we structure our number system using 5s and 10s (we have a base-ten place value system).

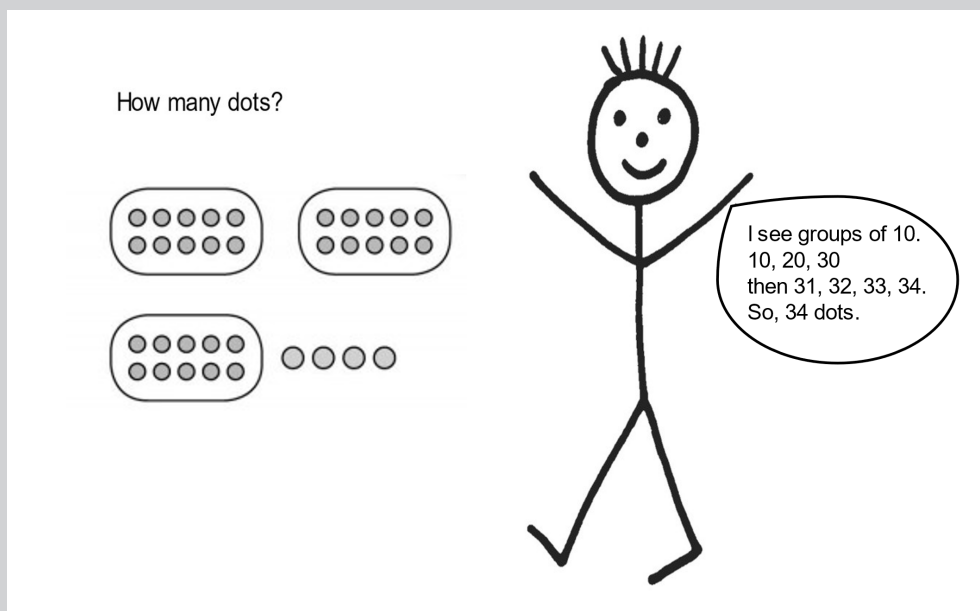
Activity for learners

How many?

Which one is easiest to quickly see how many, without counting?
Which one does not belong? Why?



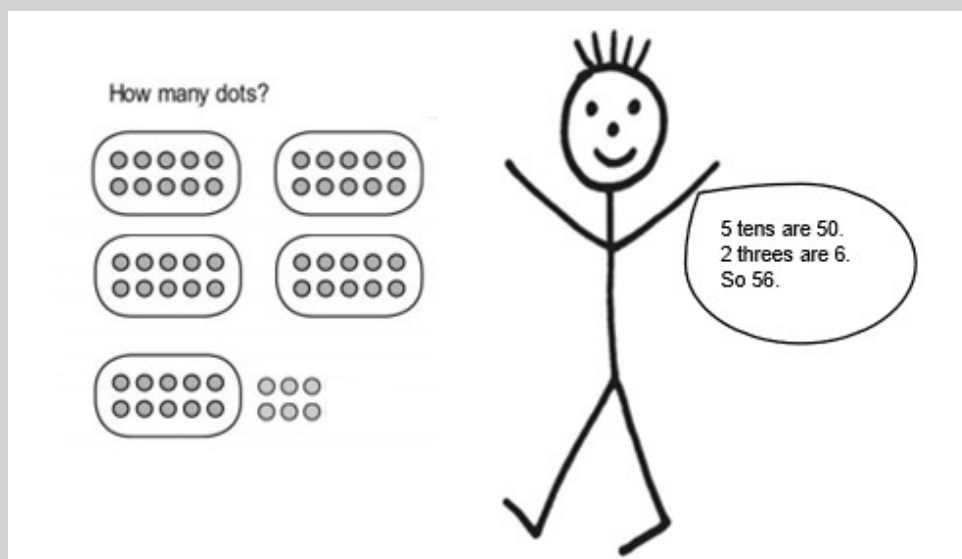
Children can subitise! They know they have 5 fingers on each hand. To show 5, they show all their fingers. There is no need to count each time. Beyond 5 they start using 2 hands.



This child uses conceptual subitising (for the 3 groups of tens, and the 4 ones). They skip count in tens (the **medium number song**) and then switch to count in ones (the **small number song**).

Key Conceptual subitising using multiplication

This drawing is done by a child.
A teacher would show '10' and not draw 10 ones



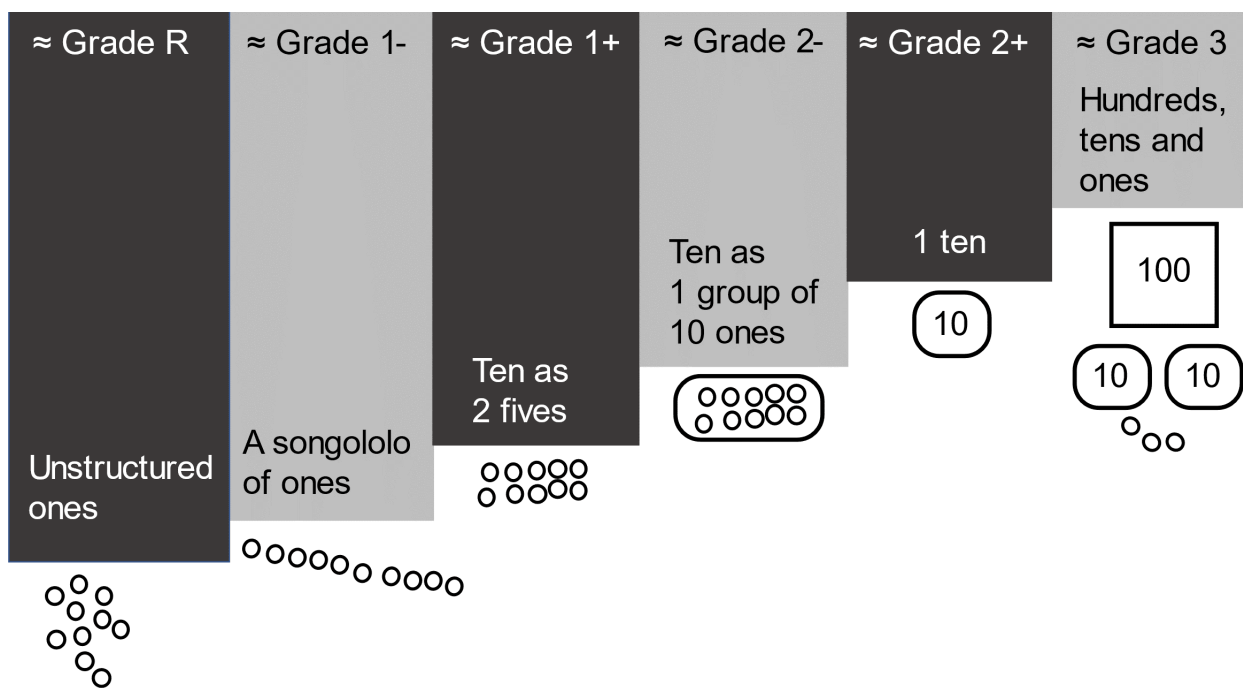
The child uses conceptual subitising with place value and multiplication
10 repeated 5 times is 50. ($10 \times 5 = 50$)
3 repeated 2 times is 6. ($3 \times 2 = 6$)
 $50 + 6 = 56$
So it is 56.

Number pictures that are structured into 1s, 10s and 100s

Children need to structure collections of discrete objects. When working with concrete materials they should arrange the objects into groups. When drawing, they slowly learn to structure bigger groups of objects using the base-ten place value system.

It is helpful to have a mental picture of the time it usually takes for children to structure numbers and work with number pictures.

Diagram: A learning trajectory for number pictures



Number pictures use the base-ten place value system.

A **one** is a small circle.

A **ten** is big circles with 10 written inside.

A **hundred** is a square with 100 written inside.

Children need to be fluent with **bonds of 10**. They re-order calculations to **find the 10s**.

Activity for learners

As teachers we ALWAYS draw by structuring in **100s, 10s** and **1s**.

Children will start drawing with **1s**.

But, as teachers, we help children to structure in **100s, 10s** and **1s**.

Find the 5. Then add.

Draw a line to find each 5.

$$\begin{array}{c} 5 \\ 1 + 3 + 4 = \end{array}$$

$$4 + 2 + 1 =$$

$$2 + 3 + 3 =$$

$$2 + 3 + 4 =$$

$$4 + 10 + 1 =$$

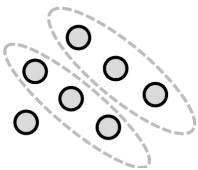
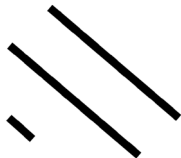
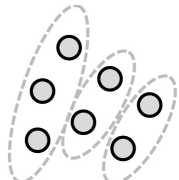
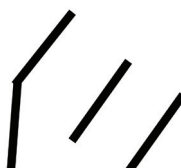
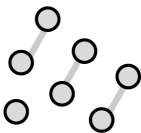
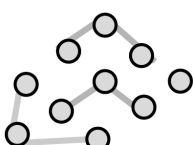
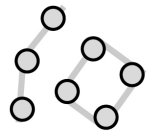
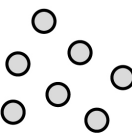
$$1 + 2 + 4 =$$

Mathematical acting and thinking

Express and imagine discrete objects and events in many different ways
A metaphor for discrete objects is a container (tomatoes on plates)
Smaller numbers are contained with bigger numbers – they form nested sets

 Task 2.2.1 Designing subitising tasks for children

1. How many different ways can you see how many dots there are in total?

Number Talk: How many?	Teacher records	
Make groups. Circle or join the dots	Draw the pattern	Write the number sentence
		$3 + 3 + 1 = 7$
		$3 + 2 + 2 = \underline{\hspace{1cm}}$
		
		
		
		

Notice that the structure of the picture, stays the same.
Each picture has 7 dots. The structure is preserved. What changes is how the elements (or units) are arranged or grouped.

The South African Numeracy Chair project has produced a guide for grade R teachers on subising. Here is an extract from their guide:

In this handbook, we refer to *regular* and *alternate* dot patterns.

Regular dot patterns, especially from 1 to 6, are the easiest to recognise. These are those most commonly seen on dice, dominoes and playing cards.

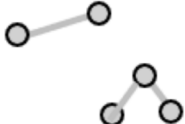
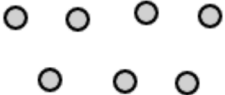
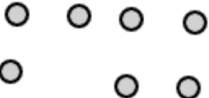
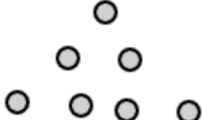
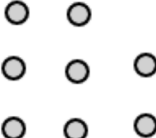
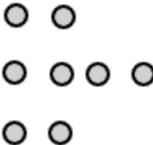
Alternate dot patterns work with other dot arrangements and are intended to help the learners to 'see' patterns in different configurations. This is because different arrangements of dots lead to different decompositions of that number.

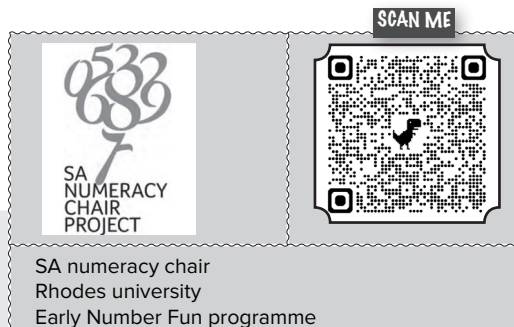
For example, here 6 dots can be seen in different ways:

Seen as 2, 2 and 2	Seen as 5 and 1	Seen as 3 and 3	Seen as 4 and 2
			

2. Number talks: How many?

Circle or join the dots to make groups. Write the number sentences.

$5 = 2 + 3$ 	$\underline{\quad} = \underline{\quad}$ 	$\underline{\quad} = \underline{\quad}$ 
$\underline{\quad} = \underline{\quad}$ 	$\underline{\quad} = \underline{\quad}$ 	$\underline{\quad} = \underline{\quad}$ 
$\underline{\quad} = \underline{\quad}$ 	$\underline{\quad} = \underline{\quad}$ 	$\underline{\quad} = \underline{\quad}$ 

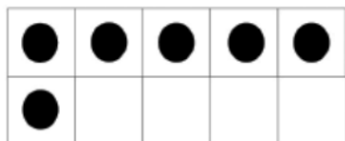


Tasks: Fluencies and structuring (in 2s, 5s and 10s)

Task 2.2.2 Fill up a ten frame

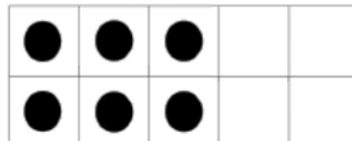
We can structure numbers by filling up a ten frame. This helps children to work with the base-ten place value. As children can subitise up to five, we use a ten-frame to picture numbers. Here is a ten-frame showing 6.

To structure in 5s, we make groups of 5. Each row has space for 5 dots.



Start on the left and fill up the top row then the bottom row.

To structure in 2s, we make groups of 2. Each column has space for 2 dots.



Start on the left and fill in pairs – one on the top, one of the bottom.

Structuring representations

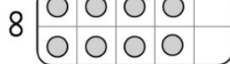
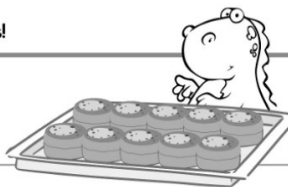
Number pictures: Using a ten-frame
Number pictures: Structuring by 2s and 5s.

1. Structure in 2s



Zibalo is very organised!
She always organises scones in 5s and 10s!

Zibalo loves to bake. She bakes scones to sell. She ALWAYS bakes scones in rows of 5 or columns of 2



2. Structure in 5s

Help Zibalo place scones on baking trays. Organise each tray into rows of 5. Do not place more than 10 on one tray!

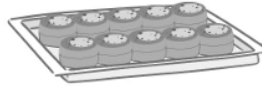


Children will start drawing with 1s.

As teachers we ALWAYS draw by structuring in 100s, 10s and 1s

1,2,3 draw! (Structure by 5s)

3. Draw using 1s, 10s and 100s



Zibalo just baked more scones.
Help her draw the numbers.



Draw the numbers.

21

8

13

53

47

35

As teachers we ALWAYS structure in **100s**, **10s** and **1s**

- For 10, draw a circle and write 10 inside it.
- For 100, draw a square and write 100 inside it.

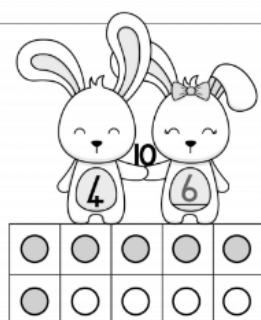
Fluencies

Make connections with bonds of 10

Task 2.2.3: Two minute tango 3

1	_____ + 2 = 10	11	100 - 80 = _____
2	_____ + 20 = 100	12	100 - _____ = 40
3	10 - 6 = _____	12	10 - _____ = 4
4	100 - 60 = _____	14	_____ + 7 = 10
5	8 + _____ = 100	15	20 + _____ = 100
6	100 - 8 = _____	16	_____ + 200 = 1 000
7	100 - 3 = _____	17	3 + _____ = 10
8	100 - 30 = _____	18	13 + _____ = 20
9	_____ + 20 = 100	19	23 + _____ = 30
10	_____ + 80 = 100	20	_____ + 3 = 20

Bonds of 10



Representations

Bonds of 10 are "friends in love"
Ten frames are trays of 10 scones.
Structure by 2s - with 2 in each column or structure by 5s - with 5 in each row.



Jumptrak

Two Minute Tango 3

Practice the bonds of 5 and bonds of 10. Add together to make 5 or 10. Subtract from 5 or 10.

4 + 6 = 10. 4 and 6 are best friends. Together they add up to make a full baking tray of 10 scones.

You can use your bonds of 10 for many other calculations.

- If **4 + 6 = 10**, then **40 + 60 = 100**
- If **4 + 6 = 10**, then **400 + 600 = 1 000**, etc.

If you know that **4 + 6 = 10**, then you can **visit the next multiple of ten**:

- If **4 + 6 = 10**, then **14 + 6 = 10 + 10 = 20**
- If **4 + 6 = 10**, then **24 + 6 = 20 + 10 = 30**
- If **4 + 6 = 10**, then **34 + 6 = 30 + 10 = 40**, etc.

Make connections

If you know one number fact (eg. a bond of 10), then use the **small, medium & large number songs**, to work out new facts



Explain the difference between **conceptual and perceptual subitising** and give examples.

Give an example of a **mistake** that you made, and what you learnt from it.

.....

UMTHAMO 2.3

Counting songs

Remember the key ideas in unit 2 were:

1. Following the **counting pathway** to learn about numbers, children **count discrete objects** or **discrete events**.
2. Children learn to 'count with meaning' by moving along a **counting trajectory**.
3. Children first learn to count in 1s. But to understand number names beyond ten, they need to **count and structure in 1s, 10s and 100s**. They also count and structure in **2s and 5s**.

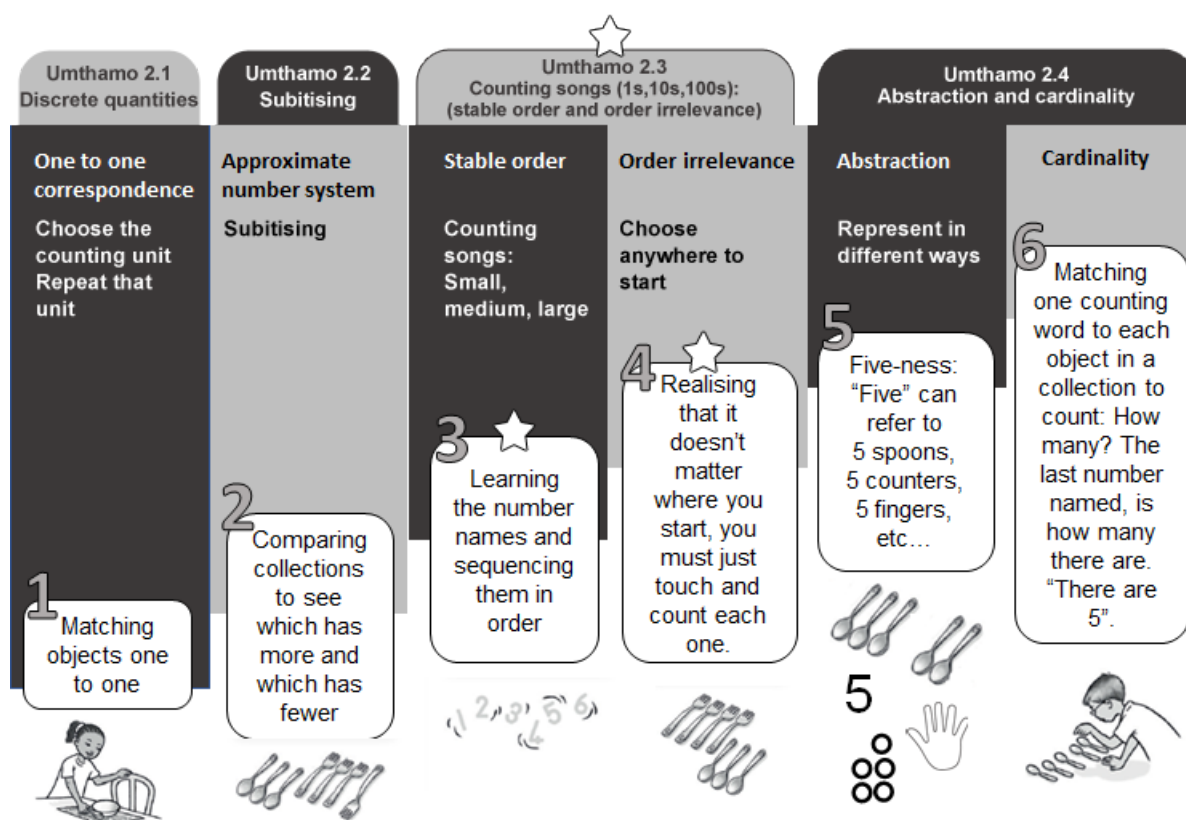
Introduction

Key Ideas

The key ideas in umthamo 2.3 are:

1. **Stage 3 of the counting trajectory is learning the small number song (in 1s)**
2. The number song is different depending on the language used. The number names in **some languages are more transparent and regular** than other languages.
3. In **Stage 4 of the counting trajectory** children realise that no matter where they start when counting a set, they get the same amount (**order does not matter**). They touch each object as they say each word of the small counting song.
4. Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s, 10s and 100s**.

There is a counting trajectory of different stages which children follow when learning to count.



In this umthamo we focus on learning the number song. When children start to match each number word to a particular discrete object or event, they then apply the order irrelevance principle.

Stage 3: Learn the number song – stable order

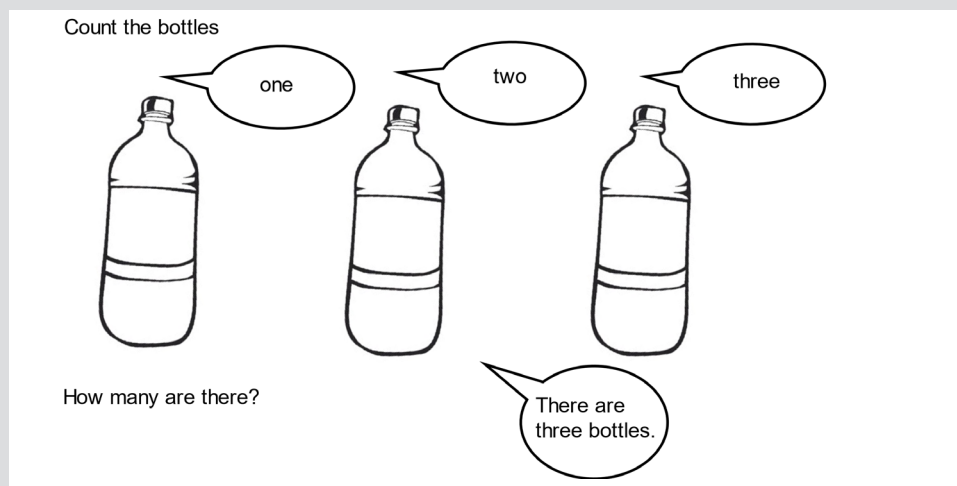
Children have reached stage 3 of the counting trajectory when they can apply the:

Stable order principle: Children know the small number song. Number names are always arranged in the same fixed order, e.g. one is followed by two, two is followed by three, three is followed by four, and so on.

To **count in 1s with meaning**, children need to;

- Know the **number song** (say the number words in sequence) starting at 1 (like a rhyme or a song).
- Match the number word with each action or object. This is **one-to one correspondence**.
- Know that the last number name that they say, is how many there are in the collection (**cardinality** of the collection).

How many bottles?



Some children **will not match** each word to each bottle. They might say “one-two-three-four”, but they don’t match each number word to each bottle. They know the forward counting song in ones, but they have not yet matched each number word to each object. Touching each bottle, and saying each word may assist.

Some children **will match** each word to each bottle. They might say “one-two-three”. But when you ask them: So, how many bottle are there? They start counting again from one. They say “one-two-three” again. This child does not yet know that the last number they say, is the number of the set of bottles. To see if they can apply the cardinality principle for four, you can also ask them: Show me 4 claps, or click 4 times or fetch me 4 pencils.

If a child cannot do this for four, then try the same activity for smaller numbers. They develop the cardinality for sets of 1,2 and 3 objects and events, before they develop their sense of cardinality for bigger numbers.

Stage 4: Match the number word to each object – order irrelevance

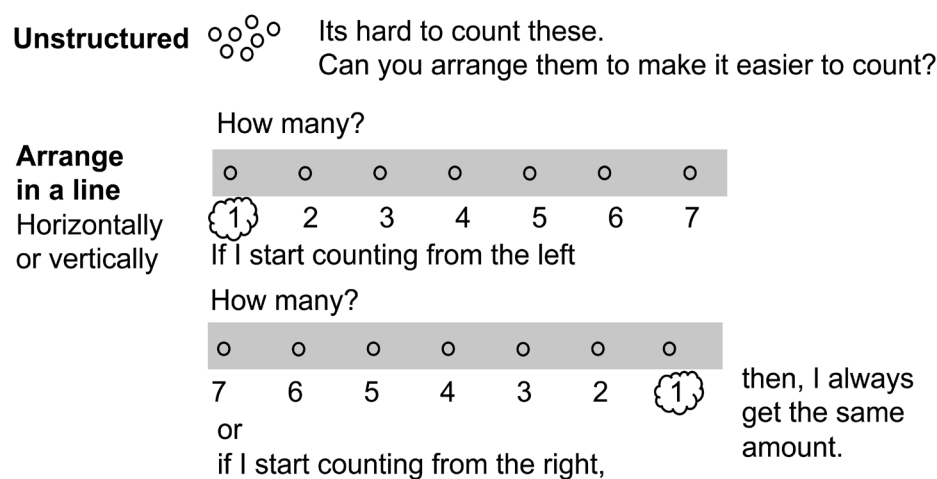
By the time children reach stage 4, they can **classify and sort** to choose what they count. They can **match objects one-to-one** (stage 1). They can tell you **which group has more, and which has less**. They know when two small groups are not the same (stage 2). They **know the number song** (possibly in more than one language) (stage 3), and they say the number words for the number song **in order** – forwards and backwards.

At stage 4 they now connect one-to-one matching with the number song. They say each number word as they touch each object **AND**, they realise that the order that they use to touch each object does not matter.

At first children don't structure groups of objects. Counting is then more difficult as they lose track of which objects they have touched and counted. They may count an object twice. They may miss out an object and not count it.

With help, children start to **arrange objects into patterns** so they can keep track of which objects they have counted. They may arrange the concrete counters in a long line (**vertically or horizontally**).

Diagram: From unstructured to arranging in lines or patterns



When children realise that it does not matter which side they start to count from (left or right, top or bottom), they apply the **order irrelevance principle**. They know that however they arrange the objects, they will always get the same amount.

Order-irrelevance principle: The order of counting the objects in a collection does not matter. Children need to understand that however we arrange the objects, the total number of objects in the collection remains the same.

The number song is different depending on the language used.

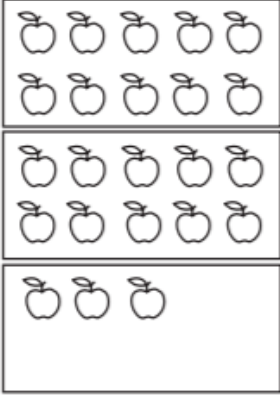
Children must learn to **say the small number song (counting in 1s) forwards and backwards**. When they get to school they start to write the numbers words (eg "two") and number symbols (eg. 2).

To **write** all numbers we only need 10 number symbols:
1; 2; 3; 4; 5; 6; 7; 8; 9 and 0.

But, to say all numbers, we don't only use 10 number words.

In most South African classrooms the number song is learnt in two languages: a home language and English.

Depending on the language we speak, we can have far more than ten words. We also have different ways of combining the words to denote the counting unit, and rules for how we say numbers that are bigger than ten.

Stimulus	English	isiXhosa	Sesotho
A visual stimulus depicting a number of objects. This could be imagined from situation or a story problem, or shown visually as concrete objects, iconic pictures of a set of objects, or a more indexical depiction of the objects.			
A chain of sounds which makes up the number word	twen-ty-three	a-ma-shu-mi -a-ma-bi-ni- a-nan-ta-thu	ma-sho-me -a-ma-be-li- a-me-tso-e- me-ra-ro
The 10-wise place value structure implicit in the number name, number symbol	2 tens and 3 units (Transparent in number words for isiXhosa and Sesotho, but not transparent for English)		
The number name written using letters of the alphabet and pronounced using a chain of sounds.	Twenty-three	amashumi amabini anantathu	mashome a mabeli a metso e meraro
A multi-digit number symbol making use of only 10 digits (0, 1, 2..., 8, 9)	23		

A number naming system is **regular** if the rules for creating big numbers are consistent (there are no exceptions). **A number** naming system is **transparent** if you can hear the counting units (in tens and hundreds)

isiXhosa and Sesotho are transparent.

How can a teacher use number names in African languages, to help build number sense?

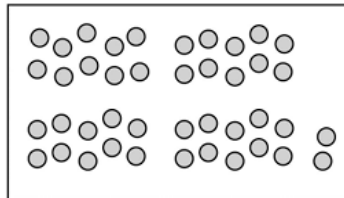
Structuring representations

Number pictures: Structuring by 1s, 10s and 100s.

Tasks: Fluencies and structuring (1s, 10s and 100s)

Task 2.3.1: Number pictures with 1s, 10s and 100s

1. Circle the tens

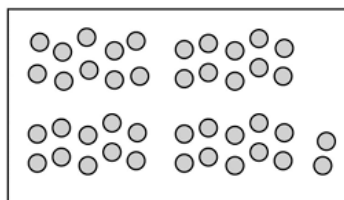


How many 10s? _____

How many 1s? _____

--

Children will start drawing with 1s.



How many 10s? _____

How many 1s? _____

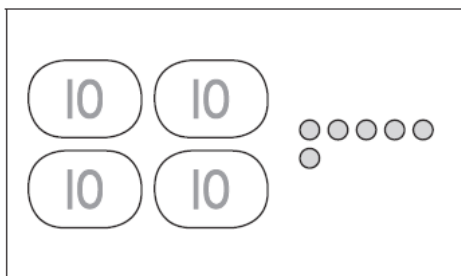
--

As teachers we help children to circle 10 ones, and then to write 10.
As teachers we ALWAYS draw by structuring in 100s, 10s and 1s

2. Quickly see and write down how many there are:

What is the number?

"10s:" means how many tens?
"1s:" means how many ones?

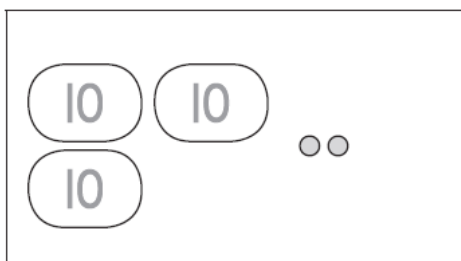


10s:	1s:
4	6

46

$$46 = 10 + 10 + 10 + 10 + 6$$

$$46 = 40 + 6$$



10s:	1s:

--

Circle the 10s. How much money?



3.

How many 100s? How many 10s? How many 1s?



Draw each number. Draw to show one. Draw to show ten. Draw to show 100.

342		<table border="1"> <tr> <td>100s?</td> <td>10s?</td> <td>1s?</td> </tr> <tr> <td>3</td> <td>4</td> <td>2</td> </tr> </table>	100s?	10s?	1s?	3	4	2
100s?	10s?	1s?						
3	4	2						
215		<table border="1"> <tr> <td>100s?</td> <td>10s?</td> <td>1s?</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>	100s?	10s?	1s?			
100s?	10s?	1s?						
305		<table border="1"> <tr> <td>100s?</td> <td>10s?</td> <td>1s?</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>	100s?	10s?	1s?			
100s?	10s?	1s?						
142		<table border="1"> <tr> <td>100s?</td> <td>10s?</td> <td>1s?</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>	100s?	10s?	1s?			
100s?	10s?	1s?						

4.

Count forward by 100s:

When we count **FORWARDS** in 100s, we add 100s.



large song (forwards)

5							
23							

Count backwards by 100s:

When we count **BACKWARDS** in 100s, we subtract 100s.



large songs (backwards)

982							
905							

4. We can use the medium number song for adding tens.

We can add big numbers by breaking down the second number. Let's draw number pictures!



Ta' Jola drove 45 kilometres. Then he drove 30 kilometres more. How many kilometres did he drive altogether?

$45\text{km} + 30\text{km} = 70\text{km}$

45

+

10

55

+

10

65

+

10

75

I write the first number.
I put it in my head.

I DRAW the second number.
30 is 3 tens!
I ADD each 10!

$45 + 20 =$

$63 + 20 =$

$57 + 30 =$

Put the first number in your head. Count forward by 10s!



35 + 10 = ____	59 + 30 = ____	37 + 40 = ____	27 + 50 = ____
61 + 20 = ____	38 + 40 = ____	64 + 30 = ____	37 + 50 = ____

5. We can use number pictures and the medium number song for subtracting tens..

We can subtract big numbers by breaking down the second number. Let's draw number pictures!



Usenathi had R75. She bought a book for R30. How much Rand does she have now?

$R75 - R30 = R45$

75

-

10

-

10

-

10

65

55

45

I write the first number.
I put it in my head.

I DRAW the second number.
30 is 3 tens!
I TAKE AWAY each 10!

$63 - 20 =$

$54 - 30 =$

$87 - 30 =$

Solve by breaking down the second number.

$$65 - 30 = 65 - \underline{10 - 10 - 10} = \underline{35}$$

$$59 - 20 = 59 - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$91 - 40 = 91 - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Task 2.3.2: Two minute tango 4

1	$33 + \underline{\hspace{2cm}} = 40$	11	$50 - 10 = \underline{\hspace{2cm}}$
2	$\underline{\hspace{2cm}} + 3 = 60$	12	$75 - 2 = \underline{\hspace{2cm}}$
3	$\underline{\hspace{2cm}} + 7 = 20$	12	$100 - \underline{\hspace{2cm}} = 90$
4	$64 + \underline{\hspace{2cm}} = 70$	14	$9 + \underline{\hspace{2cm}} = 100$
5	$97 + \underline{\hspace{2cm}} = 100$	15	Half of 14 = $\underline{\hspace{2cm}}$
6	$\underline{\hspace{2cm}} + 9 = 100$	16	$140 \div 2 = \underline{\hspace{2cm}}$
7	$83 - 3 = \underline{\hspace{2cm}}$	17	Double 90 = $\underline{\hspace{2cm}}$
8	$84 - \underline{\hspace{2cm}} = 80$	18	$98 - \underline{\hspace{2cm}} = 90$
9	$65 - \underline{\hspace{2cm}} = 60$	19	$\underline{\hspace{2cm}} + 2 = 40$
10	$16 - \underline{\hspace{2cm}} = 10$	20	$\underline{\hspace{2cm}} + 9 = 100$

Fluencies

Use bonds of 10.
Add to visit the next ten.
Subtract to visit the previous ten

Representations

Number lines to visit the next and previous 10 (or multiple of 10).

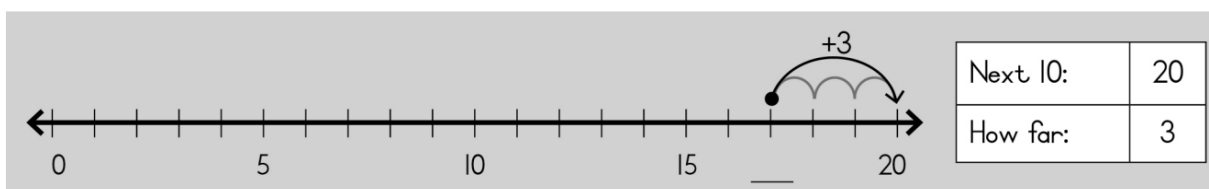
When adding we use number lines, and visit the next ten.

I tap 6 into my head.
What is the next 10?
How far to the next 10?

Zibalo LOVES 10!
When she adds she asks two questions.
What is the next 10?
How far to the next 10?

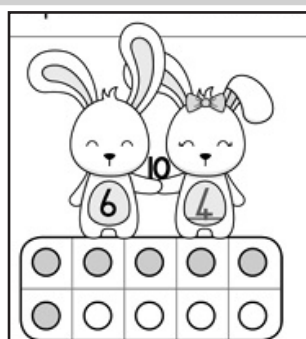
$6 + \underline{\hspace{1cm}} = 10$

So, for any number, how far is it the next 10? This uses bonds of 10.



Another mental map or picture for visiting the next 10, is filling up the 10 frame. You can ask: How many to fill up the ten?

If I know that $7 + 3 = 10$,
then I know that
 $17 + 3 = 10 + 7 + 3 = 20$



SCAN ME



Jumptrak

Two Minute Tango 4

Practice visiting the next and the previous 10.
What is the next multiple of 10?
How far to the next multiple of 10?
What is the previous multiple of 10?
How far to the previous multiple of 10?



What are **the key ideas** which have learnt so far?

Draw 11, 12 and 13 as number pictures.

Write the English number names.

Write the number names in an African language.

Discuss number names in English versus African languages:
how can they help or hinder children's learning?

UMTHAMO 2.4

Cardinality and Abstraction

Remember, the key ideas across unit 2 were:

1. Following the **counting pathway** to learn about numbers, children **count discrete objects** or **discrete events**.
2. Children learn to 'count with meaning' by moving along a **counting trajectory**.
3. Children first learn to count in 1s. But to understand number names beyond ten, they need to **count and structure in 1s, 10s and 100s**. They also count and structure in **2s and 5s**.

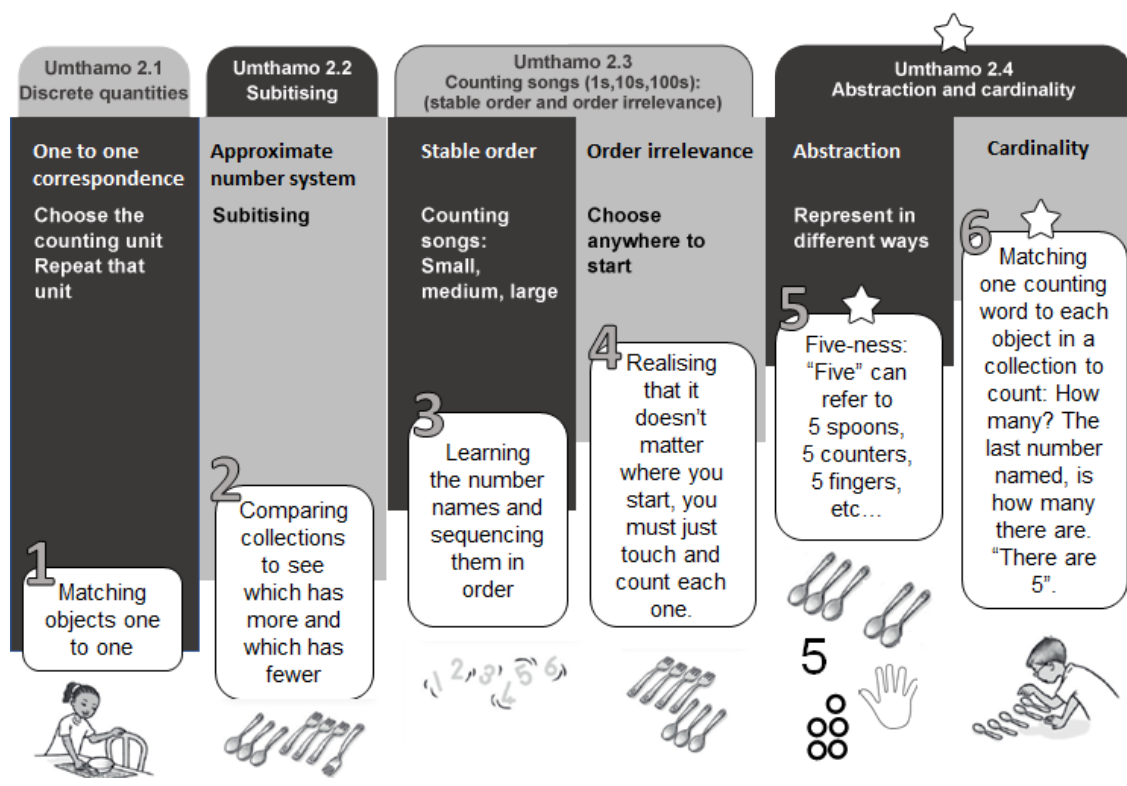
Introduction

Key Ideas

The key ideas in umthamo 2.3 are:

1. **Stage 5 of the counting trajectory is abstraction.** Children move between different representations of a number.
2. In **Stage 6 of the counting trajectory is cardinality.** Children know that the last number they name in the counting song, is how many there are.
3. Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s, 10s and 100s.**
4. You also need to be **fluent with structuring and counting in 2s**. So there are tasks and ideas for your classroom which involve:
 - The number song in 2s
 - Odd and even numbers
 - Doubling and halving
 - Investigate a number **pattern** from counting in 2s. Explore the doubling and halving **functions** and express the function using **algebra**.

Diagram: A learning trajectory for counting



In this umthamo we focus on **abstraction** and **cardinality**.

Stage 5: **Abstract** the number-ness of a number. Know that five can refer to five objects or five pictures or five events or 5 steps etc.

Stage 6: Know how many there are. Match one counting word to each object in the collection and realise that the last number named is how many there are in the set (**cardinality**).

Stage 5: Abstracting

ABSTRACTION principle: Learners understand that even if groups with the same number of objects look very different (e.g. five grapes, five people, five houses) they have the same numerosity, i.e. 'fiveness'. They realise that counting can be applied to objects, pictures, colours, shapes, or even actions or sounds.

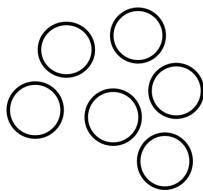


Source: Kuhne, et al. (2017) R-Maths Concept Guide

Teachers can see whether children can abstract by offering different sets, and seeing if the child can see which one does not belong.

Which one doesn't belong?

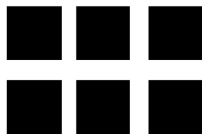
A



B



C



D



In this activity children move between different indexical representations (A, C and D) and the literal representation (B).

They need to realise that A B and C all represent 6, but that there are only 5 lines in D.

Children develop their sense of number sequentially. If a child cannot abstract the concept of 5, then try with smaller numbers. Show them 3 fingers. Ask them to fetch 3 stones. Then they could draw 3 dots.

If a child can't see which set does not belong for 5 objects, then try the same activity but with only 3 objects.

Stage 6: Cardinality

In mathematics, **cardinality** means the size of a set. The cardinality of a set is how many elements are in the set. When we think of numbers as being collections of objects which are in a container, we are using the **cardinal meaning** of number.

Counting objects and saying how many there are, is also called **rational or resultative counting**. This means that objects or events are matched with a number name. To count 'how many', learners need to realise that each object in a collection gets a number name ('one, two, three, four ...') and that you count each object only once.

Cardinality principle: The last number word said when counting a collection represents the total number in the collection. When a child is able to answer: 'How many are there?', they have reached an important counting milestone. They can now count with meaning. They realise that the last number name that they say in the number song, is also how many there are in the collection. They have found the **cardinality of the set**.

SOME IDEAS FOR YOUR CLASSROOM

Maths does not make sense if your only mental picture is counting and structuring in ones. **You need to count and structure in 1s, 10s and 100s, as well as in 2s and 5s**

Children move through the 6 stages along the counting trajectory for 1s. Children also build on their knowledge of counting in 1s and start to:

- Count and structure numbers into 2s and 5s
- Count and structure numbers in 1s, 10s and 100s

Later the counting trajectory is extended as they start to use counting for the four operations (add & subtract, multiply & divide).

Counting and structuring in 2s

Remember that **structuring** is as important as counting. Structuring builds on our ability to subitise.

Counting in 2s

Once children are secure with the number song in ones, they can learn to count in twos.

They need to realise that counting in twos is saying every second number in the counting song. The whisper the first number word, then say the second number word loudly.

Whisper; **loud**, whisper, **loud**, whisper, **loud**, whisper, loud, whisper,

One, **TWO**, three, **FOUR**, five, **SIX**, seven, **EIGHT**, nine, **TEN**, ...

1 2 3 4 5 6 7 8 9 10

Then learners need to stop whispering and just pause:

pause **2** pause **4** pause **6** pause **8** pause **10** pause **12** pause,...

They learn the counting in 2s song:

Starting at 2: **2, 4, 6, 8, 10, 12, 14**, ... and

Starting at 1: **1, 3, 5, 7, 9, 11, 13**, ...

Just as with counting in ones with meaning, children then need to match their counting in 2s song with pairs of discrete objects (cardinal meaning), or as jumps along a number line (ordinal meaning).

Structuring in 2s

Give children situations that involve pairs or jumping in twos to practicing matching each word to each pair or jump of 2.

Counting in 2s ...

How many socks are shown?

It is more efficient to count in 2s than to count in ones. The same counting principles apply, but now the counting song in twos is used.



- Say a "counting in 2s" number song
- Match each word in the counting in 2s number song to one pair of socks (one to one correspondence)
- Notice where they start and end (so they don't count the same pair of socks twice)
- Know that the last number they say in the counting song is how many socks are in the set (cardinality).

Key A doubling function

Once children can fluently count in twos forwards and backwards, they can link this to doubling. To double, you choose a number and repeat it. You take 2 groups of that number. The idea of a mirror line, or line of symmetry can help children picture what happens when you double.

Start with small numbers (less than 5), then increase the size of the numbers for children to practice doubling. Notice that numbers bigger than ten are structured in 10s. The number picture shows groups of tens – and not ones.

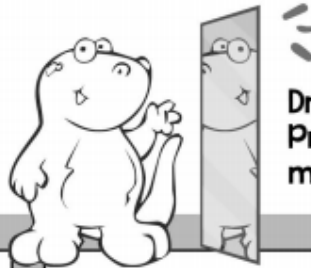
GAME OF THE WEEK:

Double!

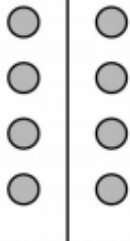
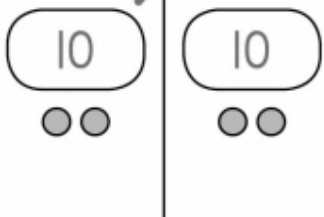
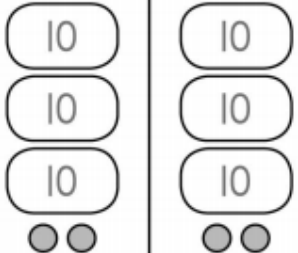
- Draw a line as a magic mirror.
- Your teacher will call a number (0 to 100)
- Draw the number on the left of the mirror
- Draw the number on the right of the mirror.



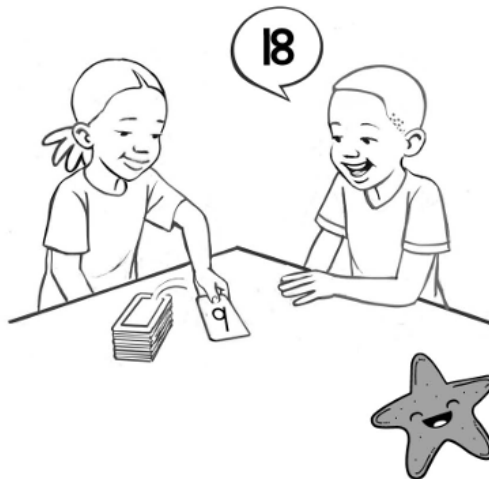
Doubling a number is about repeating a number 2 times!



Draw a line!
Pretend that it is a magic mirror! Draw double!

Double 4  Double 4 is <u>8</u> $4 + 4 = \underline{\quad}$ There are two 4s in 8.	Double 12  Double 12 is <u>24</u> $12 + 12 = \underline{24}$ There are two 12s in 24.	Double 32  Double 32 is <u>64</u> $32 + 32 = \underline{64}$ There are two 32s in 64.
---	--	---

Once children have a written method or picture for doubling, they can practice doubling mentally.



GAME OF THE WEEK: Fast Maths with Cards

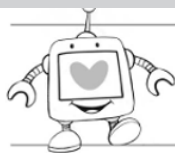
DOUBLE

- Place number cards 0 to 5 in a pile. Mix them up.
- Flip over one card.
- DOUBLE!
- Do it again! Faster!

When you get very fast, add number cards 6 to 10. Then add 11 to 15! Can you get up to 20?

If children can double, you can help them think about a doubling function.

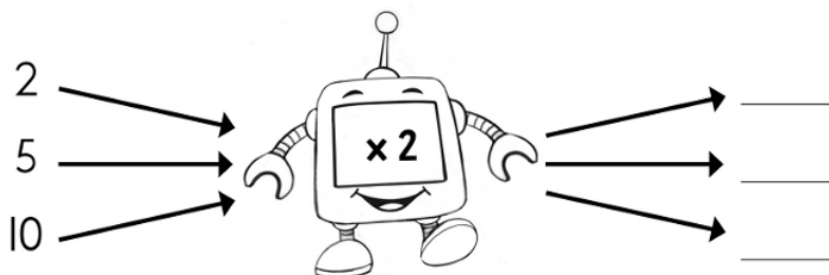
Key Doubling as a x2 function



This is a friend of Zibalo called, Function. He is a Maths Robot. His friends call him Funky. He can be programmed to produce different math functions.



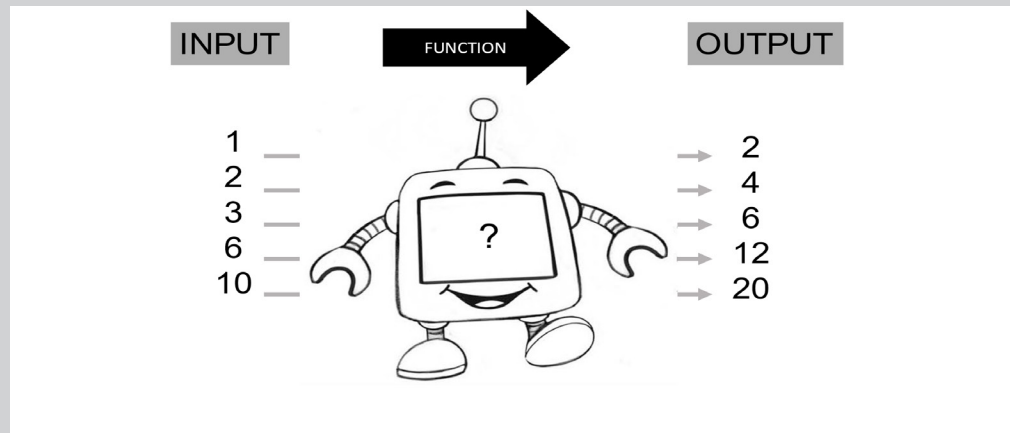
Function is now programmed to multiply 2s. When you put a 3 in to Funky, it produces 6! What will happen when we put each of these numbers into Funky?



Children can play: I am thinking of a number.

- I am thinking of a number...Double my number is 6. What's my number?
- I am thinking of a number...Double my number is 10. What's my number?
- I am thinking of a number...Double my number is 4. What's my number?
- I am thinking of a number...Double my number is 12. What's my number?

They are then ready to play: **What's my function.**



I have been programmed to be a function (like funky). Every time you give me a number (an input), I change that number in the same way.

- If the input is 2, then the output is 4
- If the input is 6, then the output is 12, etc

You can ask children to give you any input. You double it to give the output.

Key The inverse function of doubling is halving.

Sharing between two children is the same as dividing by 2!
We say, "10 hlula ka 2 ngu-5." We write $10 \div 2 = 5$. Half of 10 is 5!

5

Some things can be cut in half. We share by cutting in half!

I share 5 amagwinya equally between 2 children. Each child receives 2 and a half amagwinya.

5

Some things cannot be cut in half. When we share, sometimes we have some left over.

I share 5 marbles equally between 2 children. Each child receives 2 marbles. There is one marble left over.

Share equally between 2 children. How many does each child receive?

9

I share amagwinya!

9

I share marbles!

15

15

$9 \div 2 = 4 \text{ and } 1 \text{ half}$

$9 \div 2 = 4 \text{ and one left over}$

$15 \div 2 =$

$15 \div 2 =$

Note

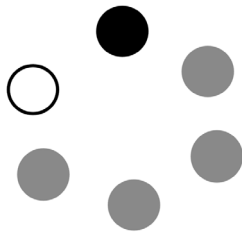
Halving is the same as sharing equally between 2. There are two ways to halve an odd number. Depending on what we are halving we get 1 left over, or 1 half.

Tasks: Conceptual understanding

Task 2.4.1: Cardinality for 'counting with meaning'

A teacher wants to see whether her grade R children can count with meaning (rational counting). She wants to see if they can apply the cardinality principle.

She puts some counters onto the mat (or desk):



Prudence starts at a grey circle. She counts: 1, 2, 3, 4, 5, 6, and carries on going round the circle...7, 8, 9, 10.

Chantal starts at the black circle.

She counts: 1, 2, 3, 4, 5, 6 as she touches each circle.

When the teacher asks: "So how many are there?" She says: "6".

Evan starts at the black circle.

He counts 1, 2, 3, 4, 5, 6 as he touches each circle. He stops.

Then he starts at the white circle and counts again, matching each word to each circle 1, 2, 3, 4, 5, 6.

Faith touches the black circle. She says: 1.

She touches the white circle. She says: 2,3.

She touches each grey circle. She says: 4, 5,... 6, 7, 8,... 9, ...10.

1. Which child does NOT YET apply the **order irrelevance** principle?
 - a. Prudence
 - b. Chantal
 - c. Evan
 - d. Faith
2. Which child does NOT YET apply the **one-to-one correspondence** principle?
 - a. Prudence
 - b. Chantal
 - c. Evan
 - d. Faith
3. Which child clearly shows they can apply the **cardinality** principle?
 - a. Prudence
 - b. Chantal
 - c. Evan
 - d. Faith

Tasks: Fluencies and structuring (2s, doubling and halving)

Task 2.4.2: Doubling and halving

- Practice doubling, and show your working using structured number pictures.

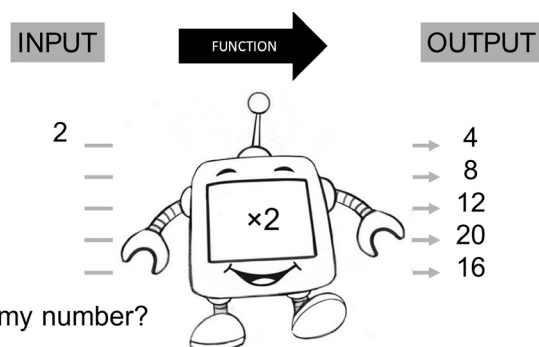
3×2	30×2	12×2
$3 \times 2 = \underline{\quad}$	$30 \times 2 = \underline{\quad}$	$12 \times 2 = \underline{\quad}$

Structuring representations

Number pictures: Using a mirror line for doubling and halving

- What is my number?

- I am thinking of a number. Double my number is 4. What's my number? _____
- Double "my number" is 8. My number is _____.
- Double "my number" is 12. My number is _____.
- Complete the doubling function diagram.



- What function is the inverse of doubling? _____
Write the function on funky's screen

- Complete

Double 5

○		○
○		○
○		○
○		○
○		○

5 = 2 + 3, so
Double 5 = Double 2 + Double ____
 $5 \times 2 = (\underline{\quad} \times 2) + (3 \times 2)$

Double 2

○		○
○		○

Double 3

●		●
●		●
●		●

- Complete

Double 26

○		○
○		○
○		○
○		○
○		○
○		○
○		○

26 = 20 + ____, so
Double 26 = Double 20 + Double ____
 $\underline{\quad} \times 2 = (20 \times 2) + (6 \times 2)$

Double 20

○		○
○		○
○		○
○		○
○		○
○		○

Double 6

○		○
○		○
○		○
○		○
○		○

3. You can play fizz-pop where the function is: Double.



Show what you would draw on the board to help Grade 1 learners work out double 8.

4. You can also double 8 like this: Count forwards in 2s, for 8 counts. Keep track of how many counts on your fingers. This works best with a context. There are 8 children, how many eyes?



Children		Eyes
1		2
2		—
3		—
4		—
5		—
6		—
7		—
8		—

5. When children know the doubles for 1-10 well, then they can make connections.

- I know that double 3 is 6. Therefore I know that double 30 is _____.
- I know that double 4 is _____. Therefore I know that double 40 is _____.
- I know that double 3 is _____ and double 10 is _____.
Therefore I know that double 13 is _____.

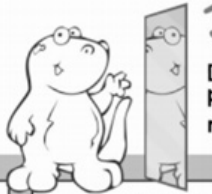
Fluencies

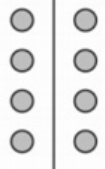
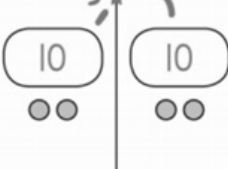
Doubling and halving

Task 2.4.3: Two minute tango 5

1	Double 2 = _____	11	$20 \div 2 =$ _____
2	$3 \times 2 =$ _____	12	Double 40 = _____
3	5 repeated 2 times = ____	12	$30 \times 2 =$ _____
4	Double 7 = _____	14	Half of 60 = _____
5	$9 + 9 =$ _____	15	$50 +$ _____ $= 100$
6	$9 \times 2 =$ _____	16	$20 \div 2 =$ _____
7	$6 \times 2 =$ _____	17	Half of 160 = _____
8	Half of 8 = _____	18	_____ $+ 400 = 1\,000$
9	$4 \div 2 =$ _____	19	$100 -$ _____ $= 70$
10	Half of 16 = _____	20	1 more than 300 is _____

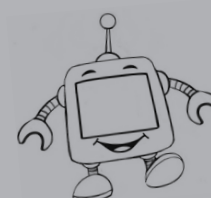
 Doubling a number is about repeating a number 2 times!

 Draw a line! Pretend that it is a magic mirror! Draw double!

Double 4  Double 4 is <u>8</u> $4 + 4 =$ _____ There are two 4s in 8.	Double 12  Double 12 is <u>24</u> $12 + 12 =$ <u>24</u> There are two 12s in 24.	Double 32  Double 32 is <u>64</u> $32 + 32 =$ <u>64</u> There are two 32s in 64.
---	--	---

Representations

Doubling uses a mirror line
Halving is sharing equally between 2.
Doubling and halving are inverse functions



Make connections

If you know one number fact (eg. a double), then use the **small, medium & large number songs**, to work out new facts

If you know your **doubles**, you know your **halves** (they are inverses)

- If **double 4** = 8, then **half** of 8 = 4
- If **double 6** = 12, then **half** of 12 = 6, etc.

If you know your doubles, then you can use this for small (1s), medium (10s) and large (100s) songs:

- If double 7 = 14, then **double 7 tens** = **14 tens** = **140**
- If double 7 = 14, then **double 7 hundred** = **14 hundred** = **1400**

SCAN ME



Jumptrak

Two Minute Tango 5

Practice doubling and halving. Multiply and divide by 2. Repeat twice, or count in 2s.



Draw a mind map of the **counting trajectory**. Show all six stages.

.....

UNIT 3:

The measurement pathway

In unit 2 we focused on the counting pathway. Now, in unit 3, we focus on the other main route to develop children's number sense: the **measurement pathway**.

Key Ideas

The key ideas in Unit 3 are:

1. **Following the measurement pathway children compare continuous quantities that can be measured.**
2. Children learn to **measure by following the same 4 steps.**
3. Children first learn to measure 1s. But very soon they use **standard units using 1s, 10s, 100s and 1000s**

Following the measurement pathway children compare continuous quantities that can be measured

Any number always occurs in a sequence of other numbers. Children learn the counting song as a list of words which is always in the same **order**. They know the sequence "one-two-three-four-five". They start to learn that:

- three comes before four; and that
- three comes after two.

Children make lists and use the ordinal numbers to plan actions: First, second, third.

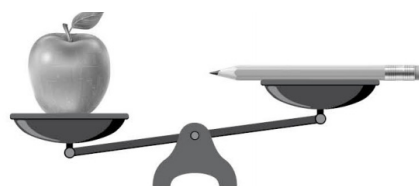
In the **measurement pathway** children reason about, and act on, **continuous quantities**. A continuous quantity is unbroken. Children **measure these quantities using attributes** such as length, height, mass, volume, capacity, time, and temperature. The measurement pathway into number starts with continuous quantities that can be compared directly and later measured.

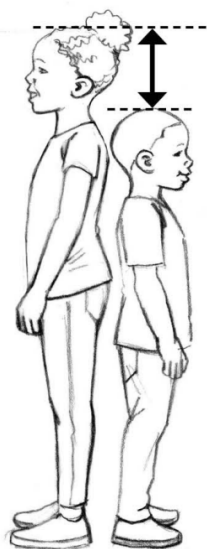
To measure we introduce a unit (a standard or a non standard unit). The measurement pathway connects number work with spatial reasoning (shape and space, and measurement contexts) in early mathematics.

To compare continuous quantities you have a few options.

First, you could compare the objects directly.

To see which is heavier – an apple or a pencil – they can be compared directly on a balance scale.





To compare continuous quantities you can compare them directly.

Children are able to order and compare objects or events by particular attributes:

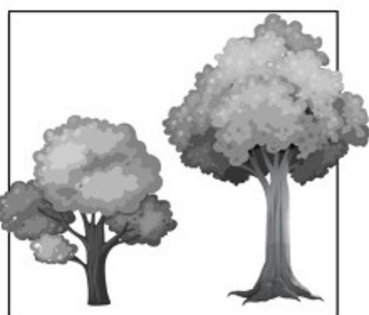
- Tall, taller, tallest.
- Heavy, heavier, heaviest.
- Late, later latest

Height, time and mass are all examples of continuous measurement contexts.

To see who is taller, children stand back to back.

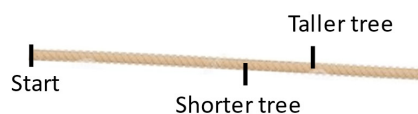
Second, you could choose a measurement unit that is bigger than the attribute.

To compare continuous quantities which cannot be moved or lined up, you could use a unit which is bigger than each quantity. So comparing the width or circumference of two tree trunks you can use a rope or a string, and mark off the circumference of each tree trunk.



Measuring the circumference:

Use a long rope to wrap around each tree.

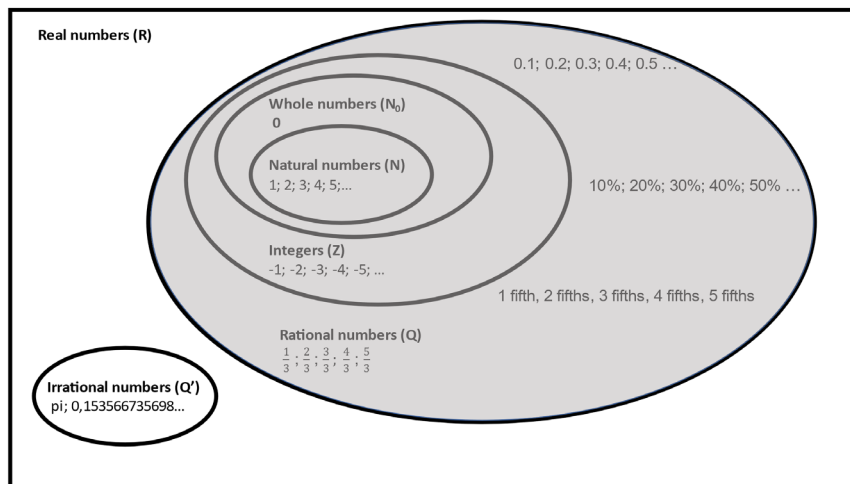


To see whether a cupboard will fit into the space between the bed and wall, you could use a dressing gown cord or belt. Mark the width of the space on the cord and compare it to the width of the cupboard.

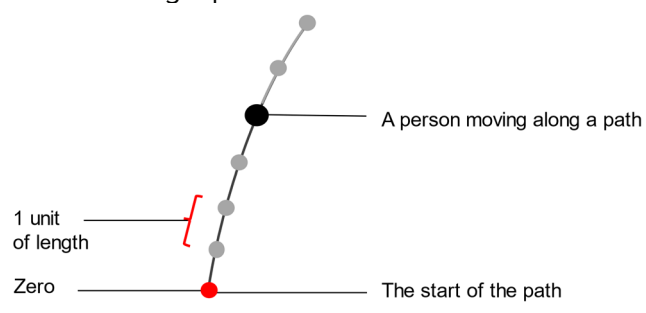
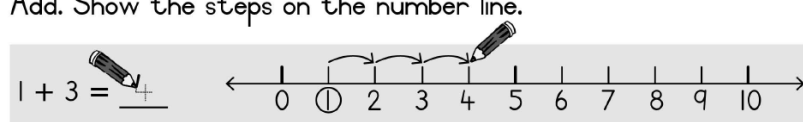
Third, you could choose a measurement unit that is smaller than the attribute. You repeat that unit to quantify the attribute.

The continuous quantity becomes countable by introducing a **unit of measurement**. At first this unit of measurement is repeated using counting or natural numbers. Very quickly they need to use parts of a unit, and rational numbers are needed.

Diagram: Number systems



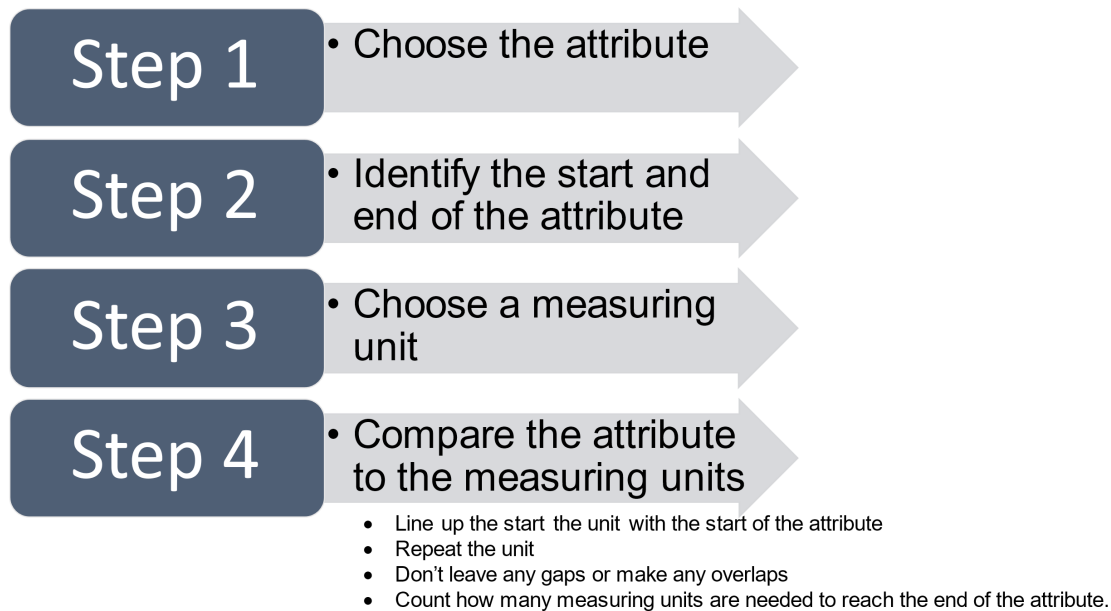
These are the main features of the measurement pathway.

Quantities	Continuous quantities that can be measured
Pathway into number sense	Measurement pathway
Unit	Unit of measurement (standard or non-standard)
Metaphor	Movement along a path 
Representations	Number lines Add. Show the steps on the number line. 
Number meaning	Ordinality Numbers are in a sequence arranged from smallest (first) to biggest (last)

When children measure they always follow the same 4 steps.

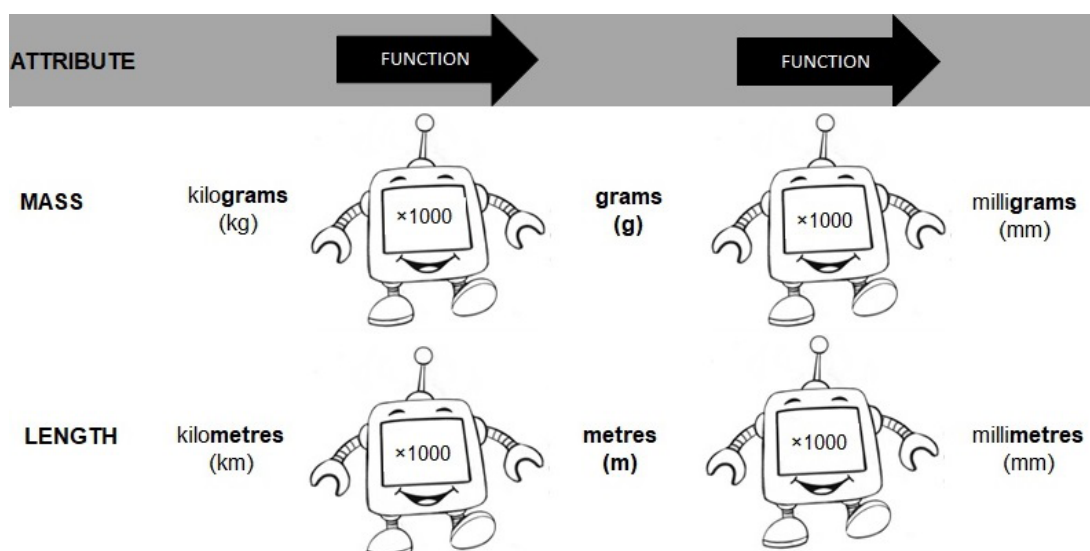
Children learn to measure, by **following these steps**:

Diagram: 4 steps for measuring



Children first learn to measure in 1s. But very soon they use standard units using 1s, 10s, 100s and 1000s

In the measurement pathway children reason about, and act on, **continuous quantities**. They **measure these quantities using attributes** such as length, height, mass, volume, capacity, time, and temperature.



CONTINUOUS MEASUREMENT

Key Ideas

The key ideas in Umthamo 3.1 are

1. **Children experience number** – not just as discrete objects or events – but also as **continuous quantities**
2. **A mental picture for measuring a continuous quantity (like length) is a person stepping along a path**
3. The ordinal meaning of number is having the numbers in a sequence
4. Whichever attribute children measure, they always follow the same 4 steps
5. We measure different attributes: **length; time; mass; capacity; volume; temperature**
6. **Maths does not make sense if your only mental picture is measuring and structuring in ones.** To understand measurement you need to understand the base-ten number system of place value and the Standard International (SI) system of measurement (in 1s, 10s, 100s, 1000s). For time there is an unusual system of measuring units involving 5s, 12s and 60s.

Continuous quantities

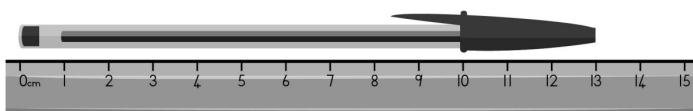
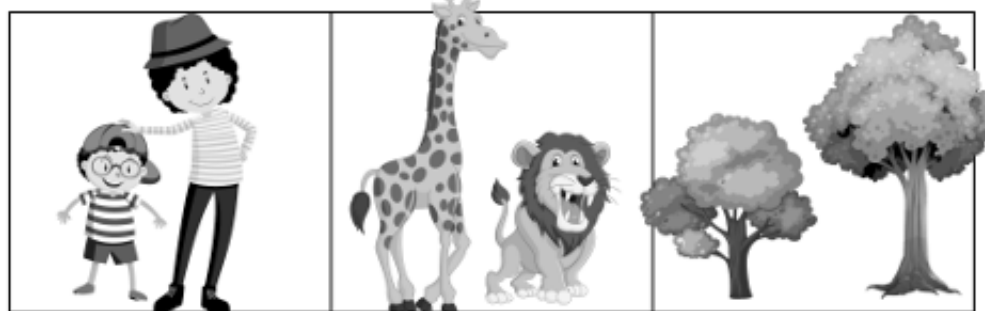
The **measurement pathway** into number starts with **continuous quantities** that can be compared directly and later measured. Only later are discrete objects introduced. To measure we introduce a unit (a standard or a non standard unit). The measurement pathway connects number work with spatial reasoning (shape and space, and measurement contexts) in early mathematics.

When children measure a continuous quantity, they must **first decide what attribute should be measured**. They must decide what are they measuring: they are measuring length, they are measuring mass. At first this is about direct comparison. Which object is taller? Which is shorter?.

Activity for learners

Length measures the HEIGHT of objects - the distance between the top to the bottom! We say an object is **TALLER** or **SHORTER** than another object.

Circle the **TALLER** object:



Inde kangakanani ipeni?		Inde kangakanani?
Inde kangakanani ipensile?		

At first children compare without measuring. At school they start using measuring instruments (like a ruler) to compare and measure.

To compare continuous quantities you can **compare them directly**. To see who is taller, children stand back to back.

Continuous quantities are unbroken. They can be measured using a measurement unit. The amount of time, the amount of ground and the amount of water, the volume and the mass are continuous. We can't count time, distance, water, or length. We need to measure using an agreed unit, as these are continuous extents.

To measure a continuous quantity children have to introduce a **measuring unit**. The unit could be standard or non-standard. Repeating the unit allows them to count the units of measurements.

Measure the LENGTH of each object using cubes.

10 cubes

2 cubes

1 cubes

1 cubes

10 cubes

1 cubes

1 cubes

10 cubes

In this learner activity the measuring unit is a cube. Cubes are smaller than the length of the objects being measured.

🔑 A metaphor for continuous measurement: 'Walking along a path'

In the measurement pathway, the **metaphor for number as a movement of a point (or object) along a line (or path)** is discussed in detail by Lakoff and Núnes (2001) in their book *Where Mathematics Comes From*.

Diagram: The 'point moving along a path' metaphor for number

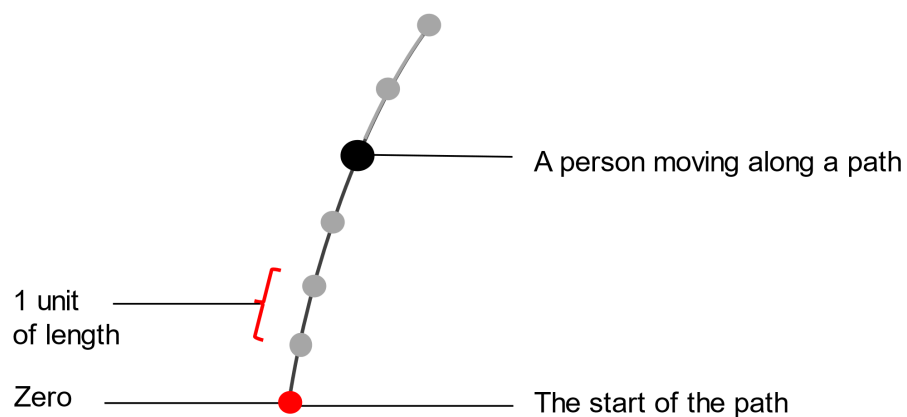


Another way that we think about a number, is using the idea of a **path**.

5 is a movement of 5 steps/sections along a line.



Diagram: Number line to show the number five is a continuous length (5 steps)



Key The ordinal meaning of number in a sequence

We take the 1st step, 2nd step, and then the 3rd step as we move along a path. There is a continuous length of 5 units (steps or metres) from the starting point (zero). The number 5 is not seen in isolation. It is thought about as having a **position (a place in the sequence of numbers)**. Five denotes the 5th step or the 5th unit of length. Five comes after 4 and before 6.

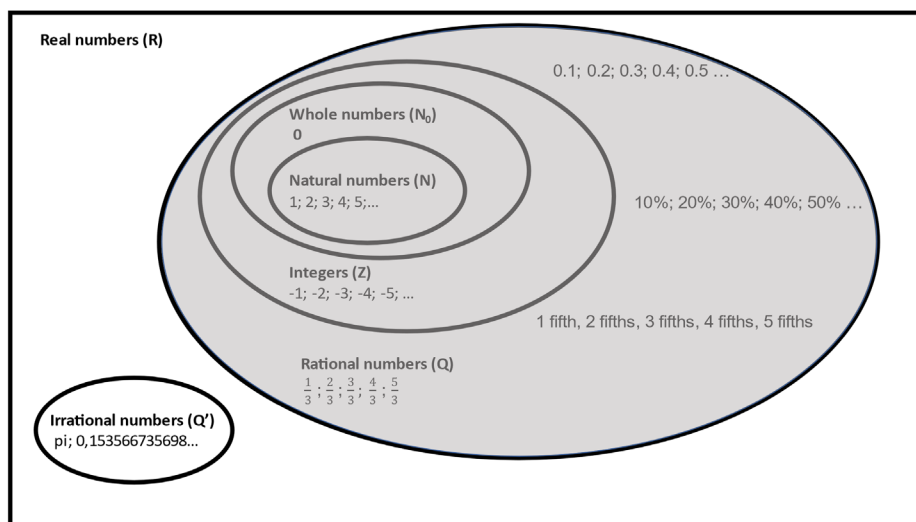
Together with developing language, **babies start to move through their environment**: first crawling or sliding, then walking. To move through the world, the baby uses their **spatial reasoning**. They need to know where they are, and plan a route to get to a new position. Humans learn how to walk and use paths to keep track of their **position relative to fixed points** (like their bed, their home, or the nearest shop). **People can imagine their own movement as if their body is a dot moving along a line**. This is a powerful mathematical idea, and is used in many communities and in further mathematics (such as dynamic geometry and calculus).

The **ordinal** use of number is also evident in the counting song. The number song is first learnt as a **sequence** or **order** of words. The words don't yet have meaning. The 1st word we say is 'one', the 2nd word is 'two', then 'three' and so on. So the number words in the counting song are in a sequence or an order. There is a 1st, 2nd, 3rd and 4th number. The number song is said within the continuous quantity of time, which we can measure. We can say it fast, or we can say it slowly. There is a rhythm or a beat to the number song.

We are using the ordinal use of number when we ask children: 'What number comes **before** 4?' or 'What number comes **after** 4?' So we think about a particular number (like 4) in relation to other numbers. **We think of the number as having a position or a place in the number sequence**. Five means the measurement (often a length) or the sequence of an event (a step along a path). Five comes after 4, and five happens before 6.

To measure we introduce a measurement unit, to make a continuous quantity countable. As we often get partial units, we use the set of **rational** numbers. The symbol for natural numbers is $\mathbb{N}$. A rational number can be represented as a **fraction**.

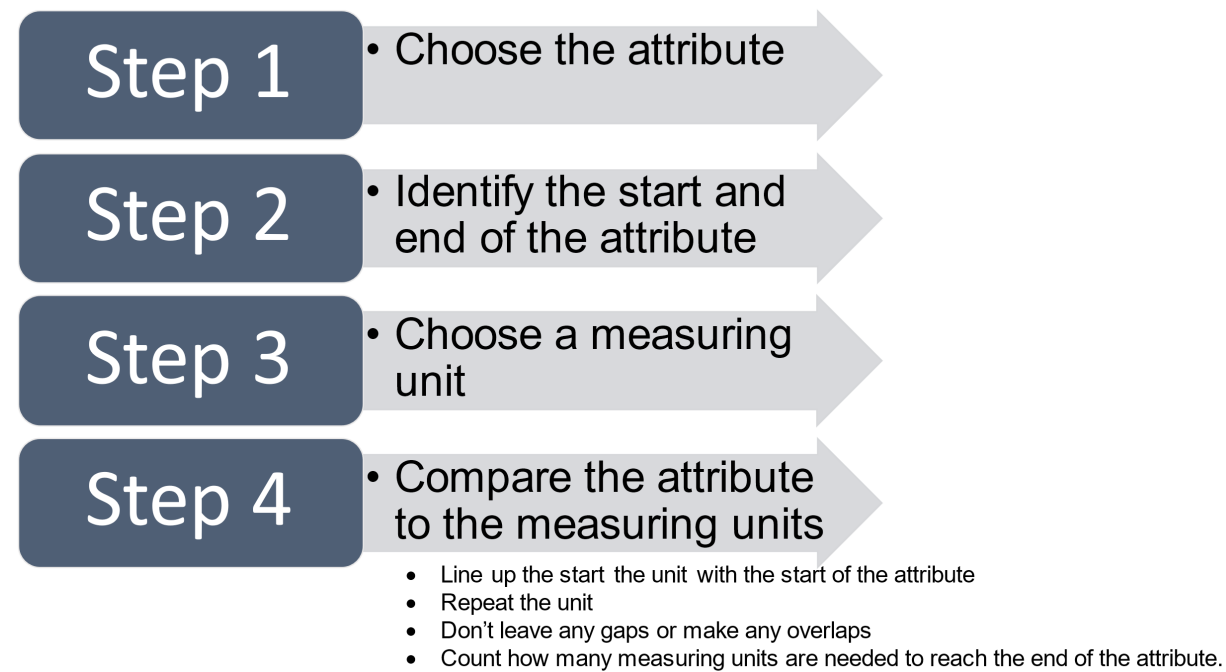
Diagram: Number systems



Key Which ever attribute children measure, they must follow the same 4 steps

Children learn to measure, by **following these steps**:

Diagram: 4 steps for measuring



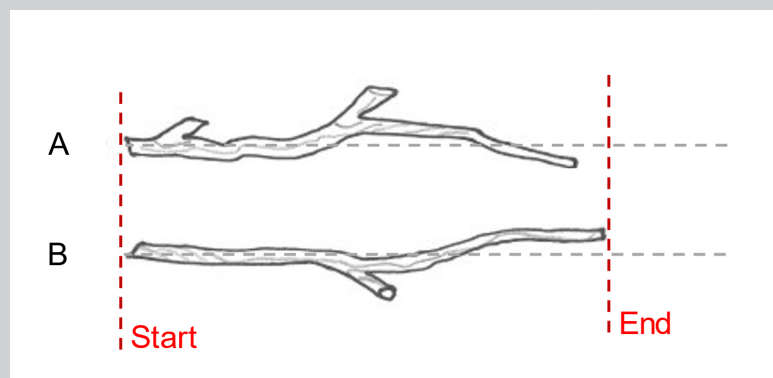
To compare two sticks the children have to decide what aspect of the sticks they are comparing. They need to choose an **attribute** to compare.

- Some children might compare the lengths. Which stick is longer?
- Some children might compare the mass. Which stick is heavier?
- Some children might choose another attribute like – which one is straighter? Which one is smoother or more flexible?

Activity for learners

Let's agree that the children will compare the lengths of the sticks.

Compare the length of two sticks

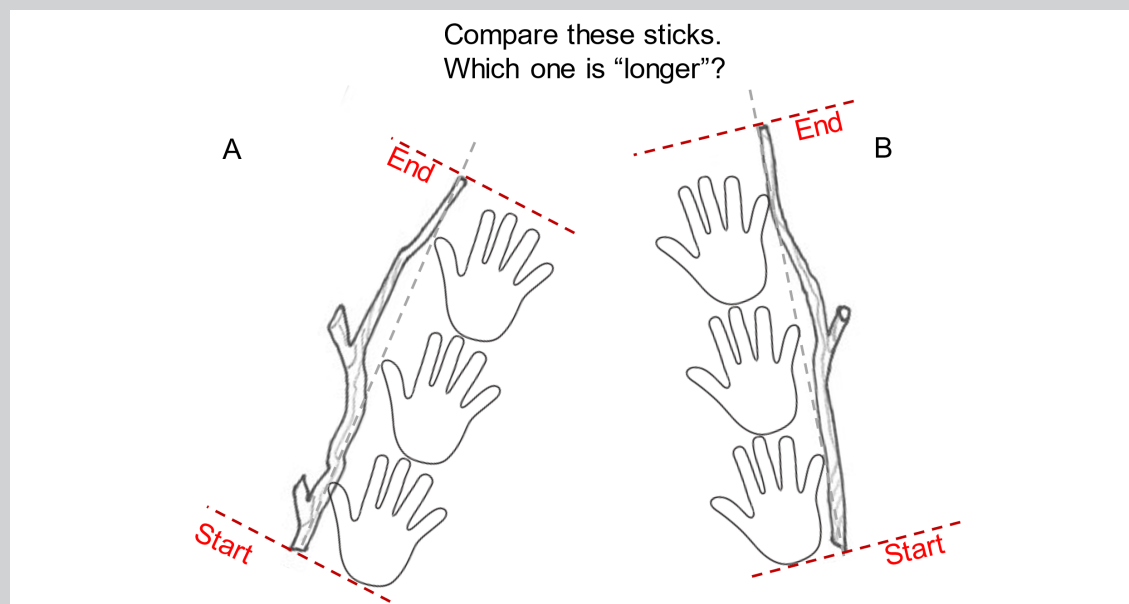


Even very young children can choose the longer stick. The attribute is length. The child must position both sticks at the **same start point**. The sticks must both be straight (so their lengths are parallel). They can see which is longer.

The children are not counting or using number as a collection of discrete objects. But they are **comparing lengths** (using their visual sketch pad). They have to **imagine the start and end lines**. They have to move the sticks into position, and orient the sticks next to each other. Then it is possible to compare lengths.

But what if they could not move the sticks? How could they compare the lengths?

Compare the length of two sticks with informal units



Perhaps the sticks are on a tree, or stuck in the ground, or drawn on the blackboard. If children can't move the sticks, the children might **introduce a unit to measure with**. The unit might be simple – like **a piece of string that is longer than either stick**. They must agree on where to start measuring – and where the end of the stick is. They line up the string on one stick. Then take the string to the other stick to compare lengths.

They might use a body part – like their arm - to create a unit for length. They line up each stick from their finger tip and see how far the stick goes up their arm.

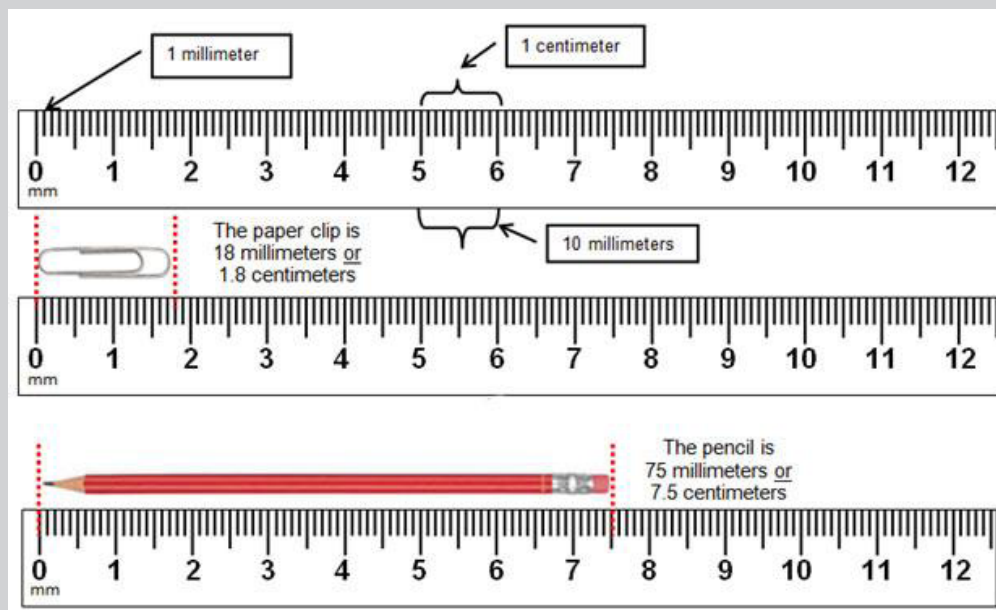
They might use a hand. They then have to carefully be sure to start at each end, and repeat the unit (their hand) with no gaps. They count how many hands each stick is. They might mark off on their hand for how much extra is needed.

Notice that now – when using a small unit like “a hand”, **they start using numbers**. The sticks are both: “three-and-a-bit hands”. Stick B has a bigger “bit” than Stick A. This is the beginning of fractions!

Measuring different attributes

When children compare and measure continuous quantities, they use the ordinal meaning of number. They choose a measurement unit and repeat it, counting in order. The measurement tools (instruments) use an **ordinal scale**.

Examples



Length and distance are **continuous**. You can't count the length of a pencil. You need to agree on a unit to measure the length of the pencil. You could use an informal unit like a paper clip.

The standard unit for measuring length is a **metre**. A bigger unit of length is a kilometre. Smaller units of length are centimetres and millimetres. We can use a **ruler** to measure length. A ruler is ordinal. A ruler has a structured number line, where

- the numbers are equally spaced (the intervals are equal),
- the numbers are arranged from smallest to biggest,
- it starts at 0.
- the line is used vertically or horizontally.



CAPACITY is a measure of the maximum amount of liquid a container can hold.
VOLUME is a measure of how much liquid a container holds now.



Colour to show full, half full and empty.

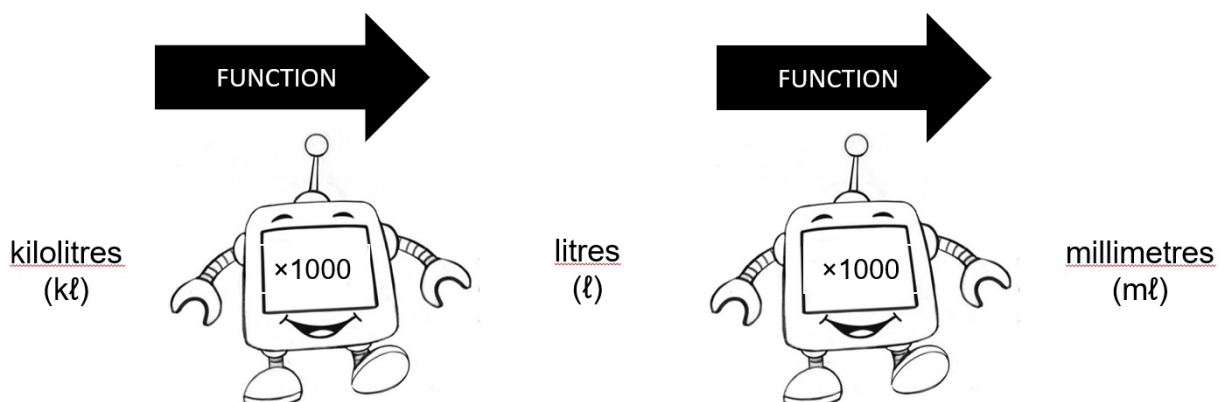


Capacity is how much a container can hold. **Volume** is the amount of liquid in a container. Both capacity and volume are **continuous quantities**.

We often use containers to measure the volume of a liquid. A liquid is continuous. To collect water from a river, you need a container. The amount you collect is measured by the size of container. To quantify the water you choose a unit - 1 **cup**, 1 **bottle** or 1 **bucket** – and repeat that unit. You count how many cups, bottles, or buckets you can fill.

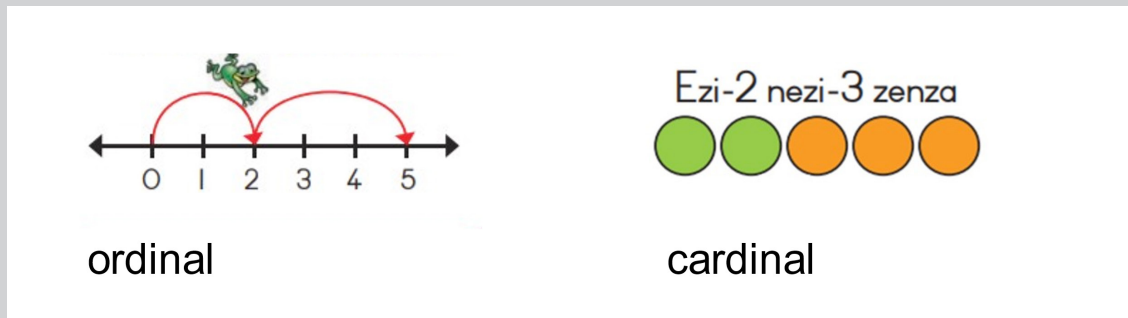
The standard unit for measuring capacity is a **litre**. Bigger units for volume are kilolitres. Smaller units for volume are centilitres and millilitres. We can use a **measuring cylinder or measuring jug** to measure an amount of liquid. A measuring jug has an ordinal scale. A measuring jug has a structured number line, where

- the numbers are equally spaced (the intervals are equal),
- the numbers are arranged from smallest to biggest,
- it starts at 0; and
- the line is used vertically.



 Task 3.1.1: Cardinal (number pictures) and ordinal (number line) representations

This example shows an ordinal and cardinal way to represent $2 + 3 = 5$.



Source: DBE Rainbow book

1. Circle the correct word or phrase.

In the ordinal example:

- a. zero is: the **place to start** OR the **empty set**.
- b. the numbers are: **poles** or **fences**?
- c. The numbers represent: **discrete objects** or **equal jumps along on a line**

2. Circle the correct word or phrase.

In the cardinal example:

- a. zero is: the place to start OR the **empty set**.
- b. the numbers are: **poles** or **fences**?
- c. the numbers represent: **discrete objects** or **equal jumps along on a line**

3. Draw a number line (ordinal example) for $20 + 30 = 50$

4. Draw a number picture for $20 + 30 = 50$.

This is an example of from a grade 1 learner workbook. It shows subtraction as take-away (tabatha)

2 Bhala isibalo soku:

	5	thabatha	1	zenza	4
		thabatha		zenza	
		thabatha		zenza	

3 Ukuthabatha 1 ngokubala ubuya umva.

Thatha ezi-3 kwezi-5



Source: DBE Rainbow book

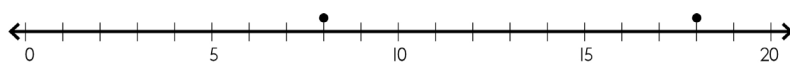
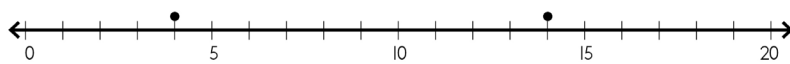
Circle the correct word:

- The triangles are an example of cardinal / ordinal representation of number.
- Is the number line an example of cardinal / ordinal representation of number.
- When a teacher shows subtractions as discrete objects being removed (or taken away), this is a cardinal / ordinal representation of number.
- When a teacher shows subtraction as jumping backwards (to the left) on a number line, this is a cardinal / ordinal representation of number.
- Draw a number picture to show $50 - 30 = 20$
- Draw a number line to show $50 - 30 = 20$.

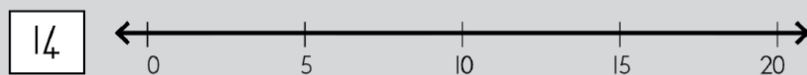
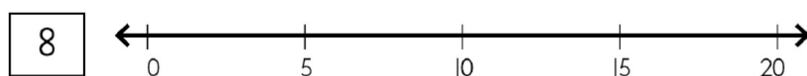
Tasks: Procedural fluencies and structuring

Task 3.1.2: Estimate 0-100 on number lines

1. Write the numbers below each dot



2. Draw a dot and write the number below the line.



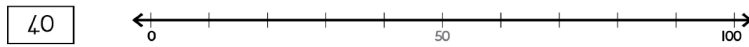
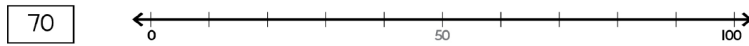
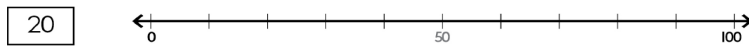
3. Write the number below the dot



Structuring representations

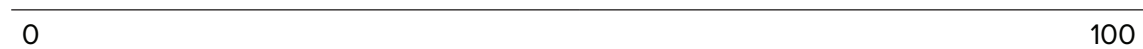
Structured number lines.

4. Draw a dot and write the number on the line.

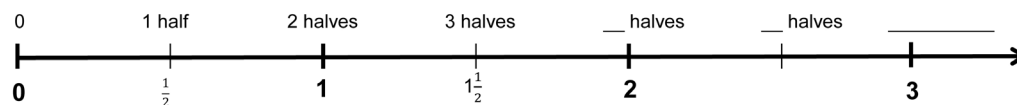


5. Look carefully at the start and end of each number line. What is the size of the interval? Mark and label half way, for each number line.

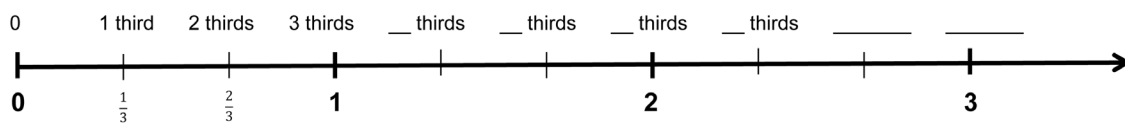
Now, draw a dot for 57 and write 57. Look carefully at each line.



6. How many equal parts in 1? _____. Name of one part: _____



7. How many equal parts in 1? _____. Name of one part: _____



Tasks: Conceptual understanding

Task 3.1.3 Investigate bottles and litres (use a number line)

- Count in 2s, on the multiples of 2.

Count by 2s from 2 to 100.

Colour each block.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Count forwards by 2s.

2	4	6				
---	---	---	--	--	--	--

22	24	26				
----	----	----	--	--	--	--

Count backwards by 2s.

14	12	10				
----	----	----	--	--	--	--

20	18	16				
----	----	----	--	--	--	--

- How many bottles, how many litres of coke? (Specialise for a particular example)

 2ℓ	 2ℓ	 2ℓ	 2ℓ	 2ℓ	How many bottles?	
					How many litres?	

- Complete the number line with equal jumps of 2 each time (Organise on a number line.)

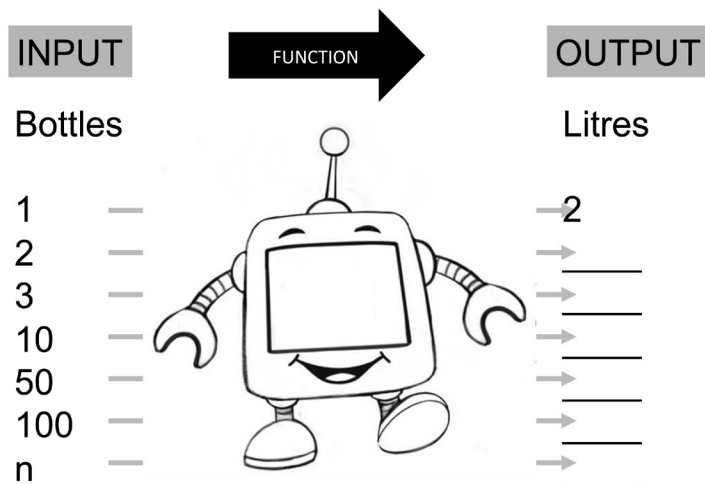


- What is the relationship between bottles and litres? (Generalise)

If I know how many bottles I must _____ to get the number of litres.'

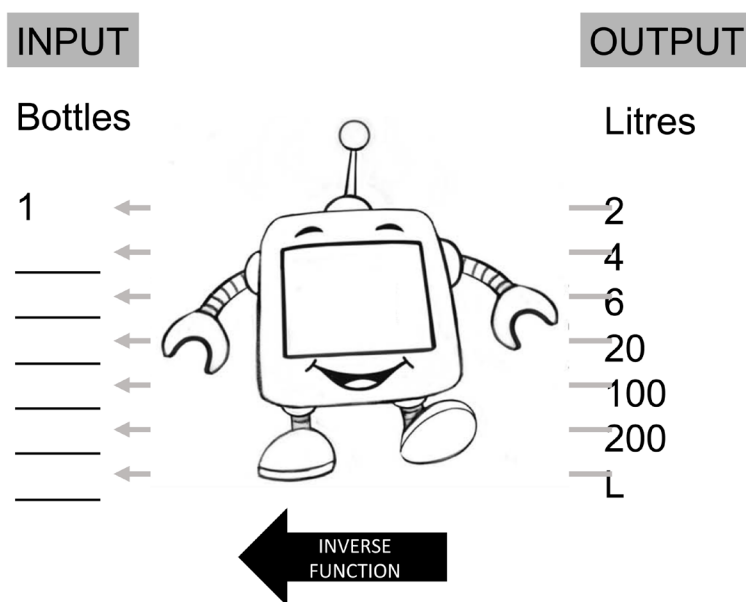
If I know how many litres, I must _____ to get the number of bottles.

5. Organise using a function machine or a flow diagram



6. The number sentence or rule, that relates the number of **litres** to the number of **bottles** is:

7. Complete the inverse function



8. The number sentence or rule, that relates the number of bottles to the number of litres is:

No. of bottles =

Let the no. of litres be L, and the no. of bottles be n.

n =

Task 3.1.4: Two minute tango 6

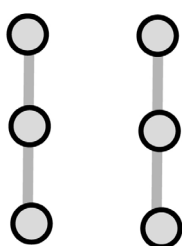
1	5 repeated 10 times = ____	11	72 ; 71 ; 70 ; ____ ; ...
2	$50 \times 10 =$ ____	12	372 ; 362 ; ____ ; 342 ; ...
3	500 multiplied by ____ = 500	12	If I decrease 500 by 1, I get ____
4	Double 18 = ____	14	$1892 + 100 =$ ____
5	Double ____ = 18	15	$(7 \times 10) + (2 \times 10) =$ ____ $\times 10$
6	800 shared equally between 8 = ____	16	$(60 \times 2) + (\text{ } \times 2) = 63 \times 2$
7	$80 \div \text{ } = 8$	17	____ + 2 = 20
8	____ $\div 10 = 80$	18	____ + 20 = 200
9	Half of 36 = ____	19	$41 + \text{ } = 50$
10	Half of 360 = ____	20	____ - 9 = 500

Fluencies

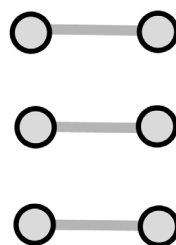
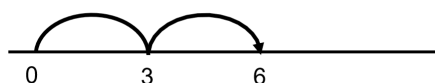
Multiply and divide by 2, by 10; by 100
To multiply a big number, you break it up and multiply each part
To divide a big number, you break it up and divide each part

Multiplication involves choosing a unit and repeating it, to find how many there are.

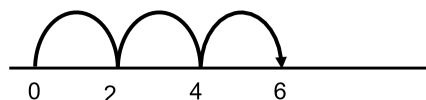
- For double 3, the unit is 3. It is repeated 2 times. The result is 6.
- For triple 2, the unit is 2. It is repeated 3 times. The result is 6.



Double 3
3 repeated 2 times
 $3 \times 2 = 6$
Jump 3. Repeat 2 times



Triple 2
2 repeated 3 times
 $2 \times 3 = 6$
Jump 2. Repeat 3 times



Representations

Multiplication as **repeating equal groups** in a **number picture**.
Multiplication as **repeating equal jumps** on a **number line**



Jumptrak

Two Minute Tango 6

Practice multiplying and dividing by 2, 10 and 100.

SUMMARY



There are 2 pathways to learn about number.

Complete the table.

Quantities	Discrete quantities that can be counted	Continuous quantities that can be measured
Pathway into number sense		
Unit		
Metaphor		
Representations		
Number meaning		

ORDINALITY

Key Ideas

1. Any number always occurs in a sequence of other numbers.
2. Continuous quantities are measured using marks (poles) to show equal units of length (fences) along a line.
3. The ordinal meaning of number describes a position along a line or a position in time.
4. Children move along a learning trajectory from structured number tracks to empty number lines.

Any number always occurs in a sequence of other numbers.

Any number always occurs in a **sequence of other numbers**. Children learn the counting sequence as an ordered list of words. They know the sequence “one-two-three-four-five”. They start to learn that:

- three comes **before** four; and that
- three comes **after** two.

They are able to **order and compare** objects by particular attributes:

- Tall, taller, tallest.
- Slow, slower, slowest.
- Heavy, heavier, heaviest.

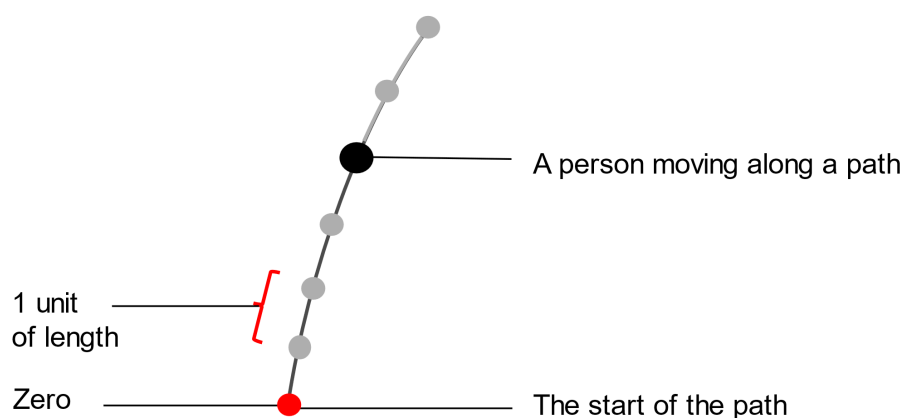
These are all **continuous measurement** contexts.

the measurement can be: mass, length, capacity, volume, time, or temperature.

Children make lists and use the ordinal numbers to plan actions: First, second, third.

This is the **ordinal meaning** of number. A number describes a sequence: 1st, 2nd, 3rd,...

Diagram: A number line showing 4 steps along a path



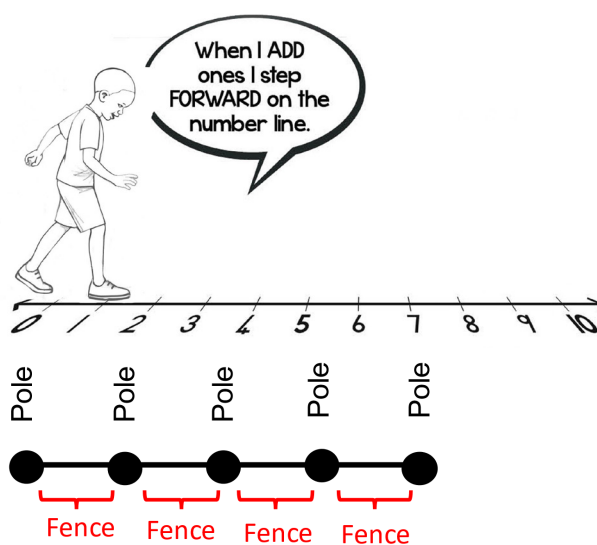
This is the **ordinal use of number**: the 1st step, 2nd step, 3rd step as we move along a path.

Continuous quantities are measured using marks (poles) to show equal units of length (fences) along a line.

The number line is a sequence of numbers placed in order along a horizontal or vertical line. Numbers are positioned or placed relative to other numbers. Calculations are done by **hopping (in ones)** or **jumping (in bigger units)**. Here **movement** - jumping forwards (to the right) or jumping backwards (to the left) is used for calculations.

When children first start using numbers, they often **confuse the cardinal and ordinal meanings**. This is especially the case when they start using number lines. If they only know the cardinal meaning of numbers, then using a number line is difficult. Using the idea of fences and poles can help remove this confusion.

Diagram: Fences and poles



The **poles** are the equally spaced markers on the number line, for each length. The first pole is 0. The second pole is 1.

The **fence** is the 'interval' between the markers on a number line. It is a measure of length. Each section of fence is the same length.

We measure continuous quantities by introducing a measuring unit and repeating that unit. With formal measurements we use an **ordinal scale** (which is a structured number line). **The spatial** idea of numbers being in an order, and arranged from smallest to biggest or numbers as moving along a line is a very important way of thinking about and using numbers.

Let's talk maths

Write down and discuss the isiXhosa words and phrases used to describe an interval (fence) and a marking (pole)?

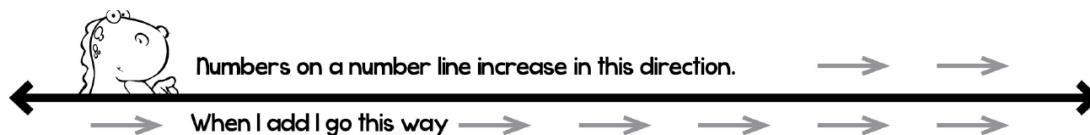
What are the various ways to describe zero? (in English and in isiXhosa). Do any of the words or phrases denote an empty set (absense) or a starting point?

Key The ordinal meaning of number describes a position along a line, or a position in time.

You have learnt that people think about and use numbers to quantify collections of discrete objects and use the idea of a container, to group those objects. This is the **cardinal meaning** of number.

But people also think about and use numbers as being in an order or as a sequence. This is the **ordinal meaning** of number.

The sequence of numbers can be arranged in space. There is a **start**. They are ordered from left to right, or from bottom to top. This is using the **bodily position sense** (to know left versus right and bottom versus top). It is using the **sight sense** and **visual sketchpad** to imagine a **line or path**.

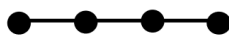


The sequence of numbers can be said as a string of words, or a number song, forwards or backwards. The words are said over time, and so a smaller number is said before a larger number. The metaphor of stepping along a path creates the dynamic idea of movement over time. Children can imagine moving their body through space and walking along a path.

Lightbulb SOME IDEAS FOR YOUR CLASSROOM

Fences and poles

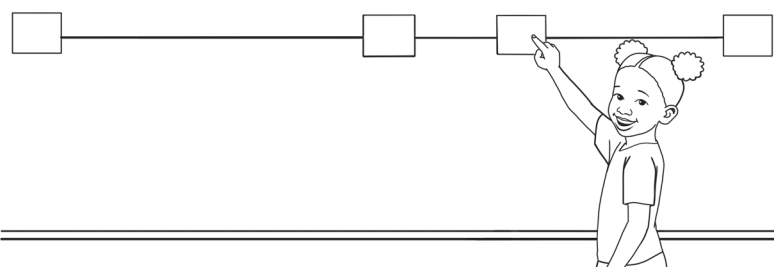
How many?



A teacher draws four dots that are equally spaced along a line. She asks: how many are there? Alan says 3, and Vusi says 4. Discuss.

Is Alan counting fences or poles?

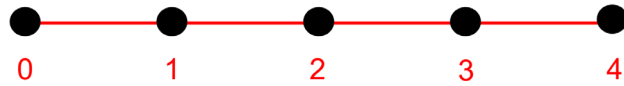
Is Vusi counting fences or poles?



Children should practice placing numbers in order on a number line . 0 on the left. 10 on the right.

Teachers should make children aware that **the number line starts at 0** (at the pole). When we count on a number line we count the **fences**.

We label the fences at the pole to the right of the fence.



The first fence needs both of its two poles to hold it up. If the pole at position 0 is removed, then the fence will fall down.

This number line is structured in ones. There are 3 important features of all structured number lines:

- They all start at 0.
- Each interval (fence) is the same size. In this case each fence is 'one' long)
- Horizontal number lines have the smallest number on the left, and biggest on the right.
- Vertical number lines have the smallest number at the bottom, and biggest number at the top.

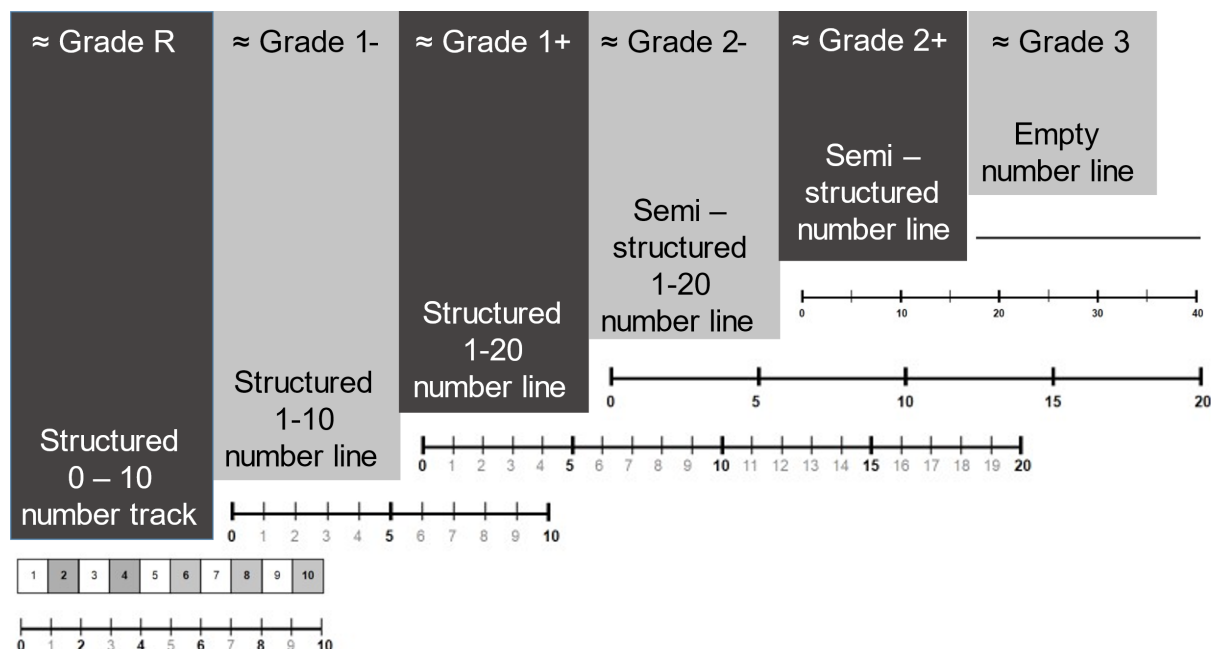
 Children move along a learning trajectory from structured number tracks to empty number lines.

From number tracks to structured number lines to empty number lines

Length is key representation for children. It uses the metaphor of a path. The path could be vertical (from top to bottom) or horizontal (from left to right).

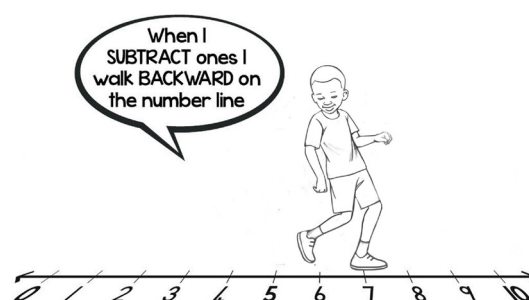
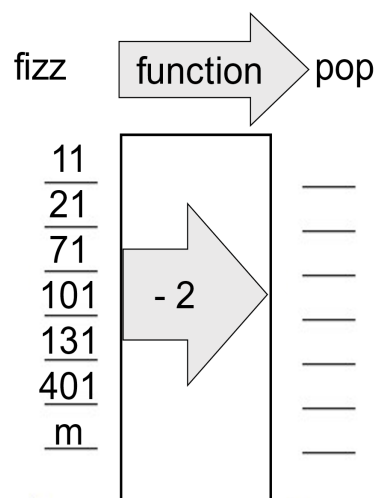
At first children use a structured number track for the whole numbers 0 to 10. Then then use a structured number line from 0 to 10. Over time children get used to working with less structured number lines. They should stop counting in ones. They stop hopping in ones, and make big jumps.

Diagram: A learning trajectory for number lines



 Task 3.2.1

1. Complete a function machine for a fizz pop rule of '2 less than function':



2. The counting unit is 'ones' (small number song).

- a. Draw a number line to represent $7 - 2 = 5$. Start at 7. Circle the 7. Jump back 2.



- b. The measuring unit is 'tens' (medium number song).

Draw a number line to represent: $70 - 20 = 50$. The number line does not start at 0. The last pole on the right is 100.



- c. Draw a number line to represent 2 hundred less than 7 hundred = 5 hundred. The last pole on the right is 800.



The counting unit is _____.

Task 3.2.2 Counting on, and back, in tens (use a number line)

- The medium number song is counting on in tens, starting at any number. Start at 0. Count on in tens.

Count by 10s from 10 to 100.
Colour each block.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Count forwards by 10s.

10	20					
----	----	--	--	--	--	--

40	50					
----	----	--	--	--	--	--

Count backwards by 10s.

100	90					
-----	----	--	--	--	--	--

70	60					
----	----	--	--	--	--	--

- Start at 9. Jump forward in tens. Draw it on the number line.



We can use the medium number sequence for adding.

53 + 30 =

We can add 10s to any number!

53 + 30 = 83

We can add big numbers using an EMPTY number line!

- I draw an empty number line.
- I label the 53.
- There are 3 tens in 30!
- I jump forward by 10 3 times!
- I label each jump!

Notice that you start on the left, and jump forward in 10s to the right.

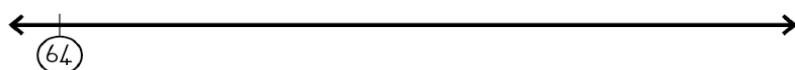
3. Solve by showing on a number line.

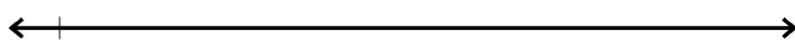
$$20 + 40 =$$

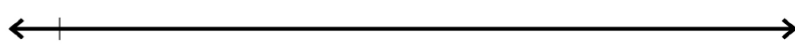

$$50 + 30 =$$


$$70 + 30 =$$


4. Solve by showing on a number line


 $64 + 20 = \underline{\quad}$


 $73 + 20 = \underline{\quad}$


 $57 + 30 = \underline{\quad}$

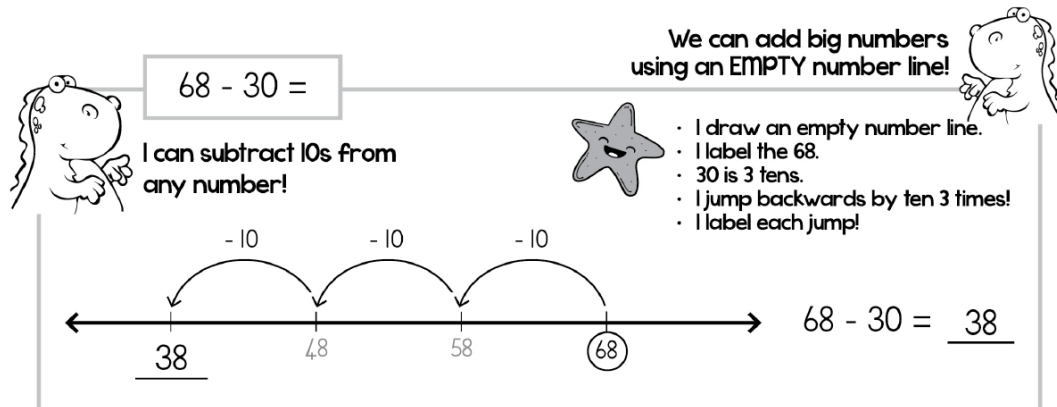
5. Draw number lines. Show how to jump forwards in tens and hop forwards in ones for addition

a. $64 + 75$

b. $73 + 28$

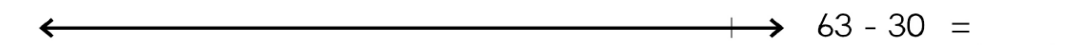
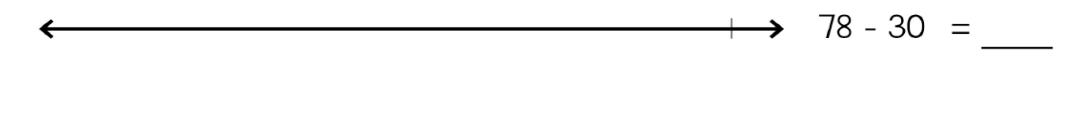
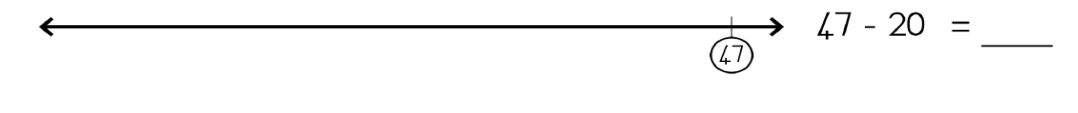
c. $57 + 36$

We can use the medium number sequence for subtracting.



Notice that you start on the right, and jump back in 10s to the left.

6. Solve by showing on a number line



7. Draw number lines. Show how to jump backwards in tens and hop backwards in ones for subtraction

a. 64 - 17

b. 52 - 33

c. 78 - 32

 Task 3.2.3 Show and explain a calculation (with number pictures and number lines)

As a maths teacher you need to be able to calculate and explain you calculation to primary school children.

Look back at task 1.3 on page 15. You had to show and explain how to do the calculation $92 - 58 = \dots$

That was near the beginning of the Emergent Number Sense module. You were using your mental maps that you learnt at school. You did not yet know about the counting pathway and the measurement pathway to learn about number.

1. USING NUMBER PICTURES (from the counting pathway), show how to calculate $92 - 58 = \dots$
2. Now write a paragraph to explain what to do to a child. Use the words, metaphors and mental pictures from the COUNTING PATHWAY.

- 6.** Create your own subtract calculation that is like $92 - 58 = \dots$
Show three different ways to calculate.
You can use number pictures, number lines and number symbols)

 Task 3.2.4: Two minute tango 7 – bring it all together

1	363 ; 362 ; 361 ; 369 ; ____	11	Double 14 = ____
2	372 ; 362 ; ____ ; 342 ; ...	12	Double ____ = 140
3	If I decrease 700 by 10, I get ____	12	73 - ____ = 70
4	3 872 - 100 = ____	14	73 + ____ = 80
5	40 + ____ = 100	15	60 ÷ ____ = 6
6	1000 - ____ = 400	16	____ ÷ 10 = 60
7	____ + 7 = 20	17	600 ÷ ____ = 6
8	____ + 70 = 200	18	____ + 400 = 800
9	377 + ____ = 380	19	(8×10) + (3×10) = ____×10
10	____ is half way between 1 000 and 900	20	double 30 + double ____ = double 37

You should be fluent with all of the following:

Fluencies

Multiply and divided by 2, by 10; by 100
To multiply a big number, you break it up and multiply each part
To divide a big number, you break it up and divide each part

Fluencies

Make connections with bonds of 10

Fluencies

Doubling and halving

Fluencies

Bonds of 5
Bonds of 10

Fluencies

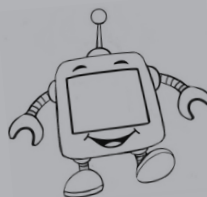
Make connections with bonds of 10

Fluencies

Use bonds of 10.
Add to visit the next ten.
Subtract to visit the previous ten

Representations

Doubling uses a mirror line
Halving is sharing equally between 2.
Doubling and halving are inverse functions



Representations

Multiplication as **repeating equal groups** in a **number picture**.
Multiplication as **repeating equal jumps** on a **number line**

Representations

Number pictures and number lines for the

- small (1s)
 - medium (10s)
 - large (100s)
- number songs

Representations

Number lines to visit the next and previous 10 (or multiple of 10).

Representations

Bonds of 10 are “friends in love”
Ten frames are trays of 10 scones.
Structure by 2s - with 2 in each column or Structure by 5s - with 5 in each row.

Representations

A whole-part-part diagram for a family of number sentences

Representations

If you know one number fact, then use the **small, medium & large number songs**



Jumptrak

Two Minute Tango 7

Bring it all together. Practice all the mental fluencies that you have worked on so far.



What are the key ideas for umthamo 3.2?

Show the go-to strategy for subtraction. Use $63 - 27$ as your example.

.....

UNIT 4:

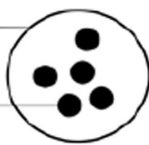
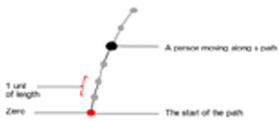
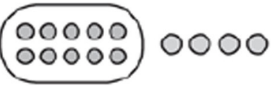
Go-to strategies

Key Ideas

A go-to strategy is an easy strategy that every child can master. It always works, for any numbers. It builds understanding and number sense.

1. The go-to strategies for addition and subtraction for **addition** is: Put the first number in my head. Break down the second number into tens and ones. **Count forwards** in tens. **Count forwards** in ones.
2. The go-to strategy for **subtraction** is: Put the first number in my head. Break down the second number into tens and ones. **Count backwards** in tens. **Count backwards** in ones.
3. The go-to strategy multiplication is: Say which unit that must be repeated. **Count up in that unit**. Keep track of the counts using your fingers. Draw an array or use a t-table.
4. The go-to strategy for division is: Say which unit that must be repeated. **Count up in that unit to reach the target**. Keep track of the counts using your fingers. Draw an array or use a t-table.

There are two pathways into number: The counting pathway and the measurement pathway

Quantities	Discrete quantities that can be counted	Continuous quantities that can be measured
Pathway into number sense	Counting pathway	Measurement pathway
Unit	Counting unit (often in 1s, 2s, 5s or 10s)	Unit of measurement (standard or non-standard)
Metaphor	A container 	
Representations	Number pictures 	Number lines 
Number meaning	Cardinality Number is the size of a set/collection of discrete quantities.	Ordinality Numbers are in a sequence arranged from smallest (first) to biggest (last)

Children use both pathways for adding, subtracting, multiplying and dividing. A lot of the primary school work on measurement is about multiplying and dividing. Children choose a unit and repeat that unit.

UMTHAMO 4.1

GO-TO-STRATEGIES: ADD AND SUBTRACT

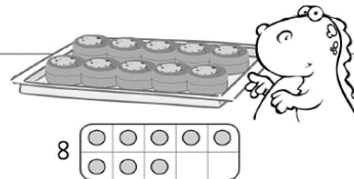
A go-to strategy always works, for any numbers. It works in a similar way for addition and subtraction. We emphasise 1s, 10s, and 100s. You build up the skills for the go-to strategies slowly.

IDEA 1:

Draw numbers from 0 to 10

Children need lots of practice arranging numbers in two rows of 5.

Zibalo loves to bake. She bakes scones to sell. She ALWAYS bakes scones in rows of 5. She never places more than 10 scones on one baking tray.

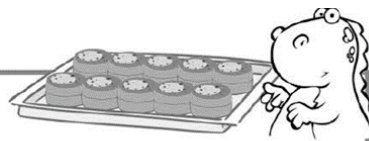


IDEA 2:

Draw numbers from 10 to 20

Children need lots of practice circling each ten. They don't need to draw each scone inside the tray of 10. They just write 10 and circle it.

Zibalo decides to start a scone business. She starts to bake more scones. Her friend Infinity suggests that she writes down how many scones she bakes every day. Zibalo likes to draw the numbers.



Today she baked 16 scones. She drew 16:



Infinity explains to Zibalo that she does not have to draw every scone. She can record ten scones like this:



Zibalo likes this suggestion. She draws 16 like this:

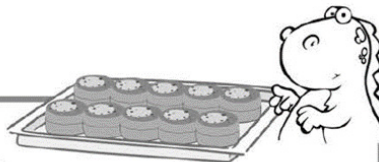


IDEA 3:

Draw numbers to 100

Towards the end of Grade 2 and into Grade 3, children now draw bigger numbers. They should not draw each scone inside the tray of 10. They just write 10 and circle it.

Zibalo's scone business is growing! She continues to draw the number of scones she bakes each day. She thinks about the numbers in trays of 10! Today she baked 37 scones! She drew the number:



IDEA 4:

Add 1s by counting forwards in 1s. Subtract 1s by counting backwards 1s.

Children must know the **small number song** backwards and forwards. They should count backwards and forwards **in 1s**, starting at any number.

$34 + 5 =$

When we add, numbers get bigger!

- I put the biggest number in my head.
- I tap to make sure it is in there.
- I add by counting forwards.
- I use my fingers to keep track.

34, 35, 36, 37, 38, 39

When you add, you count forwards in ones.

$32 - 4 =$

When we subtract, numbers get smaller!

- I put the first number in my head.
- I tap my head to make sure it is in there.
- I count backwards in ones to take away ones.
- I use my fingers to keep track.

32, 31, 30, 29, 28

When you subtract, you count backwards in ones.

IDEA 5:

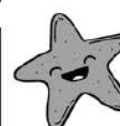
Add and subtract a 10

Children need lots of practice with adding and subtracting a 10 from any number. Play Fizz-Pop: Add 10. Play Fizz-Pop: Subtract 10. Use the 100 chart.

When we add 10 numbers get bigger. We move one row up on this number chart. Can you see that the units stay the same. The tens decrease by one ten. What happens when we go UP one column?



91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10



When we subtract 10 numbers get smaller. We move one row down on this number chart. Can you see that the units stay the same? The tens go decrease by one ten. What happens when we go DOWN a column?

IDEA 6:

Add 10s by counting forwards in 10s

Children must know the **medium number** song backwards and forwards. They should count backwards and forwards **in 10s**, starting at any number. Use the 100 chart.

Zibalo baked 34 scones in the morning. In the afternoon she baked 30 more. How many scones did she bake altogether?

$34 + 30 = 64$

34

+

10

+

10

+

10

+

64

I write the first number.
I put it in my head.

I DRAW the second number.
30 is 3 tens!
I ADD each 10!

IDEA 7:

Subtract 10s by counting backwards

Practice counting backwards in **10s**. Start at any number. 64, 54, 34, Use the **medium number song** to subtract multiples of ten.

Zibalo baked 73 scones. She sold 30 scones. How many scones remain?

$73 - 30 = 43$

73

-

10

-

10

-

10

-

43

I write the first number.
I put it in my head.

I DRAW the second number.
30 is 3 tens!
I TAKE AWAY each 10!

When children can add and subtract 10s fluently, then they can add and subtract 10s and 1s. they first use the medium song to add or subtract the 10s. then they use the small song to add or subtract the 1s.

IDEA 8:

Add by counting forwards in 10s then in 1s

Children must know the **small number song** in 1s forwards.

Children must know the **medium number song** in 10s forwards.

Then they can bring them together.

First they count forwards in 10s, then they count forwards in 1s.

Zibalo baked 35 scones in the morning. In the afternoon she baked 28 more.
How many scones did she bake altogether?

We can add BIG numbers by BREAKING DOWN the SECOND number!
DRAW the second number!

I write the first number.
I put it in my head.

35 + 28 = 63

35 + 10 + 10 + 8 = 63

45 55 63

Zibalo draws the second number.
She adds each ten.
Then she ADDS the ones!

IDEA 9:

Subtract 2 digit numbers: Subtract by counting forwards in 10s then in 1s

Children must know the **small number song** in 1s backwards.

Children must know the **medium number song** in 10s backwards.

Then they can bring them together.

First they count backwards in 10s, then they count backwards in 1s.

Zibalo baked 82 scones yesterday. She sells 23. How many scones remain?

We can subtract BIG numbers by BREAKING DOWN the SECOND number!
DRAW the second number!

I write the first number.
I put it in my head.

82 - 23 = 59

82 - 10 - 10 - 3 = 59

72 62 59

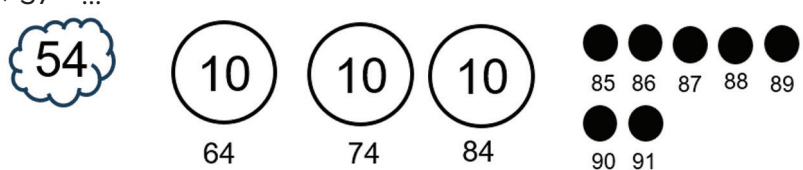
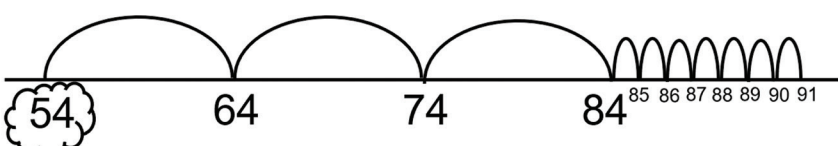
Zibalo draws the second number.
She subtracts tens by counting backwards in 10s.
Then she subtracts the ones by counting backwards in 1s!

Children can represent the go-to strategy in different ways:

Using a number picture, a number line or number sentences.

COUNT FORWARDS FOR ADDING

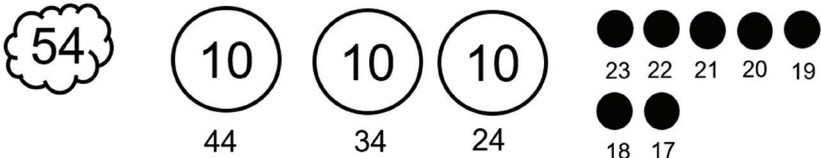
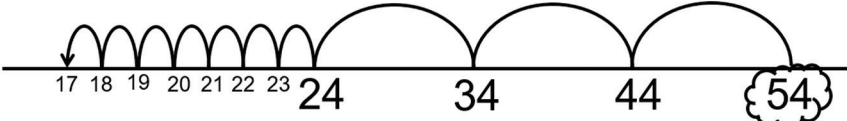
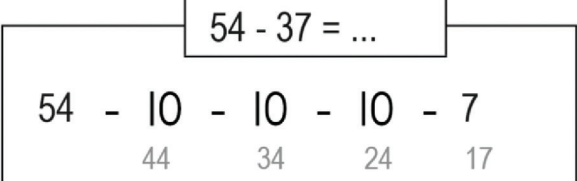
What does the **jump strategy** - where you **break down the second number** - look like in a number picture or a number line?

Number picture	54 + 37 = ...  54 + 37 = 91 Put 54 in my head Break down the second number into 30 and 7 Count forwards in 10s. Count forwards in 1s
Number line	54 + 37 = ...  54 + 37 = 91 Start at 54. Leave space on the right. Break down 37 into 30 and 7. Jump forwards in 10s Jump forwards in 1s
Number sentences	 I add the same way as Zibalo. But I write numbers! 54 + 37 = 91 54 + 10 + 10 + 10 + 7 64 74 84 91 Write down 54. Break down the 37 in 3 tens and 7 ones. Count on in 10s. Count on in 1s.

The **break down the second number strategy** is also called the **jump strategy** (as you hop in ones, and jump in 10s and leap in 100s).

COUNT BACKWARDS FOR SUBTRACTING

What does the **jump strategy** where you **break down the second number** look like in a number picture or a number line?

Number picture	54 - 37 = ...  54 - 37 = 17 Put 54 in my head Break down the second number into 30 and 7 Count backwards in 10s. Count backwards in 1s
Number line	54 - 37 = ...  54 - 37 = 17 Start at 54 (on the right). Leave space on the left. Break down the second number into 30 and 7. Jump backwards in 10s. Jump backwards in 1s.
Number sentence	54 - 37 = ...  I subtract the same way. But I just write numbers!

It takes time for children to start to draw confidently and use number pictures and number lines to show their working.

Game



1,2,3 Show!

SCAN ME

Children practice quick addition with their fingers. You can also do subtraction.

GO-TO-STRATEGIES: MULTIPLY AND DIVIDE

A go-to strategy always works, for any numbers.

It works in a similar way **for multiplication and for division**. It builds understanding and number sense. In Foundation Phase we always using a real context: apples and bags, children and eyes, chairs and legs. We emphasise 2s, 5s, and 10s.

The go-to strategy for multiplying and dividing uses strategic counting. So learners skip count in a unit (for example in 2s, or 5s, or 10s) and keep track of how many counts on their fingers. They can then write down what they are doing using an array or a T-table.

IDEA 10:

Doubling

The first introduction to multiplication is doubling.

To double we start with a number and then repeat it.

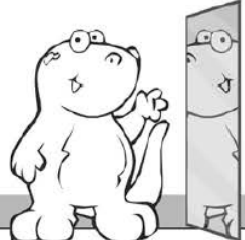
Double 4 = 4 + 4. Children can imagine doubling as using a mirror line.

The draw the number (in 10s and 1s) and then repeat that number).

- Draw a line as a magic mirror.
- Your teacher will call a number (0 to 100)
- Draw the number on the left of the mirror
- Draw the number on the right of the mirror.

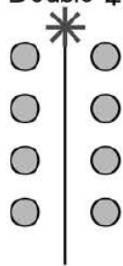


Doubling a number is about repeating a number 2 times!



**Draw a line!
Draw a star * at the top.
Now it is a magic mirror!
Draw double!**

Double 4

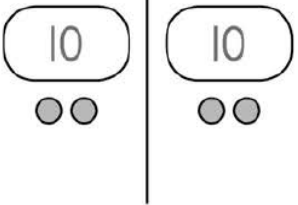


Double 4 is 8

4 + 4 =

There are two 4s in 8.

Double 12

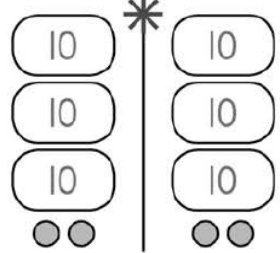


Double 12 is 24

12 + 12 = 24

There are two 12s in 24.

Double 32

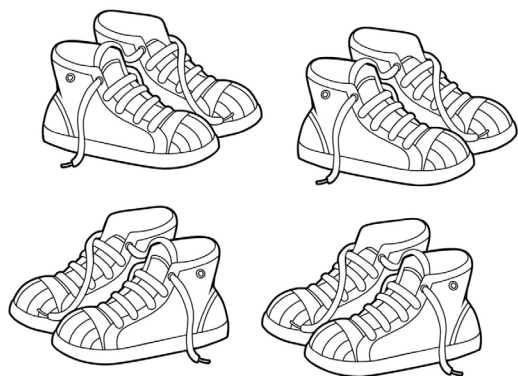


Double 32 is 64

32 + 32 = 64

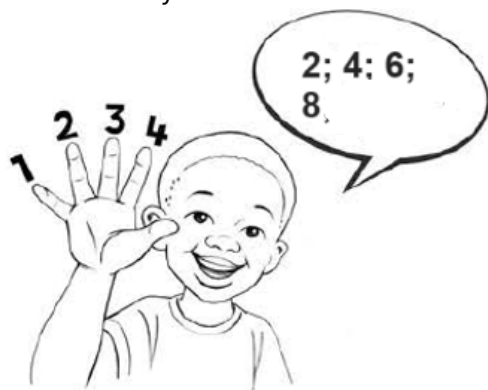
There are two 32s in 64.

You can also double 4 like this: Count forwards in 2s, for 4 counts. Keep track of how many counts on your fingers. This works best with a context. There are 4 children, how many shoes?



Pairs		Shoes
1	● — ●	2
2	● — ●	4
3	● — ●	6
4	● — ●	8

To double 4: Count forwards in 2s, for 4 counts. Keep track on your fingers. What number do you reach?



IDEA

Halving is the inverse of doubling.

To halve 8: Count forwards in 2s, until you reach 8. Keep track on your fingers.

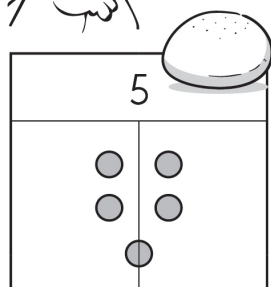
How many counts?

Children first understand half as sharing equally into two groups.

When you halve an odd number you get a half, or one left over.

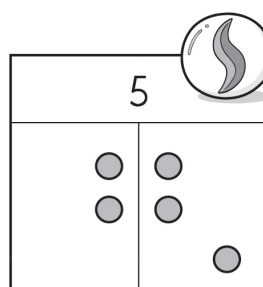


Sharing between two children is the same as dividing by 2!
We say, "10 hlula ka 2 ngu-5." We write $10 \div 2 = 5$. Half of 10 is 5!



Some things can be cut in half. We share by cutting in half!

I share 5 amagwinya equally between 2 children. Each child receives 2 and a half amagwinya.

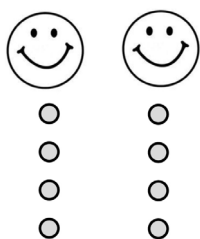


Some things cannot be cut in half. When we share, sometimes we have some left over.

I share 5 marbles equally between 2 children. Each child receives 2 marbles. There is one marble left over.

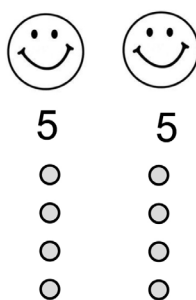
Sharing context for halving: There are ... sweets. We share them equally between two children. How many sweets does each child get?

Share 8 equally between 2 children



Share 8 equally between 2.
Each child gets 4

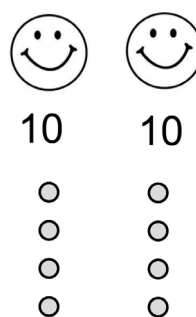
Share 18 equally between 2 children



Share 18 equally between 2.
Break up 18 into 10s and 1s
Share 10 equally.
Each child gets 5
Share 8 equally.
Each child gets 4.

Therefore, each child gets 9

Share 28 equally between 2 children



Share 28 equally between 2.
Break up 28 into 10s and 1s
Share 20. Each child 10
Share 8. Each child gets 4.

Therefore, each child gets 14

You can also half by counting how many twos (or pairs) there are. Count the pairs.

Pairs context for halving: There are 8 shoes. How many pairs?

8 shoes
How many pairs?

Pairs	Shoes
1	2
2	4
3	6
4	8

1 pair is 2 shoes
Count forwards in 2s to reach a target of 8.
Keep track of the counts
4 pairs is 8 shoes.

18 shoes
How many pairs?

Pairs	Shoes
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

1 pair is 2 shoes
Count forwards in 2s to reach a target of 18.
Keep track of the counts.
9 pairs is 18 shoes.

28 shoes
How many pairs?

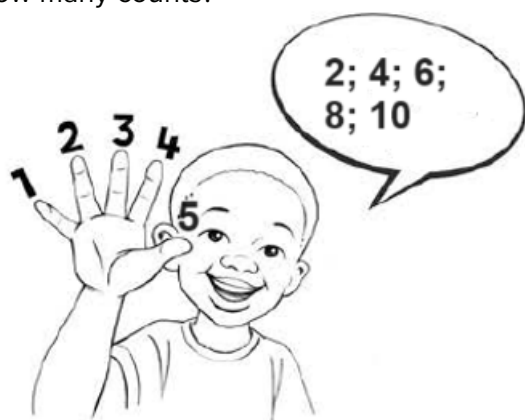
Pairs	Shoes
1	2
2	4
3	6
4	8
10	20
14	28

1 pair is 2 shoes
Count forwards in 2s to reach a target of 28. Draw a T-table. Use known facts...
I know 1 pair is 2 shoes, Therefore I know 10 pairs is 20 shoes
14 pairs is 28 shoes.

There are **two ways to double** and to **two ways to halve**

To double 5: Repeat 5 two times. **OR** Count forwards in 2s, for 5 counts.

To half 10: Share equally into 2 groups. **OR** Count forwards in 2s and reach a target of 10. How many counts?



Strategic counting for multiplying and dividing



IDEA

Multiplication involves choosing a unit and repeating it. So $5 \times 3 = 5$ repeated 3 times.

In Foundation Phase we don't use the $\times$ sign until Grade 3.

We do lots of oral counting (skip counting).

Children should practice counting in 2s, 5s and 10s. When they count, they should keep track of how many counts using their fingers.



Keeping track of the number of counts (using fingers), is key skill for multiplying and dividing.

During mental maths, you can practice the skills for multiplying and dividing by doing strategic counting.

Count forwards in 2s, 5s and 10s. Keep track of the number of counts on your fingers.

Count backwards in 2s, 5s and 10s. Keep track of the number of counts on your fingers.

 Task 4.1: The go-to strategies

 IDEA

The go-to strategies for adding

Put the first number in your head. Break down the second number into tens and ones. Count forwards to add the tens. Then count forwards to add the ones.

Example: $108 + 35$

Show the go-to strategy for addition using a number picture

Show the go-to strategy for addition using a number line

 IDEA

The go-to strategy for subtracting

Put the first number in your head. Break down the second number into tens and ones. Count backwards to subtract the tens. Then count backwards to subtract the ones.

Example: $104 - 38$

Show the go-to strategy for subtraction using a number picture

Show the go-to strategy for subtraction using a number line

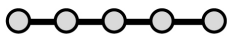
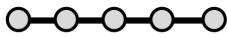

IDEA

The go-to strategy for multiplying

Example: There are 6 hands. How many fingers?

Find the unit that must be repeated. Count up in that unit. Keep track of the counts using your fingers. Draw an array or a t-table.

1. Complete the array

Hands	Fingers
1	 5
2	 _____
3	 _____
4	 _____
5	_____
6	_____

2. Complete the T-table

Hands	Fingers
1	5
2	_____
3	_____
4	_____
5	_____
6	_____

If there are 6 hands, then there are _____ fingers.

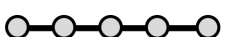

IDEA

The go-to strategy for dividing

Example: There are 30 fingers. How many hands?

Find the unit that must be repeated. Count up in that unit to reach the target. Keep track of the counts using your fingers. Draw an array or use a t-table.

3. Complete the array

Hands	Fingers
1	 5
_____	 10
_____	_____ 15
_____	_____ 20
_____	_____ 25
_____	_____ 30

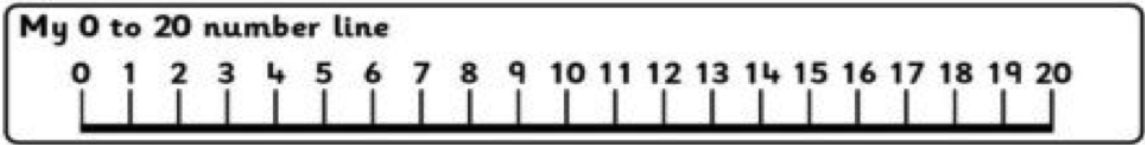
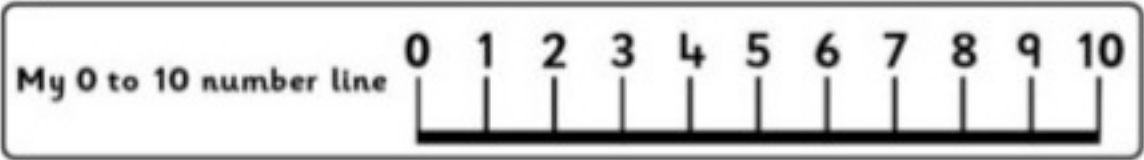
4. Complete the T-table

Hands	Fingers
1	5
_____	10
_____	15
_____	20
_____	25
_____	30

If there are 30 fingers, then there are _____ hands.

Photocopy and use these with your class.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	8	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

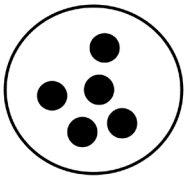


Two pathways into number focuses on how to teach the following **Curriculum and Assessment Policy Standards (CAPS)** topics in mathematics, to South African children, in Foundation Phase classrooms.

1	Numbers, operations and relationships	Number concept: Discrete quantities - Choose the counting unit. - Repeat that counting unit.
1.1	Count objects	Count with whole numbers in 1s, 10s and 100s. Start at any number.
1.2	Count forwards and backwards	
1.3	Number symbols and number names	Represent whole numbers
1.4	Describe compare and order objects	Describe compare and order whole numbers
1.5	Break down into 100s, 10s and 1s	Place value. Large, medium small counting units.
1.6	Solve problems in context	Problem solving
1.7	Addition and subtraction	Put the larger number in your head. Break down the second number into 10s and 1s. Count forwards or count backwards in 10s and 1s
1.8	Repeated addition leading to multiplication	The counting unit is ____. Count forwards in the counting unit. Keep track of the counts.
1.9	Grouping and sharing leading to division	Share equally between ____. The counting unit is ____. Make equal groups ____. The counting unit is ____. Count forwards in the counting unit. Keep track of the counts.
1.10	Sharing leading to fractions	Share marbles. Share amagwinya
1.11	Problem solving techniques	Representations: - concrete apparatus e.g. counters - drawing number pictures - drawing number lines
1.16	Mental mathematics	Number concept, rapidly recall and calculation strategies - Put the larger number first in order to count forwards or count backwards - Building up and breaking down - Use the relationship between addition and subtraction - Use the relationship between multiplication and division.
4	Measurement	Measurement concept: Continuous quantities - Choose the measuring unit - Repeat the measuring unit.
4.1	Time	Time passing: hours, minutes, days, weeks, months, years
4.2	Length	Non standard and standard units: metres, centimetres
4.3	Mass	Non standard and standard units: grams, kilograms
4.4	Capacity/volume	Non standard and standard units: millilitres, litres,

Maths4Primary teachers is a learning series for teachers in training, and to support the ongoing professional development of educators in schools. It is relevant for teachers of both Foundation Phase and Intermediate Phase. Each module includes online materials, games, practice tasks, and summaries.

This **Two pathways to number** module works alongside lesson starters – which guide teachers in how to make use of the mental starters section of their maths lessons.

The counting pathway	The measurement pathway
 5 is a collection of 5 discrete objects,  One way to think about number is using the idea of elements in a container. You arrange discrete objects into a set. You arrange tomatoes on a plate. This is the cardinal meaning of number. We represent cardinality using number pictures.	 5 is a movement of 5 steps/sections along a line.  Another way to think about number is using the idea of movement along a path. You measure equal intervals along a line. You take equal steps along a path. This is the ordinal meaning of number. We represent ordinality using a number line.

Underlying both ways of thinking about number is a structural approach involving patterns, functions and algebra.

This **Two pathways to number** module draws on the expectations outlined in the CAPS. It provides teacher guidance on:

- ▶ **Mental mathematics**, consolidation of concepts and allocation of independent activities for at least 20 minutes per day at the start of the mathematics lesson.
- ▶ **How the teacher works orally and practically** with the learners, engaging in such activities as counting, estimation, number concept development and problem-solving activities, as well as activities concerning pattern and measurement.
- ▶ Examples of calculation strategies and representations available for problem solving activities.
- ▶ Group sessions which are very interactive and where learners are encouraged to “**do, talk, demonstrate and record**” their mathematical thinking.

Officials in the provincial departments and district offices can contact the authors if they would like to offer this training module in their schools.

Maths4Primary teachers: Two pathways to number

Target: Foundation Phase educators in mathematics

Undertaken by



education

Department of
Education
FREE STATE PROVINCE

in collaboration with



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